Topographic Influence of Longwall Mining on Ground-Water Supplies

by D. Elsworth and J. Liu

Abstract
The extent of potential aquifer dewatering resulting from underground longwall mining is determined through application of a nonlinear finite-element model. The model represents the form of the body strain field that accompanies mining-induced subsidence, and uses strain magnitudes to define the modified hydraulic conductivity field. The model is applied to test the sensitivity of the induced strain field to ground surface topography. The location and extent of three characteristic zones of conductivity enhancement are defined, representing gravitational detachment above the panel, shear failure above the abutment, and extensive deformation at the ground surface. Correspondingly, well completion locations are ranked with their potential for dewatering representing relatively high potential in upland areas and relatively low potential in valley base locations. These results amplify and offer a phenomenological explanation of observational data. Modeling results are compared with several documented studies in the Appalachian coal fields and favorable agreement achieved.

Introduction
A number of studies have demonstrated the considerable effects of longwall mining on overlying aquifers by monitoring the change of hydraulic conductivity during and after mining (Bruhn, 1986; Walker, 1988; Booth et al., 1992; Hasenfus et al., 1990). Several observational studies have also shown the extensive impacts of longwall mining on local water supplies (Cifelli and Rauch, 1986; Leavitt and Gibbens, 1992; Donohue, 1993). However, few of these studies document the processes on the basis of physical or phenomenological behavior (Ouyang and Elsworth, 1993). A nonlinear finite-element (FE) model (Elsworth et al., 1994) representing strain softening behavior is used to evaluate the physical relationship between mining and the potential for strata dewatering. The nonlinearity is realized through changing the elastic constants for overburden elements subject to extension and in-panel elements subject to full compression and closure. These behavioral models are based on stress and strain criteria, respectively. Because the processes representing the failure and subsequent deformation of the overburden are accurately represented, the resulting subsidence profiles are relatively insensitive to the material parameters of strength and deformation modulus. This rather surprising conclusion results from the "displacement control" applied to the subsidence process by complete closure between floor and roof of the mined-out panel. For "displacement control" the resulting strain distribution is predominantly moderated by the thickness of the extracted panel, rather than the elastic material parameters. Subsequently, the induced strain field is evaluated without need for complex material parameters, and still represents a reliable depiction of the true situation. Consequently, the mining-modified conductivity field is determined through the unique relationship linking induced strains and hydraulic conductivities. Based on this modified hydraulic conductivity field, the effects of longwall mining on ground-water supplies are evaluated under different mining environments.

Finite-Element Formulation
The resulting finite-element discretization for the nonlinear FE model is written as

\[
\begin{bmatrix}
K_s & 0 \\
0 & K_f
\end{bmatrix}
\begin{bmatrix}
d \\
h
\end{bmatrix}
=
\begin{bmatrix}
f \\
q
\end{bmatrix}
\]

(1)

in which

\[
K_s = \int_V B^T D B \, dv
\]

(2)

\[
K_f = \int_V A^T K A \, dv
\]

(3)

where \( K_s \) and \( K_f \) are stiffness matrices for solid and fluid, respectively; \( d \) is a vector of nodal displacements; \( h \) is a vector of nodal heads; \( f \) is a vector of nodal forces; \( q \) is a vector of nodal fluxes; \( B \) is a geometric matrix linking displacements and strains; \( D \) is the constitutive matrix; \( A \) is a matrix of element shape function derivatives; \( K \) is the element conductance matrix; integration is through the
volume of element, $v$; and superscript $T$ is matrix transpose. The relationship linking induced strain and hydraulic conductivity is defined as (Ouyang and Elsworth, 1993)

$$K = \begin{bmatrix} K_{0x} R_{ex} & 0 \\ 0 & K_{0y} R_{ey} \end{bmatrix} \tag{4}$$

in which

$$R_{ex} = \left[1 + \frac{b_0 + s_0 (1 - R_m)}{b_0} \Delta \varepsilon_x\right]^{3} \tag{5}$$

$$R_{ey} = \left[1 + \frac{b_0 + s_0 (1 - R_m)}{b_0} \Delta \varepsilon_y\right]^{3} \tag{6}$$

