Illustration 1. Calculate the energy that a human body (75 kg) possesses on top of the Empire State Building (1250 feet tall).

Solution.
Assuming that the above height is given with reference to sea level, the potential energy ($E_p$) that the human body has is

$$E_p = m g h,$$

where $m$ is the mass, $g$ is the acceleration due to gravity (9.8 m/s/s, approximated here as 10 m/s$^2$), and $h$ is the height. Thus,

$$E_p = (75 \text{ kg}) (10 \frac{\text{m}}{\text{s}^2}) (1250 \text{ ft}) (\frac{0.3 \text{ m}}{1 \text{ ft}}) = 281,250 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

The above unit of energy, kg m$^2$/s$^2$, is defined as one joule. Therefore, the answer can be expressed as

$$E_p = 281,250 \text{ J} = 2.8 \times 10^5 \text{ J} = 280 \text{ kJ (approx.)}$$
### TABLE 1
Classification of energy into kinetic and/or potential forms (with examples)

<table>
<thead>
<tr>
<th>Energy Form</th>
<th>Potential</th>
<th>Kinetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational</td>
<td>Yes (by definition)</td>
<td>Not applicable (N/A)</td>
</tr>
<tr>
<td>Mechanical (work)</td>
<td>Body at rest</td>
<td>Body in motion</td>
</tr>
<tr>
<td>Electric</td>
<td>Charged battery</td>
<td>Battery being discharged</td>
</tr>
<tr>
<td>Thermal (heat)</td>
<td>N/A</td>
<td>Yes (by definition)</td>
</tr>
<tr>
<td>Solar (radiation)</td>
<td>N/A</td>
<td>Yes (by definition)</td>
</tr>
<tr>
<td>Nuclear</td>
<td>Yes (by definition)</td>
<td>N/A</td>
</tr>
<tr>
<td>Chemical</td>
<td>Yes (by definition)</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Illustration 2.** Calculate the kinetic energy \( E_k \) of an automobile (3200 pounds) traveling at 60 miles per hour.

**Solution.**
For easy definition of energy units, in this calculation we need to convert miles to meters (1 mile = 1600 m), pounds to kilograms (1 kg = 2.2 lb) and hours to seconds:

\[
E_k = \frac{mv^2}{2} = \frac{1}{2} (3200 \text{ lb}) (60 \frac{\text{mi}}{\text{h}})^2 \left(\frac{1 \text{ kg}}{2.2 \text{ lb}} \right) \left(\frac{1600 \text{ m}}{1 \text{ mi}} \right)^2 \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = \\
= 5.2 \times 10^5 \frac{\text{kg m}^2}{\text{s}^2} = 5.2 \times 10^5 \text{ J} = 520 \text{ kJ} \text{ (approx.)}
\]

Note that this quantity of energy is similar to the energy calculated in Illustration 1.
**FIGURE 1.** The motion of the pendulum is a familiar example of the interconversion between potential energy and kinetic energy.

**Illustration 3.** If the pendulum shown in Figure 1 has a mass of 1 kilogram and its length is 0.5 meters, calculate the speed of the pendulum as it reaches its lowest point (position 2).

**Solution.**

The potential energy of the pendulum is obtained as follows:

\[ E_p = (1 \text{ kg}) (10 \frac{m}{s^2}) (0.5 \text{ m}) = 5 \text{ J} \]

At the lowest point, all potential energy has been converted into kinetic energy. Therefore,

\[ E_p = 5 \text{ J} = E_k = \frac{mv^2}{2}, \quad \text{so} \quad v^2 = \frac{2E_k}{m}. \]

Finally,

\[ v = \left( \frac{2 \left(5 \frac{\text{kg}}{\text{s}^2}\right)}{1 \text{ kg}} \right)^{0.5} = 3.2 \text{ m/s} \]
Illustration 4. Calculate the work done by a person lifting a mass of 100 pounds to a height of 10 feet. If this work is done in a period of 10 seconds, how much power is this person exerting? Does this person's energy increase or decrease in the process?

