ANALYTICAL EVALUATION OF POST-EXCAVATION HYDRAULIC CONDUCTIVITY FIELD AROUND A TUNNEL

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ABSTRACT

Changes in the hydraulic conductivity field around a tunnel that accompany underground excavation are difficult to characterize due to the nature of hydromechanical coupling processes in fractured rock masses. A methodology is developed to represent changes in the post-excavation hydraulic conductivity as a result of excavation-induced strains. Relationships linking changes in hydraulic conductivity and excavation-induced strain are defined through the assumption of an equivalent porous medium. Analytical solutions for the post-excavation hydraulic conductivity field are obtained by applying these relations to fractured rock surrounding a circular tunnel. The analytical solutions are then used to develop a dimensionless chart which defines the relationship between normalized ratios of post-excavation to pre-excavation hydraulic conductivities and the dimensionless radium. This chart is verified against field results from Buffer Mass Test at the Stripa.

KEYWORDS

Hydraulic Conductivity • Excavation • Deformation • Analytical Methods • Tunnelling • Fluid Flow

INTRODUCTION

A knowledge of changes in hydraulic conductivity due to the redistribution of stresses or strains, resulting from underground excavation, is crucially important in many engineering fields such as radioactive waste disposal ([1–2]), tunnelling ([3–5]), and methane emission from coal seams ([6–8]). The spread of aqueous and colloidal contaminants as a result of thermoporomechanical coupling in a radioactive waste repository must be controlled and limited. Ground water inflow into a tunnel may create difficult and unsafe tunnelling conditions and a slow rate of advance. The stress state within a coal seam may affect the diffusion and flow of methane, thus influencing the rate of emission of coal-bed methane into both underground mine workings and to the environment as a greenhouse gas.

Statement of the Problem

Increases of hydraulic conductivity in the rock mass adjacent to excavations have been reported by a number of investigators ([4; 3; 1; 9]) although there is little quantitative data. The relationship between
rock mass hydraulic conductivity \( (K_e) \) at a specific effective normal stress \( (\sigma_e) \) and rock mass hydraulic conductivity \( (K_0) \) at zero effective normal stress \( ([9]) \) may be defined as:

\[
\frac{K_e}{K_0} = \frac{1}{[A(\frac{\sigma_e}{\xi})^t + 1]^3}
\]

where \( A, \xi \) and \( t \) are empirical parameters. Changes in rock mass hydraulic conductivity occurring in response to stress relief are predicted by calculating the changes in stress for an assumed circular opening and then relating the stress changes to changes in hydraulic conductivity using equation 1. A number of simplifying assumptions are made: 1. Fracture aperture and spacing are statistically uniform through the rock mass prior to excavation; 2. There is no change in fracture spacing or continuity in response to excavation; 3. Stress changes can be calculated using elastic or elastoplastic, closed-form solutions for circular openings; 4. Hydraulic conductivity is determined by fractures perpendicular to specified stress orientations; and 5. Changes in hydraulic conductivity result from normal displacements across fractures (representing both true normal displacement and dilation during shearing.)

The elastic stress distribution in the radial and tangential directions around a circular opening, and the corresponding hydraulic conductivities, obtained from equation 1 with \( K_e \) established at a stress of 28\( M \) Pa, are illustrated in Figure 1. It is apparent that the maximum increase in hydraulic conductivity is significantly less than two orders of magnitude. However, it is concluded from the Buffer Mass Test at Stripa that the increase in hydraulic conductivity may have been as high as three to four orders of magnitude within a 0.5 – 1.0m wide zone adjacent to the excavation ([1]). A quantitative understanding of this effect is of importance in defining hydrology in repositories for high level radioactive waste products since these zones may serve as hydraulic “superconductors”.

New Approach

A new methodology is proposed in this study to develop the theoretical relationship between the excavation-induced stress and the post-excavation hydraulic conductivity using a strain-based relation for conductivity changes. The constitutive relation is incorporated into solutions for strains induced around a tunnel of circular cross-section. The analytical solution is verified against field measurements from Buffer Mass Test at Stripa.

METHODOLOGY

The potential impact of underground excavation on the hydraulic conductivity field is evaluated through the following straightforward steps: a. The strain field that develops around a tunnel as a result of excavation is analyzed using elastic theory; b. From this predicted strain field, and from knowledge of the pre-excavation hydraulic properties of the overlying strata, changes in hydraulic conductivity that result from the strain field are determined; c. With the modified conductivity field determined, the post-excavation hydrologic system is subsequently defined through application of a ground water model. The post-excavation hydraulic conductivity field is the key to define the underground excavation induced hydromechanical coupling.