where $K_{0x}$ and $K_{0y}$ are the initial conductivities in the horizontal and vertical directions; $s_0$ and $b_0$ are initial fracture spacing and aperture, respectively; $R_m$ is modulus reduction factor; and $\varepsilon_x$ and $\varepsilon_y$ are induced strains in the $x$ (horizontal) and $y$ (vertical) directions. This is an equivalent porous medium representation incorporating orthogonal fracture sets embedded within an impermeable matrix (Ouyang and Elsworth, 1993). Both fractures and matrix exhibit elastic stiffnesses characterized by the modulus reduction ratio, $R_m$, that varies between zero and unity.

Equation (1) is uncoupled between displacements and heads in any direct mode, and each equation may be solved independently. This assumption is reasonable in considering the small influence that poroelastic or effective stress effects have (Bai and Elsworth, 1993; 1994) in comparison to the influence of mining induced deformations incorporated in equations (5) and (6). Coupling is indirect and apparent in one direction only, as conductivities are influenced by the evaluated strain field through equation (4). Equation (1) is actually solved sequentially rather than concurrently. Once the displacement field is determined from equation (1), the spatial distribution of strains may be evaluated and these strains used to define the resulting conductivity field. This model has been carefully tested and calibrated against field results (Liu et al., 1994; Matetic et al., 1994).

**Model Parameters and Geometries**

The topographic influence of longwall mining on ground-water supplies in three topographic settings, namely, subplateau, subhilltop, and subvalley settings is evaluated using three different mining depths for each of the idealized geometries illustrated in Figure 1. The long axis of the panel parallels the strike of the slope, where included. Panel depth is varied between 300 to 900 feet for a panel width of 600 feet and coal seam thickness of 5 feet. Premining hydraulic conductivities, $K_{0x}$ and $K_{0y}$, fracture spacing, $s_0$, topography, mining geometry, and material parameters, are required in the analysis. A homogeneous initial hydraulic conductivity of $K_0 = 0.1 \times 10^{-2}$ ft/sec, is assumed representative of the overburden for all analyses, together with an elastic modulus of $E = 2.0 \times 10^5$ psi and a Poisson ratio of $\mu = 0.28$. A fracture spacing, $s_0$, of 1.0 ft is applied in the analyses, with $b_0$ evaluated from equations (4) through (6) with $\Delta \varepsilon_x = \Delta \varepsilon_y = 0$. Under this constraint, absolute magnitudes of conductivity change will depend on the choice of $s_0$; however, the form of the null contour separating zones of conductivity increase from zones of decrease [$\log(K/K_0) = 0$] is invariant with choice of $s_0$. Consequently, the results are not sensitive to these choices of parameters; rather, they are influenced by the choice of modulus reduction ratio, $R$, applied within the elements subject to extension (Liu, 1993). The magnitude of this important parameter is determined by matching maximum subsidence in the model with that predicted from empirical predictions through the NCB (1966) method. This method avoids the need for complex strength data that are typically unavailable, and provides the most reasonable means of reliably evaluating train distributions.

It is assumed that the variety of settings selected may be treated as symmetric problems with respect to the centerline of the panel. Vertical displacements are restrained along the mesh base, and horizontal displacements are restrained at the sides. Interpenetration of the roof and floor around the mined panel is prohibited through the method developed by Elsworth et al. (1994).

![Fig. 1. Selected two-dimensional geometry to represent (a) subplateau and (b) subhilltop and subvalley settings. Long axis of panel strikes parallel to strike of slope.](image-url)
Fig. 2. Contours of ratio of postmining vertical hydraulic conductivity to premining conductivity \([\log(K_v/K_0)]\). The null influence contour is represented as a log ratio of zero.

**Conceptual Models of Overburden Hydraulic Response**

**Definition of Hydraulic Effect**

The changes in hydraulic conductivities defined by the strain field induced within the overburden are evaluated. Since change in water supply will be most influenced where conductivities are increased, the most obvious index to define zones of potential hydraulic impact is the ratio of postmining to premining hydraulic conductivity \((K_v/K_0)\). These ratios are reported for changes in vertical and horizontal conductivities in the nine distinct mining geometries in Figures 2 and 3, respectively.