Solution.

\[
\text{Work} = \text{[Force] x [Distance]} = (100 \text{ lb}) \left(10 \frac{\text{m}}{\text{s}^2}\right) \left(10 \text{ ft}\right) \left(\frac{1 \text{ kg}}{2.2 \text{ lb}}\right) \left(\frac{0.3 \text{ m}}{1 \text{ ft}}\right) = 1.4 \text{ kJ}
\]

\[
\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{1400 \text{ J}}{10 \text{ s}} = 140 \text{ W}
\]

The energy of this person decreases by 1.4 kilojoules. In thermodynamic terms, the energy of the system (the person's body) decreases as a result of work being done by the system.

Illustration 5. Calculate the energy added to 1 cubic foot of water by heating it from 60 to 200 °F.

Solution.

A quantity of water is normally measured in units of volume, such as the cubic foot (ft\(^3\) or cf). To convert volume into mass, we need to know the density. The density of water is 1 gram per cubic centimeter (g/cc). This is equivalent to 62.8 pounds per cubic foot. Therefore,

\[
\text{Mass} = \text{[Density] x [Volume]} = (62.8 \text{ pounds per cubic foot}) \times (1 \text{ cubic foot}) = 62.8 \text{ pounds}
\]

The temperature difference (in °C) is (approximately) 93 – 15 = 78 °C. We can now determine the amount of heat transferred. From Table 2, the heat capacity of water is 1 cal/g/°C. Therefore, we have:

\[
\text{Heat transferred} = (62.8 \text{ lb}) \left(1 \frac{\text{cal}}{\text{g} \text{ °C}}\right) (78 \text{ °C}) \left(\frac{1000 \text{ g}}{2.2 \text{ lb}}\right) = 2.2 \times 10^6 \text{ cal} = 2200 \text{ kcal}
\]
Illustration 6. The weight shown in Figure 9 (assume 1000 kg) falls over a distance of 10 ft. If all its (mechanical) energy is converted to heat, by warming the water (half a gallon) in the vessel, by how much will the temperature of the water increase?

Solution.
The postulated equivalence of heat and work allows us to write the following energy balance:

Heat transferred to the water = Work done by the falling weight

In other words,

\[ m \ C \ \Delta T = m \ g \ h . \]

The work done by the falling weight is (approximately):

\[ m \ g \ h = (1000 \text{ kg}) (10 \text{ m/s}^2) (3 \text{ m}) = 30,000 \text{ J} = 30 \text{ kJ}. \]

Therefore:

Temperature increase = \( \Delta T = \frac{m \ g \ h}{m \ C} = \frac{(30000 \text{ J}) \left( \frac{1 \text{ BTU}}{1055 \text{ J}} \right)}{(0.5 \text{ gal}) \left( \frac{62.8 \text{ lb}}{1 \text{ gal}} \right) \left( \frac{1 \text{ BTU}}{1 \text{ lb} \cdot ^\circ \text{F}} \right)} = 0.9 ^\circ \text{F} \]
ENTROPY: DEVELOPMENT OF A CONCEPT

Carnot developed the first theory of the conversion of heat to work. He got interested in heat engines after realizing that France lost the Napoleonic wars because of British supremacy in power technology. In his posthumously recognized classic, “Reflections on the Motive Power of Fire,” he was guided by the false but insightful analogy between a water wheel and a heat engine. Thinking of heat as a substance that is conserved, like water, he concluded in 1824 that the amount of work produced is dependent only on the temperature difference of the bodies between which the heat flows. Although based on false assumptions, this revolutionary conclusion turned out to be correct. It was reinterpreted and developed further by Kelvin and Clausius, two decades later, after Kelvin learned from Joule about the quantitative equivalence of heat and work. Both as rivals and complementing each other’s findings, Kelvin and Clausius were not the first to realize that heat was not a substance; but they were indeed the first to reconcile Carnot’s conclusion with the energy conservation principle. It took another two decades to find the key to this reconciliation. They analyzed two extremes in the design of heat engines and realized that heat cannot be conserved. At one extreme is the maximum-efficiency Carnot engine in which the amount of work produced is equal to the temperature difference between the hot and cold body (reservoir) divided by the temperature of the high-temperature reservoir. At the other extreme is the zero-efficiency engine which does no work at all, and heat is simply transferred from the hot reservoir to the cold reservoir. In 1865 Clausius introduced the term ‘entropy’ (from the Greek trope, meaning transformation, and by analogy with energy) to measure the energy that is made unavailable (for conversion to work) in the latter case.

