Scattering Theory - General

(Read Roe, section 1.2)

Definitions of flux, scattering cross section, intensity

Flux $J$ - for plane wave, energy/unit area/sec
  - no. photons/unit area/sec
Scattering Theory - General

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$$J = A^*A = |A|^2$$
where $A$ is the amplitude of the wave
Scattering Theory - General

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For a spherical wave scattered by a point:

$J$ is energy/unit solid angle/sec
Scattering Theory - General

For a spherical wave scattered by a point:

\[ J \] is energy/unit solid angle/sec

\[ J_0 \] is flux incident on point scatterer

Then we are interested in \( J/J_0 \) and

\[ \frac{J}{J_0} = \frac{d\sigma}{d\Omega} = \text{# photons scattered into unit solid angle/sec incident beam flux} \]

\[ = \text{differential scattering cross section} \]

\[ = \text{intensity } I \]
Interference and diffraction

X-rays scattered by electrons

Consider 2 identical points scattering wave $s_o$ in the direction of $s$

Phase difference $\Delta \phi$ betwn scattered waves is

$$\Delta \phi = \frac{2\pi}{\lambda} (s_o \cdot r - s \cdot r)$$

Diffraction vector $S = (s - s_o)/\lambda$
Scattering Theory - General

Interference and diffraction

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Phase difference $\Delta \phi$ between scattered waves is

$$\Delta \phi = \frac{2\pi}{\lambda} \left( s_o \cdot r - s \cdot r \right)$$

$$|S| = \frac{(2 \sin \theta)}{\lambda}$$

Diffraction vector $S = (s - s_o)/\lambda$
Scattering Theory - General

Interference and diffraction

Spherical waves scattered by each pt:

\[ A_1 = A_o \, b \, \exp (2\pi i (\nu t - x/\lambda)) \]
\[ A_2 = A_1 \exp (i\Delta\phi) = A_o \, b \, (\exp (2\pi i (\nu t - x/\lambda))) \exp (-2\pi i S \cdot r) \]

\( b \) = ‘scattering length’
Spherical waves scattered by each point:

\[ A_1 = A_o \cdot b \exp \left(2\pi i \left( \nu t - x / \lambda \right) \right) \]
\[ A_2 = A_1 \exp(i\Delta\phi) = A_o \cdot b \left( \exp \left(2\pi i \left( \nu t - x / \lambda \right) \right) \right) \exp \left(-2\pi i S \cdot r \right) \]

and

\[ A = A_1 + A_2 = A_o \cdot b \left( \exp \left(2\pi i \left( \nu t - x / \lambda \right) \right) \right) \left(1 + \exp \left(-2\pi i S \cdot r \right) \right) \]

\[ b = \text{‘scattering length’} \]
Scattering Theory - General

Interference and diffraction

\[ A = A_1 + A_2 = A_0 \ b \ (\exp (2\pi i(\nu t-x/\lambda))) \ (1 + \exp (-2\pi i \ S \ r)) \]

\[ J = A^*A = A_0^2 b^2 \ (1 + \exp (2\pi i \ S \ r)) \ (1 + \exp (-2\pi i \ S \ r)) \]

So:

\[ A(S) = A_0 \ b \ (1 + \exp (-2\pi i \ S \ r)) \]
Scattering Theory - General

Interference and diffraction

\[ A = A_1 + A_2 = A_o \ b \ (\exp (2\pi i (vt-x/\lambda))) \ (1 + \exp (-2\pi i \ S \ r)) \]

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So:

\[ A(S) = A_o \ b \ (1 + \exp (-2\pi i \ S \ r)) \]

If \( n \) identical scatterers:

\[ A(S) = A_o \ b \ \sum_{j=1}^{n} \exp (-2\pi i \ S \ r_j) \]
Scattering Theory - General

Interference and diffraction

If $n$ identical scatterers:

$$A(S) = A_o b \sum \exp (-2\pi i S \cdot \mathbf{r}_j)$$

For a continuous assembly of identical scatterers:

$$A(S) = A_o b \int n(r) \exp (-2\pi i S \cdot \mathbf{r}_j) \, dr$$

where $n(r) = \#\text{ scatterers in } dr$
Scattering Theory - General

Scattering by One Electron (Thomson)

Electrons scatter x-rays

\[ E_z = E_{oz} \frac{e^2}{mc^2}R \]
\[ E_y = (E_{oy} \frac{e^2}{mc^2}R) \cos 2\theta \]
Scattering Theory - General

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For an unpolarized beam:

\[ E_o^2 = \langle E_{oy}^2 \rangle + \langle E_{oz}^2 \rangle \quad \text{and} \quad \langle E_{oy}^2 \rangle = \langle E_{oz}^2 \rangle = \frac{1}{2} E_o^2 = J_o/2 \]
Scattering Theory - General

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For an unpolarized beam:

\[ E_o^2 = \langle E_{oy}^2 \rangle + \langle E_{oz}^2 \rangle \quad \text{and} \quad \langle E_{oy}^2 \rangle = \langle E_{oz}^2 \rangle = 1/2 \quad E_o^2 = J_0/2 \]

Thus, the scattered flux is:

\[ J_o \left( \frac{e^2}{mc^2R} \right)^2 (1 + \cos^2 2\theta)/2 \]
Scattering Theory - General

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Thus, the scattered flux is:

\[ J_o \left( \frac{e^2}{mc^2}R \right)^2 \frac{(1 + \cos^2 2\theta)/2}{2} \]

polarization factor for unpolarized beam
Scattering Theory - General

Atomic Scattering Factor

\[ f(S) \equiv \text{scattering length} \]

and

\[ f(S) = \int n(r) \exp(-2\pi i S \cdot r) \, dr \]

where \( n(r) \) = average electron density distribution for an atom \( k \)
Scattering Theory - General

Atomic Scattering Factor

If electrons are grouped into atoms:

\[ f(S) \equiv \text{atomic scattering factor} \]

and

\[ f_k(S) = \int n(r) \exp(-2\pi i S \cdot r) \, dr \]

where \( n(r) \) = average electron density distribution for an atom \( k \)
Scattering Theory - General

Scattering Amplitude \( A(S) \)

Finally, for N atoms:

\[
A(S) = A_o b_e \sum f_k(S) \exp (-2\pi i S \cdot r_k))
\]
Scattering Theory - General

Scattering Amplitude $A(S)$

Finally, for $N$ atoms:

$$A(S) = A_o b_e \sum f_k(S) \exp(-2\pi i S \cdot r_k))$$

Or, for a continuous distribution of identical atoms whose centers are represented by $n_{at}(r)$:

$$A(S) = A_o b_e f_k(S) \int n_{at}(r) \exp(-2\pi i S \cdot r_k)) \, dr$$