Apparent from these figures is that the main influence on the hydraulic conductivity is both close to the panel, and at the ground surface. Three distinct zones develop, namely gravitational detachment above the panel, shear failure in the abutment region and in the near surface zone. The zones of conductivity change are predominantly above panel elevation, but also penetrate to displacements induced below the panel. These subpanel displacements are manifest as floor heave in the panel and as abutment shear failure. Changes in conductivity resulting from elastic (rather than failure per se) displacements diminish rapidly with depth and away from the panel. Based on these parametric results, the conceptualized representations of overburden hydraulic response to longwall mining in these three mining environments are presented in the following. Water wells are classified as three different categories, namely, C1 (little change in phreatic elevations), C2 (anticipated drop in phreatic elevations), and C3 (unchanged or recharged). The bases for these distinctions are two factors, namely, the presence or absence of conductivity enhancement, and the hydrogeologic environment of the zone relative to recharge or discharge. Correspondingly, in upslope recharge locations where conductivities are also increased, it is hypothesized that phreatic elevations will be reduced (C2). In downslope (recharge) or
valley locations where conductivities are increased, flow throughput or phreatic elevations are likely increased (C3). Where only small changes in conductivity result, changes in phreatic elevations may only be induced by changes in flow conditions in the surrounding medium (C1) and changes are likely less severe than in the alternate environments (C2 and C3).

Based on these conceptual models and the critical parameters derived from the FE modeling results, the distribution of these behavioral categories may be determined in response to longwall mining.

Conceptual models representing idealized overburden hydraulic response to longwall mining are illustrated in Figures 4(a) through (c) describing zones of increased conductivity. The parametric results are derived from the FE modeling results listed in Table 1. The enhancement of overburden hydraulic conductivity can be represented by three distinct zones, namely, Zone 1: the suprapanel zone, suffering gravitational detachment; Zone 2: the abutment shear zone; and Zone 3: the surface zone, as illustrated in Figure 4. As identified in Table 1, these zones respond quite differently in the three different mining settings. The conceptual models of the overburden hydraulic response to longwall mining in the three mining environments are presented in detail as follows.

The extent of the zones of potential hydraulic impact within the overburden is controlled both by mining geometry and by topography. In evaluating the true hydraulic effect, ground-water recharge and transmissive characteristics must be considered together to give a definitive evaluation of dewatering potential. However, in the absence of this information, some qualified evaluation of dewatering potential may be estimated. For the same geometric setting, the impacts of longwall mining on water supplies are mainly

Fig. 3. Contours of ratio of postmining horizontal hydraulic conductivity to premining conductivity \[\log(K_z/K_0)\]. The null influence contour is represented as a log ratio of zero.
controlled by topography. The resulting water well classifications in the three mining environments are illustrated in Figures 5(a) through (c) and are described in the following.

**Parametric Results**

**Subplateau Mining Environment**

**Results:** The conceptual model of the overburden hydraulic response to longwall mining in the subplateau mining environment is illustrated in Figure 4(a). The average heights of Zone 1, Zone 2, and Zone 3 of hydraulic conductivity enhancement are 140, 380, and 170 feet or 28t, 73t, and 33t (t is the thickness of coal seam, taken as 5 ft in this example), respectively. The enhancement of the hydraulic conductivity in Zone 1 is in the horizontal direction (as a result of vertical strains) as shown in Figures 3(a) through (c). The enhancement of the hydraulic conductivity in Zone 2 is predominantly in the vertical direction as shown in Figures 2(a) through (c). The enhancement of the hydraulic conductivity in Zone 3 is predominantly in the horizontal direction as shown in Figures 3(a) through (c).