In any spontaneous energy conversion, the entropy of the system increases. Regardless of the nature of energy conversion, the entropy of the universe tends toward a maximum.
Another very important practical consequence of the Second Law needs to be introduced here. It is best explained by recalling the probabilistic nature of the concept of entropy and the interpretation of the Second Law in terms of probability. Using intuition, as well as the defining expression for entropy, we can conclude the following:

*Heat is spontaneously transferred from a hot place (“high-temperature reservoir,” in the terminology of thermodynamics) to a cold place (“low-temperature reservoir.”)*

The validity of this statement can be confirmed with the help of elementary probability arguments. These are illustrated in Figures 2, 3 and 4. We can think of the hot place (system) as being in the center of the box. For simplicity let's assume that it consists of 16 ‘hot’ molecules (H1), marked as filled squares in Figures 2 and 3. Two cases will be considered for the cold place (surroundings): the smaller one consists of 48 molecules (C2, Figure 2) while the larger one contains 84 molecules (C3, Figure 3); these ‘cold’ molecules are marked as blank squares. We shall idealize the heat transfer from the system to the surroundings as a simple exchange of hot and cold molecules. Entropy changes during this process are calculated in the tables accompanying Figures 2 and 3. They are shown also in graphical form in Figure 4. The calculation involves use of the Boltzmann equation \( S = k \ln W \), which says that entropy \( S \) is proportional to the natural logarithm of the number of ways a system can be arranged without an external observer being aware that a rearrangement has occurred \( W \). (For simplicity, let's assume that \( k=1 \).) The reason for the logarithmic relationship is straightforward. When two systems are combined, the entropy \( S \) of the combined system is the *sum* of the entropies...
of the two subsystems (S1 and S2); in contrast, the probability for the combined system (W) is the product of the probabilities of the two subsystems (W1 and W2). And from simple math, \( S = S1 + S2 = \ln W1 + \ln W2 = \ln[(W1)(W2)] \).

When all 16 hot molecules are in the system, there is only one way of arranging them, so \( W1 = W2 = 1 \), and \( S1 = S2 = \ln 1 = 0 \). When one molecule is transferred from the system to the surroundings, there are 16 ways to arrange C1 (or H1) in the system, 48 ways to arrange the smaller surroundings and 84 ways to arrange the larger surroundings;

![Molecular arrangements](image)

<table>
<thead>
<tr>
<th>H1</th>
<th>C1</th>
<th>H2</th>
<th>C2</th>
<th>W1</th>
<th>W2</th>
<th>S1</th>
<th>S2</th>
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<td>45</td>
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<td>48x47x46/3!</td>
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<td>9.76</td>
<td>16.1</td>
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<td>44</td>
<td>16x15x14x13/4!</td>
<td>48x47x46x45/4!</td>
<td>7.51</td>
<td>12.2</td>
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<td>(etc.)</td>
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<td>6</td>
<td>38</td>
<td>0.00</td>
<td>28.4</td>
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</tr>
</tbody>
</table>

**FIGURE 2.** As the number of molecules transferred to the 8x8 surroundings increases from 0 to 16, the number of ways that they can be arranged increases. Entropy is proportional to this number.
FIGURE 3. As the number of molecules transferred to the 10x10 surroundings increases from 0 to 16, the number of ways that they can be arranged increases. Entropy is proportional to this number.

