§1. Introduction

Brandt et al have carried out detailed experiments on the galvanomagnetic properties of graphite in magnetic field ranged from 200 Oe to 500 kOe. (1) They observed that in fields \( H \geq 200 \text{ kOe} \) \(|\sigma_{xy} H|\) decreases sharply with \( H \). (see Fig.1) In strong magnetic field \( \sigma_{xy} H \) is given by

\[
\sigma_{xy} H = \frac{e}{\hbar}(p - n), \quad (e > 0), \quad (1.1)
\]

where \( p \) and \( n \) denote hole and electron concentrations, respectively. Charge neutrality needs the following condition:

\[
p - n = \sum_{i} \left( \frac{z_i}{e} \right) n_i, \quad (1.2)
\]

where \( z_i \) denotes the valence of the \( i \)-th impurity and \( n_i \) its concentration.

Brandt et al ascribed the decrease of \( |\sigma_{xy} H| \) to the freeze-out of carriers. They conjectured that the similar effect predicted by K\&A theory in semiconductors (2), can occur in graphite. Namely, \( \alpha^* < r_s \) holds, where \( \alpha^* \) denotes the Bohr radius of an impurity electron and \( r_s \) is the screening radius of current carriers. In graphite the condition \( \alpha^* < r_s \) is not satisfied in zero magnetic field, then bound states are not formed, but intense magnetic field changes the condition necessary for occurrence of magnetic freeze-out. As \( H \) increases, the effective Bohr radius \( \alpha^*_H \) decreases and the binding energy \( E_b \) increases, accordingly the bound states, which trap current carriers, are formed.

However, it seems to be doubtful to apply the theory of the freeze-out effect in semiconductors to semimetallic graphite. In this article we present a theory describing an impurity state in graphite in the quantum limit by extending the theories of Koster-Slater, Koster, Wolf, and Clogston. (3-6)

As will be stated later, we can show that feature of the Landau states in graphite make appearance of the bound states possible for \( H > H_c \), where \( H_c \) (\( \approx 70 \text{ kOe} \)) correspond to the quantum limit field strength.

§2. Scattering of Landau Electrons

In a field of an impurity potential \( V(r) \) an incoming state \( \psi_{l} \) is related to an outgoing scattered wave as follows:

\[
\psi = \psi_{l} + (\varepsilon - \lambda + \lambda^2) V \psi, \quad (2.1)
\]

where \( \lambda \) indicates a Landau state and \( \lambda^2 \) is the unperturbed Hamiltonian. (7)

Let us expand \( \psi \) by the Wannier function (3-6),

\[
\psi = N^{-1/2} \sum_{t=1}^{4} \sum_{n} U_{l}(R_n) \psi_{t}(r-R_n), \quad (2.2)
\]

where \( t = 1-4 \) corresponds to the bands \( E_1, E_2, E_3, \) and \( E'_3 \) (8), and \( N \) is a number of the unit cell.

By neglecting \( \gamma_3 \) the Landau state \( \psi_{l} \) is given by

\[
\psi_{l} = \sum_{t=1}^{4} c(\lambda_t) u_{n_{l}}(x-x) e^{i k_{x} x} \phi_{l}(t), \quad (2.3)
\]

where

\[
x = \frac{\varepsilon}{s}, \quad s = \frac{eH}{\hbar},
\]

and \( \phi_{l}(t) \) is the Bloch function at \( k_{x} = k_{y} = 0 \).

By expanding \( V \psi \) into the Landau states and operating

\[
\int d \tau \bar{\psi}_{t}(r-R_n) \psi_{l},
\]

we get...
\[ u_t(k_0) \equiv A(\lambda_t) + N^{-1} \sum_\mu |c(\omega)|^2 (\epsilon - \epsilon_\mu + i \tau)^{-1} \times \sum_s V_{ts} U_s(k_0), \quad (2.4) \]

where

\[
A(\lambda_t) = c(\omega) u_{nt}(x_0 - x)e^{-ik_0 z_0},
\]

\[
\int d\epsilon w_\epsilon(r-k_m) V(r) v_\epsilon(r-k_n) \approx \sum_{s,s} \delta_{mn}.
\]

In deriving (2.4) we assumed that \( V(r) \) is well localized around \( R_s(x_s, y_s, z_s) \). Due to the presence of \( C(\mu) \) it is difficult to solve the simultaneous equation (2.4). To obtain the condition of the bound state formation is equivalent to find in what situation \( U_s(k_0) \) becomes infinite.

3. Bound state in graphite

It is more easier to treat \( U(k_0) = \sum_s U_s(k_0) \) instead of \( U_s(k_0) \). For simplicity we put

\[ \sum_s V_{ts} U_s(k_0) = V_0 \sum_s U_s(k_0) = V_0 U(k_0), \quad (3.1) \]

and assume \( V_0 \) as a constant. Thus, we get

\[ \frac{A_\lambda}{D(\epsilon)} = A_\lambda = \sum_t A(\lambda_t). \quad (3.2) \]

Introducing a quantity

\[ \eta = \frac{c_0 k_z}{2}, \quad c_0: \text{unit cell height}, \quad (3.3) \]

\( \epsilon(\xi) \) is represented by

\[ D(\epsilon) = 1 - V_0 \left\{ \frac{\Omega}{2\pi^2 c_0} \int d\eta \frac{1}{\epsilon - \epsilon_\mu(\eta)} \right\}, \quad (3.4) \]

\( \epsilon_\mu(\eta) \): density of state related to \( \mu \)-th level, \( \Omega \): unit cell volume.

The condition of the bound state formation is given by

\[ \text{KeD}(\epsilon) = 0, \quad g(\epsilon) = \sum_\mu g_\mu(\epsilon) = 0. \quad (3.5) \]

Carriers enter into the quantum limit at \( H = 70 \) kOe. (9, 10) Landau levels at \( H = 70 \) kOe is shown in Fig.2, which indicates that any energy \( \epsilon \) does not satisfy \( g(\epsilon) = 0 \). Since \( \epsilon^*_{\mu}(1) = E_3 \) is a level which is independent of \( H \), with increasing \( H \), \( \epsilon^*_{\mu}(1) \) goes down, and we can find \( \epsilon \) satisfying \( g(\epsilon) = 0 \) between \( \epsilon^*_{\mu}(1) \) and \( \epsilon^*_{\mu}(1) \).

In strong magnetic field the quantity

\[ a = |\text{KeD}(\epsilon) - 1| \]

decreases with \( H \). Then, if

### References

8. Slonczewski J. C., and Weiss P. R., Phys. Rev. 102, 272 (1956).