**Interpretation:** The water well classification in the subplateau mining environment is illustrated in Figure 5(a).

### Table 1. Parametric Results of FE Modeling for Models (a) Through (i)

<table>
<thead>
<tr>
<th>Mining Setting</th>
<th>Model</th>
<th>Panel Depth</th>
<th>$h_1$ (R_1)</th>
<th>$h_2$ (R_2)</th>
<th>$h_3$ (R_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sub-plateau</td>
<td>a</td>
<td>500</td>
<td>120(24)</td>
<td>300(60)</td>
<td>200(40)</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>700</td>
<td>150(30)</td>
<td>400(80)</td>
<td>200(40)</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>900</td>
<td>150(30)</td>
<td>400(80)</td>
<td>100(20)</td>
</tr>
<tr>
<td></td>
<td>average</td>
<td>700</td>
<td>140(28)</td>
<td>380(73)</td>
<td>170(33)</td>
</tr>
<tr>
<td>sub-hilltop</td>
<td>d</td>
<td>500</td>
<td>100(20)</td>
<td>300(60)</td>
<td>200(40)</td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>700</td>
<td>120(24)</td>
<td>400(80)</td>
<td>300(60)</td>
</tr>
<tr>
<td></td>
<td>f</td>
<td>900</td>
<td>300(60)</td>
<td>400(80)</td>
<td>250(50)</td>
</tr>
<tr>
<td></td>
<td>average</td>
<td>700</td>
<td>170(35)</td>
<td>380(73)</td>
<td>250(50)</td>
</tr>
<tr>
<td>sub-valley</td>
<td>g</td>
<td>300</td>
<td>150(30)</td>
<td>700(140)</td>
<td>600(120)</td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>500</td>
<td>225(45)</td>
<td>900(180)</td>
<td>200(40)</td>
</tr>
<tr>
<td></td>
<td>i</td>
<td>700</td>
<td>300(60)</td>
<td>1100(220)</td>
<td>300(60)</td>
</tr>
<tr>
<td></td>
<td>average</td>
<td>750</td>
<td>225(45)</td>
<td>900(180)</td>
<td>400(80)</td>
</tr>
</tbody>
</table>

$h_1$: height of zone 1; $R_1$: ratio of $h_1$ to seam thickness
$h_2$: height of zone 2; $R_2$: ratio of $h_2$ to seam thickness
$h_3$: depth of zone 3; $R_3$: ratio of $h_3$ to seam thickness

depth units: feet
Only two types of well, C1 and C2, are likely to develop in this mining environment. The critical parameter for this classification is the elevation separation between mine panel and water well bottom. The critical magnitude is of the order of 500 feet. If the elevation separation is greater than 500 feet, the well is classified as category C1, otherwise C2. In other words, if the well is completed outside the zone 500 feet over the panel, the impact of longwall mining on the well is likely temporary and recoverable. If the well is completed inside the zone 500 feet over the panel, the impact of longwall mining on the well may be permanent and less likely recoverable.

**Subhilltop Mining Environment**

**Results:** The conceptual model of the overburden hydraulic response to longwall mining in the subhilltop mining environment is illustrated in Figure 4(b). The average heights of Zone 1, Zone 2, and Zone 3 of the hydraulic conductivity enhancement are 170, 380, and 250 feet or 55, 73t, and 50t (t is the thickness of coal seam, taken as 5 ft in this example), respectively. The enhancement of the hydraulic conductivity in Zone 1 is both in the vertical and horizontal directions as shown in Figures 2(d) through (f) and Figures 3(d) through (f). The enhancement of the hydraulic conductivity in Zone 2 is predominantly in the vertical direction as shown in Figures 2(d) through (f). The enhancement of the hydraulic conductivity in Zone 3 is predominantly in the horizontal direction as shown in Figures 3(d) through (f).

**Interpretation:** The water well classification in the subhilltop mining environment is illustrated in Figure 5(b). The critical parameter for this classification is the elevation separation between mine panel and water well bottom. The critical magnitude is of the order of 500 feet. If the elevation separation is greater than 500 feet, the well completed within upland areas is classified as category C1, otherwise C2. Wells completed within the valley base are classified as category C3. In other words, if the well is completed outside the zone 500 feet over the panel, the impact of longwall mining on the well is temporary and likely recoverable. If the well is completed inside the zone 500 feet over the panel, the impact of longwall mining on the well may be permanent and less likely recoverable. If the well is completed within the valley base, the well may not be affected or may be quite likely recharged.