Therefore S1=2.77, S2=3.87 and S3=4.43. When two molecules are transferred, there are 120 ways (16x15/2) to arrange the system, 1128 ways (48x47/2) to arrange the smaller surroundings and 3486 ways (84x83/2) to arrange the larger surroundings; therefore S1=4.79, S2=7.03 and S3=8.16. When three molecules are transferred, there are 560 ways to arrange the system, 17,296 ways to arrange the smaller surroundings and 95,284 ways to arrange the larger surroundings; the reader can also verify that S1=6.33, S2=9.76 and S3=11.5.

To convince yourself that this is so, think of the analogous card dealing experiment. Say you want to calculate the probability of being dealt three or four consecutive kings from a deck of 16 cards. You first count the number of ways of getting three of the four
kings in the deck; this is $4 \cdot 3 \cdot 2 / 3! = 4$, where $3!$ is shorthand for $3 \times 2 \times 1$. Indeed, the four ways are either in the first draw, or the second, or the third, or in neither of the three. Obviously, there is only one way to draw four consecutive kings, and that is $4 \cdot 3 \cdot 2 / 4! = 1$. Then you count the number of ways of drawing three or four cards out of 16 cards in the deck; for three cards, this is $16 \cdot 15 \cdot 14 / 3! = 560$, and for four cards it is $16 \cdot 15 \cdot 14 \cdot 13 / 4! = 1,820$. Therefore, the probability of drawing three kings in three draws is greater ($4 / 560 = 1 / 140$) than the probability of drawing four kings in four draws ($1 / 1,820$).

An alternative way to obtain the same probability is the following. The probability that the first card is a king is $4 / 16$ because 4 of the 16 cards in the original deck are kings. After the first king has been dealt, the probability that the second card drawn is a king is $3 / 15$. After the second king has been dealt, the probability that the third card drawn is a king is $2 / 14$. And the probability that the fourth card drawn is a king is $1 / 13$. When these four probabilities are multiplied, we get indeed $(4 / 16) \cdot (3 / 15) \cdot (2 / 14) \cdot (1 / 13) = 1 / 1,820$. So in this case there are $16 \cdot 15 \cdot 14 \cdot 13 / 4!$ equivalent ways of arranging four kings in a deck of 16 cards. The kings in the deck are analogous to hot molecules in Figures 2 and 3.

The probabilistic definition of temperature tells us that it is inversely proportional to the logarithm of the ratio of the number of cold molecules ($N_c$) to that of hot ones ($N_h$),

$$\text{Temperature} = \frac{k}{\ln \frac{N_c}{N_h}}$$

where $k$ is again a constant that need not concern us here. Thermal equilibrium is reached when total entropy, which is the sum of the entropies of the system and the surroundings, reaches its maximum value. In the arrangement shown in Figure 2, this occurs when 12 hot molecules have been transferred from the system to the surroundings (see Figure 4a); therefore, the ratio $N_c/N_h$ is $12/4 = 3$ in the system and $36/12 = 3$ in the surroundings and indeed the temperature is the same everywhere. In the arrangement shown in Figure 3, the same occurs when a larger number of molecules is transferred to the surroundings (see Figure 4b). This makes intuitive sense: more heat must be ‘expended’ to keep a larger room warm.

The transfer of heat from a cold place to a hot place is also possible, of course. In fact, this is what a refrigerator or an air conditioner does. But we need to do work to achieve this.

In summary, then, we have shown here that the heat loss from a warm environment (our home in winter) is the consequence of spontaneous mixing of ‘hot’ and ‘cold’ atoms or molecules. These elementary constituents of matter prefer to be well mixed, and thus warm everywhere, rather than remaining segregated into a hot system (our home) and a cold surroundings.

We conclude this chapter by restating the two laws of thermodynamics in the following familiar terms:
FIGURE 4. An illustration of the Second Law of Thermodynamics. The temperatures of the system and the surroundings become equal when the total entropy reaches a maximum.

*You can't get something for nothing (First Law); the best that you can do is break even, and in conversion from heat to work, you can't even do that (Second Law).*

The first part of this statement is easy to understand. Energy is conserved in all processes, and therefore it cannot be created (nor can it be destroyed). The second part leads us to the concept of efficiency of energy conversion.