Relationship between Strain and Conductivity

Ratios of post-excavation to pre-excavation hydraulic conductivities for the plane strain problem are
defined through the relationships between excavation-induced strain and post-excavation hydraulic conductivity ([10–11]) as

\[ R_x = \frac{K_x}{K_0} = (1 + \beta_x \Delta \varepsilon_y)^3 \]  \hspace{1cm} (2)

\[ R_y = \frac{K_y}{K_0} = (1 + \beta_y \Delta \varepsilon_x)^3 \]  \hspace{1cm} (3)

where

\[ \beta_x = 1 + \frac{1 - R_m}{b_x / s_x} \]  \hspace{1cm} (4)

\[ \beta_y = 1 + \frac{1 - R_m}{b_y / s_y} \]  \hspace{1cm} (5)

and \( b_x/s_x \) and \( b_y/s_y \) may be defined as a function of equivalent fracture porosity or density. \( R_m \) is defined as the ratio of the elastic modulus of the rock mass to that of the intact rock. \( b_x \) and \( s_x \) are fracture aperture and spacing, respectively. \( K_0 \) is pre-excavation hydraulic conductivity. \( K_x \) and \( K_y \) are post-excavation hydraulic conductivities in the x- and y-directions respectively. \( R_x \) and \( R_y \) are ratios of post-excavation to pre-excitation hydraulic conductivities in the x- and y-directions, respectively.

The modulus reduction ratio, \( R_m \), is a measure of scale effect which is defined as the variation of test results with specimen sizes. When the modulus reduction ratio, \( R_m = 1 \), the rock mass and intact rock material moduli are identical and the strain is uniformly distributed between fractures and matrix. This results in the smallest possible change in conductivity. When \( R_m = 0 \), the strain is applied entirely to the fracture system and precipitates the largest possible change in conductivity. These values bound the possible ranges in behavior of the system in a natural, and mechanistically defensible manner. Voight and Pariseau ([12]) surmised that for application to rock masses at field scale, the laboratory values of deformation and rigidity moduli should be reduced by at least an order to magnitude. Berry ([13]) also drew the same conclusion by comparing field data of subsidence magnitudes and elastic calculations for transversely isotropic ground. These two studies suggest that \( R_m \) is less than 0.1. This scale effect of elastic modulus on three types of rock has been statistically studied ([14]). These research results may be used to estimate the magnitudes of \( R_m \) values. Equations 2 through 3 are used to determine the post-excavation hydraulic conductivity field. The post-excavation hydraulic conductivity field is determined from: 1. The excavation-induced strain field \((\Delta \varepsilon_x \text{ and } \Delta \varepsilon_y)\); 2. The pre-excavation hydraulic conductivity \((K_x^0 \text{ and } K_y^0)\); and 3. Fracture parameters \((b_x \text{ and } b_y; s_x \text{ and } s_y; \text{ and } R_m)\). The excavation-induced strain field can be obtained through the analytical solution for a circular tunnel and pre-excavation hydraulic conductivities are typically obtained from \textit{in situ} hydraulic tests. The remaining fracture and material parameters are determined from the pre-excavation hydraulic conductivity field as described in the following section.

\textit{Determination of Fracture Parameters}
The rock mass is assumed to possess \( N \) families of plane, parallel and constant width fractures with different orientations and of very high density, such that the discontinuous media can be substituted by an equivalent continuous one. The intact rock is assumed to be impermeable and Darcy’s law is valid within a single fracture. Under these assumptions, Castillo ([15]) derived the following relationship

\[
K_{\mathbf{n}} = \sum_{i=1}^{N} \lambda_{i} K_{i} \cos \alpha_{i}
\]

(6)

where \( K_{\mathbf{n}} \) is the hydraulic conductivity in the direction of \( \mathbf{n} \), \( N \) is the number of fracture families, \( \lambda_{i} = \frac{b}{s_{i}} \) is fracture density, \( K_{i} \) is hydraulic conductivity of a single fracture in the \( i \)th fracture family, \( \alpha_{i} \) is the angle between the direction of the unit normal, \( \mathbf{n} \) and the direction of strike of the \( i \)th fracture family. If there are three identical sets of orthogonal fractures in a rock mass, substituting \( \lambda_{1} = \lambda_{2} = \lambda_{3} = b/s \) and \( K_{1} = K_{2} = K_{3} = K_{0} \) and \( \cos^{2} \alpha_{1} + \cos^{2} \alpha_{2} + \cos^{2} \alpha_{3} = 1.0 \) into equation 6 yields

\[
K_{\mathbf{n}} = \frac{bK_{0}}{s}.
\]

(7)

Equation 7 demonstrates that the hydraulic conductivity in any direction is constant for a rock mass with three identical sets of orthogonal fractures. In other words, an isotropic continuous medium can be substituted by a discontinuous medium with three identical sets of orthogonal fractures and vice versa. If the \textit{in situ} hydraulic conductivity is defined as \( K_{0} \) (\( K_{x0} = K_{y0} = K_{0} \)), the rock mass can be represented by a discontinuous medium with two identical sets of orthogonal fractures and the fracture aperture is obtained as

\[
b = b_{x0} = b_{y0} = \left[ \frac{12\mu_{k}sK_{0}}{g} \right]^{\frac{1}{3}}
\]