**Subvalley Mining Environment**

**Results:** The conceptual model of the overburden hydraulic response to longwall mining in the subvalley mining environment is illustrated in Figure 4(c). In this case Zone 2, the lateral zone of hydraulic conductivity enhancement, spreads directly into the upland area. The average heights of Zone 1, Zone 2, and Zone 3 of hydraulic conductivity enhancement are 300, 900, and 300 feet or 60, 180t, and 60t (t is the thickness of coal seam, taken as 5 ft in this example), respectively. The enhancement of hydraulic conductivity in Zone 1 is both in the horizontal and vertical directions as shown in Figures 2(g) through 2(i) and Figures 3(g) through (i). The enhancement of hydraulic conductivity in Zone 2 is predominantly in the vertical direction as shown in Figures 2(g) through 2(i). The enhancement of the hydraulic conductivity in Zone 3 is predominantly in the horizontal direction as shown in Figures 3(g) through (i).

**Interpretation:** The water well classification in the subvalley mining environment is illustrated in Figure 5(c). The critical parameter for this classification is the distance between drainage base and water well bottom. The critical magnitude is of the order of 300 feet. Wells completed within upland areas are classified as category C2. If the vertical separation is greater than 300 feet, wells completed within the valley base are classified as category C2, otherwise C3. In other words, if the well is completed within the upland area, or within the valley base deeper than 300 feet beneath the drainage base, the impact of longwall mining on the well may be permanent and less likely recoverable. If the well is completed within the valley base shallower than 300 feet beneath the drainage base, the well may not be affected or may be quite likely recharged from the increased through-flow of the near surface aquifer.

**Summary of Results**

Three zones may be defined that exhibit a different hydraulic response to longwall mining. These differences appear controlled by both mining geometry and topography. Under the same geometric mining setting, the impacts of longwall mining on water wells are mainly con-

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Fig. 5. Water well classifications in different mining environments. C1 represents the least influence on the phreatic surface, C2 represents the greatest potential for lowering the phreatic surface, and C3 represents the possibility of a raised phreatic surface.
trolled by topography. By comparison among the three conceptual models and the parametric results derived from the FE modeling, the topographic influence of longwall mining on three different category wells may be summarized as follows.

1. Under the three topographic settings, the conductivity field in the upland areas is modified most while the conductivity field in the valley area is modified least due to longwall mining. This implies that water wells (categories C1 and C2) completed within upland areas should be affected most while water wells (category C3) completed within the valley base should be affected least or unaffected due to longwall mining. Therefore, wells located on hilltops and hillsides seem likely to experience some loss or shift in water supply location downslope.

2. The topographic influence on the mining-modified conductivity field diminishes as the mining depth increases in the subhiltop mining environment. For panel depths of 900 ft, the revised conductivity pattern is nearly identical for subhiltop and subplateau environments. Any measured differences in hydraulic response will result from change in hydraulic baseline conditions, alone. The critical parameter may be the distance between the drainage base and the mine panel. The critical separation is of the order of 500 feet. However, topography still plays a dominant role in the mining-modified conductivity field even when the mining depth is 700 feet beneath the valley base in the subvalley mining environment. The conductivity field in upland areas, instead of around the panel, is most changed in this mining environment.

3. The topographic influence of longwall mining on water well category C1 is temporary and likely recoverable in both subplateau and subhiltop mining environments because these wells are located in the zone which is hydraulically isolated from the panel. The topographic influence of longwall mining on water well category C2 is permanent and less likely recoverable in all three mining environments because these wells are within a zone which becomes hydraulically connected with the panel. The topographic influence of longwall mining on water category C3 is temporary and likely recoverable in all three mining environments. Actually, these wells are quite likely recharged rather than dewatered.