(8)

where \( s \) (assuming \( s_{x} = s_{y} = s \)) is the pre-excavation fracture spacing, \( g \) is gravitational acceleration, and \( \mu_{k} \) is kinematic viscosity of the fluid. Changes in the fracture aperture, \( \Delta b_{x} = \beta_{x}\Delta \varepsilon_{x} \) and \( \Delta b_{y} = \beta_{y}\Delta \varepsilon_{y} \), are determined by excavation-induced strains and the modulus reduction ratio, \( R_{m} \).

The basic equation for the elastic deformation of a two-component medium in the direction perpendicular to the fracture is defined as ([16])

\[
\frac{b + s}{E_{m}} = \frac{sd\sigma}{E_{r}} + \frac{bd\sigma}{\xi E_{r}}
\]

(9)

where \( b \) is aperture, \( s \) is spacing, \( d\sigma \) is the normal stress acting perpendicular to the fracture, and \( \xi \) is the mineral contact area on the fracture wall. For a rock mass containing fractures forming different angles with respect to the horizontal plane, the modulus reduction ratio, \( R_{m} \), is obtained ([16]) as

\[
R_{m} = \frac{1}{1 + \sum_{i=1}^{N} \eta_{i}(1 - \sin^{4} \beta_{i})}
\]

(10)
and

\[ \eta_i = \frac{b}{s\xi_i} \quad (11) \]

where \( i \) is an index of fracture sets, \( \beta_i \) is the angle between the fracture and the area perpendicular to the direction of the desired modulus, \( \xi_i \) is geometric characteristic of the \( i \)th fracture set, and \( N \) is the number of the fracture sets in the rock mass. It is suggested ([16]) that \( \xi_i = 3 \times 10^{-4} \) be applied as a constant. Assuming three sets of orthogonal fractures coincident with the x-, y-, and z-directions, respectively, then \( N \) is equal to 3. Furthermore, assuming \( \eta_1 = \eta_2 = \eta_3 = b/s\xi \) and \( \xi = 3 \times 10^{-4} \), then equation 10 becomes

\[ R_m = \frac{1}{1 + \frac{10^4 b}{3s}} \quad (12) \]

The theoretically derived equation 12 can be used to determine the magnitude of \( R_m \).

**EVALUATION OF POST-EXCAVATION CONDUCTIVITY**

Ratios of post-excavation to pre-excavation hydraulic conductivities can be evaluated based on the methodology, developed previously. The procedures are as follows: 1. The fracture parameters \( (s, b \) and \( R_m) \) for an equivalent fractured medium are determined through equations 8 and 12; 2. The excavation-induced strains \( (\Delta \varepsilon_x \) and \( \Delta \varepsilon_y) \) are derived through the elastic theory; 3. The ratios of post-excavation to pre-excavation hydraulic conductivities \( (R_x \) and \( R_y) \) are determined through equations 2 and 3. The results are reported in the following sections.

**Evaluation of Excavation-induced Strain Field**

As shown in Figure 2, the excavation induced strains in the x- and y-directions can be obtained ([11]) as

\[ \Delta \varepsilon_x = \frac{P(1 + \mu)a^2}{Er^2} (\cos^2 \theta - \sin^2 \theta) \quad (13) \]

\[ \Delta \varepsilon_y = -\frac{P(1 + \mu)a^2}{Er^2} (\cos^2 \theta - \sin^2 \theta) \quad (14) \]

where \( P \) is the uniform far field stress, \( E \) is the elastic modulus, \( \mu \) is the Poission ratio, and \( \Delta \varepsilon_x \) and \( \Delta \varepsilon_y \) are excavation-induced strains in the x- and y-directions, respectively. The remaining parameters, \( a, r, \) and \( \theta \), are tunnel radius, radius of interest and polar angle above the horizontal, respectively, defined as in Figure 2.