4. The impacts of longwall mining on well categories

<table>
<thead>
<tr>
<th>Study</th>
<th>N</th>
<th>Topographic Setting</th>
<th>h (ft)</th>
<th>Findings and Results</th>
<th>Category (this study)</th>
<th>Correspondence (category with finding)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cifelli and Rauch (1986)</td>
<td>19</td>
<td>hilltop and hillside</td>
<td>&lt;400</td>
<td>complete dewatering 288 ft above mined seam; partial dewatering from 288-360 ft above coal seam</td>
<td>C2</td>
<td>1</td>
</tr>
<tr>
<td>Bruhn (1986)</td>
<td>4</td>
<td>hilltop</td>
<td>350</td>
<td>significant water level declines</td>
<td>C2</td>
<td>1</td>
</tr>
<tr>
<td>Hasenfus et al, (1990)</td>
<td>4</td>
<td>hilltop</td>
<td>&gt;500</td>
<td>lowered and then recovered</td>
<td>C1</td>
<td>Partial</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>hilltop</td>
<td>&lt;100</td>
<td>went dry</td>
<td>C2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>hilltop</td>
<td>400</td>
<td>unaffected</td>
<td>C2</td>
<td>0</td>
</tr>
<tr>
<td>Cifelli and Rauch (1989)</td>
<td>4</td>
<td>hilltop</td>
<td>&gt;500</td>
<td>10 (29%) survive</td>
<td>C1</td>
<td>.71</td>
</tr>
<tr>
<td></td>
<td>74</td>
<td>hillside</td>
<td>&gt;500</td>
<td>49 (66%) survive</td>
<td>C1</td>
<td>.34</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>valley</td>
<td>53</td>
<td>53 (82%) survive</td>
<td>C3</td>
<td>.82</td>
</tr>
<tr>
<td>Donohue (1993)</td>
<td>7</td>
<td>hilltop</td>
<td>&gt;500</td>
<td>2 affected</td>
<td>C1</td>
<td>.71</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>hillside</td>
<td>&gt;500</td>
<td>2 went dry</td>
<td>C1</td>
<td>.33</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>valley</td>
<td></td>
<td>2 affected</td>
<td>C3</td>
<td>.60</td>
</tr>
</tbody>
</table>

N is the number of surveyed or monitored wells.

h is the average elevation separation between mine and well (ft).

Correspondence of modeling category with field findings are rated as:
1 = 100% correspondence; 0 = 0% correspondence.
C1 and C3 are a function of both topography and mining geometry while the impacts of longwall mining on category C2 wells are a function of only mining geometry.

The available observations and analyses reported for the Appalachian coalfields are compared with the FE modeling results on the basis of the well classifications defined above in the following.

**Comparisons with Observational Results**

A number of observational results (Cifelli and Rauch, 1986; Leavitt and Gibbens, 1992; Donohue, 1993) are reported for the Appalachian coalfields. These results are compared with the predicted responses on the basis of the previously defined well classifications through the following straightforward steps: (1) The surveyed or monitored wells are classified in different categories on the basis of the critical parameter magnitudes derived from the FE modeling results. (2) The impacts of longwall mining on the wells are predicted based on these classifications. (3) These predicted impacts are compared with the observational results. These comparisons are listed in Table 2. It is apparent that the predicted impacts are coincident with the observational results. The FE modeling results confirm and extend the mechanisms of dewatering.

An appraisal of the reliability of the FE observations is available where the known outcome of field monitored experiments are compared with the deterministic evaluation of well classification. This is reported in Table 2, sampling types of well classifications C1 through C3. The small number of available case studies, together with uncertainties in characterizing well types for these case studies hinders any definitive conclusions regarding the certainty of predictions as a function of well classification. However, as apparent in Table 2, the positive correlation between prediction and observation averages about 80% across the categories. This correspondence is encouraging and gives some confidence in using these techniques in a predictive capacity. This is especially encouraging where it is realized that all results are obtained for a homogeneous system, where results are independent of material elastic parameters, and only mildly influenced by the hydraulic parameters selected for the system.

**Conclusions**

The nonlinear FE model which enables hydraulic conductivity magnitudes to be defined as unique functions of the strain field is a useful tool to evaluate the topographic effects of longwall mining on ground-water supplies. Model results have confirmed the mechanisms of strata dewatering potential reported for the Appalachian coalfields, and may be extrapolated to other mining environments. The qualitative hydraulic responses of the overburden to longwall mining may be predicted on the basis of the well classifications defined through this study. More definitive predictions of the overburden hydraulic responses to longwall mining may be realized if the water budget is simultaneously evaluated through use of the nonlinear FE model.

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**References**