**Evaluation of Post-excavation Conductivity Field**

Post-excavation hydraulic conductivities in the \( r \)- and \( \theta \)-directions, \( K_r \) and \( K_\theta \), as illustrated in Figure 2, are defined (assuming \( K_{xy} = 0 \)) as
\[ K_r = K_x \cos^2 \theta + K_y \sin^2 \theta + 2K_{xy} \sin \theta \cos \theta = R_x K_0 \cos^2 \theta + R_y K_0 \sin^2 \theta \]  
(15)

\[ K_\theta = K_x \sin^2 \theta + K_y \cos^2 \theta - 2K_{xy} \sin \theta \cos \theta = R_x K_0 \sin^2 \theta + R_y K_0 \cos^2 \theta \]  
(16)

\[ K_{r\theta} = K_{xy} (\cos^2 \theta - \sin^2 \theta) - (K_x - K_y) \sin \theta \cos \theta = -(K_x - K_y) \sin \theta \cos \theta \]  
(17)

or

\[ R_r = \frac{K_r}{K_0} = R_x \cos^2 \theta + R_y \sin^2 \theta \]  
(18)

\[ R_\theta = \frac{K_\theta}{K_0} = R_x \sin^2 \theta + R_y \cos^2 \theta \]  
(19)

\[ K_{r\theta} = \frac{K_{r\theta}}{K_0} = -(R_x K_0 - R_y K_0) \sin \theta \cos \theta \]  
(20)

where \( R_x \) and \( R_y \) are defined by Equations 2, 3, 13 and 14, \( R_r \) and \( R_\theta \) are ratios of post-excavation to pre-excavation hydraulic conductivities in the \( r \)- and \( \theta \)-directions, respectively. Substituting equations 2 and 3, 13 and 14 into 18 and 19, equations 21 and 22 are obtained as

\[ R_r = \{1 - \left[ \frac{\beta P(1+\mu)\alpha^2}{E r^2} \right] \} \cos^2 \theta + \{1 + \left[ \frac{\beta P(1+\mu)\alpha^2}{E r^2} \right] \} \sin^2 \theta \]  
(21)

Equations 21 and 22 can be used to evaluate the ratios of post-excavation to pre-excavation hydraulic conductivities.

When \( \theta = 0 \), equation 18 may be written as

and if \( \beta P(1-\mu)/E \alpha^2/r^2 < 1.0 \), equation 19 may be written as

where \( R_c \) is defined as the dimensionless conductivity ratio, and \( \beta \) is defined as

\[ R_c \] is plotted as a function of the fracture parameter, \( \beta \), as shown in Figure 3.

**VERIFICATION OF ANALYTICAL SOLUTION**

The Buffer Mass Test was conducted in the Stripa mine over the period 1981–1985. The effect of excavation on the fracture pattern surrounding the test drift is illustrated in Figure 4. The radius of the tunnel is about 2.5m. The average spacing of the two sets of fractures is assumed as 1.0m. The initial stress field is anisotropic with a high horizontal stress component and the conductivity of the virgin rock is about \( 10^{-10} \) m/sec. The excavation of the test drift produced a dramatic increase in axial hydraulic conductivity in a narrow zone adjacent to the periphery of the drift. The conductivity increase is
estimated to be of the order of 1000 times. It is observed that the piezometric head is 40–90m of water at
about a 2m distance from the periphery of the drift and 80–130m at 10m distance. The hydraulic gradient
is approximately 5. It was concluded from the pressure gradient that a “skin” of lower radial permeability
surrounds the drift. This skin effect determines that water flowing towards the drift may be effectively
discharged through the rock in the axial direction of the drift.

Assuming that $K_{r_0} = K_{\theta_0} = 10^{-10}m/sec.$, $E = 2.5 \times 10^{11}Pa$, $P = 15 \times 10^{6}Pa$, $\mu = 0.25$, $s = 1m$, $\beta = \beta_x = \beta_y = 1.8 \times 10^5$, the conductivity ratios of equations 18 and 19 are plotted as shown in Figures 5 and 6,
respectively. As shown in Figure 5, the hydraulic conductivity ratio in the $\theta$-direction may increase by as
much as 3000 times in the surface of the drift ($r = a$ and $\theta = 0$). The depth of influence may extend 4
radii into the drift wall. As shown in Figure 6, the hydraulic conductivity in the radial direction
dramatically decreases within the region $r < 6a$. The “skin” may be as thick as 15 m ($r = 6a$). These
theoretical results are consistent with the experimental results reported for the Buffer Mass Test in Stripa ([1]).

**CONCLUSIONS**

Analytical relationships between underground excavation induced strain and post-excitation hydraulic
conductivity are defined through the assumption of an equivalent porous medium. These relationships
may be used to represent a whole spectra of fractured rock masses by assigning different $R_m$ (modulus
reduction ratio) values. When the modulus reduction ratio, $R_m = 1$, the rock mass and intact rock material
moduli are identical and the strain is uniformly distributed between fractures and matrix. This results in
the smallest possible change in conductivity. When $R_m = 0$, the strain is applied entirely to the fracture
system and precipitates the largest possible change in conductivity. These values bound the possible
ranges in behavior of the system in a natural, and mechanistically defensible manner. The dimensionless
chart, developed for a circular tunnel, defines the relationships between normalized ratios of
post-excitation and pre-excitation hydraulic conductivities with dimensionless radius, and may be
applied in a variety of engineering fields, including geotechnical and petroleum engineering.

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