5_1 Simple Quantitative Models - of Geothermal Reservoirs

Recap:

Fluid Flow - Defines rates of fluid transmission - controlled by Darcy's Law
Reservoirs may be liquid or vapor or mixed (multiphase) - Relative permeability
Geometry of flow matters (flow nets)

Movies:

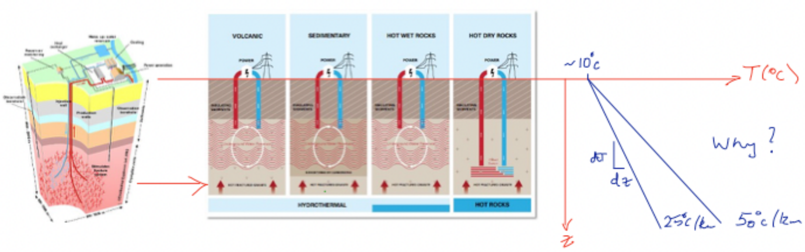
??:

Resources: AG2&3

Mass/Energy Balance: https://www.youtube.com/watch?v=P-wSRZIPJcg&feature=youtu.be

Motivation:

1. **Motivation [10%]** Provide context for the topic. *Use of relevant public domain videos* are a useful method for this. Why is this particular topic or sub-topic important in the broad view of geothermal energy engineering?



What changes in P&T result in a reservoir during production?
How do these changes impact rates and longevity of production of hot fluids?
What simple models describe these systems? Describing output (Power) and duration?
What are intrinsic differences between Lumped (box) and Distributed Parameter models?

Scientific Questions:

2. **Scientific Questions to be Answered/Outline [10%]** What questions arise from the motivation. What are the sub-topical areas that address these scientific questions.

Define rates of fluid flow and energy production with time - thus productivity and longevity

1. Overall behavior of geothermal reservoirs - under production - Mechanisms of depletion
2. Lumped parameter models - pressure and heat recovery
3. Distributed parameter models - radial flow
4. Thermal breakthrough
5. Limits of heat rate recovery

1. Overall behavior of geothermal reservoirs - under production - Mechanisms of depletion

General form of reservoir:

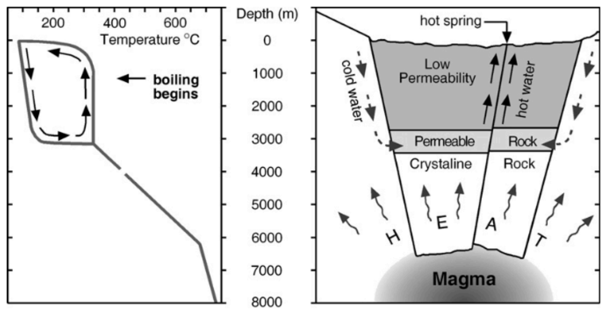


FIGURE 2.2 Model of the large-scale circulation of fluid in the natural state of a geothermal system. *Source: White, D.E., 1967 "Some principles of geyser activity, mainly from Steamboat Springs, Nevada" Am. J. Sci. 265, pp641–684.*

Pressure versus Depth

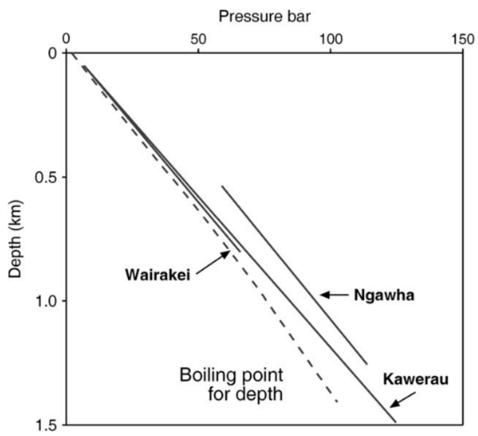


FIGURE 2.6 Pressure distribution with depth in three New Zealand geothermal fields. *Source: Grant 1981.*

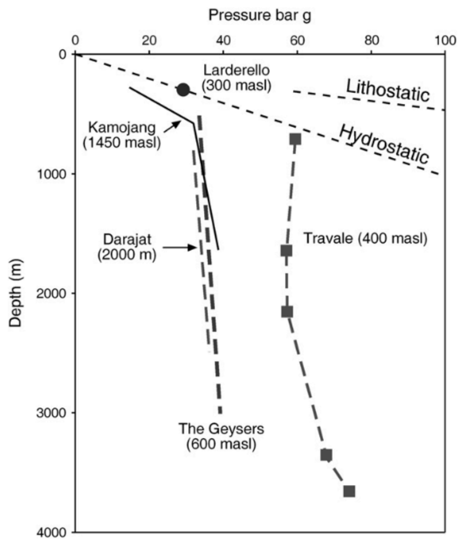
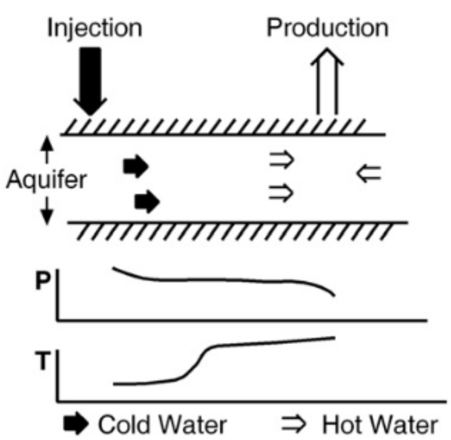


FIGURE 2.7 Reservoir pressure in vapor-dominated systems (Travale is part of greater Larderello). *Source: Allis, 2000.*

Impact of Depletion and Reinjection



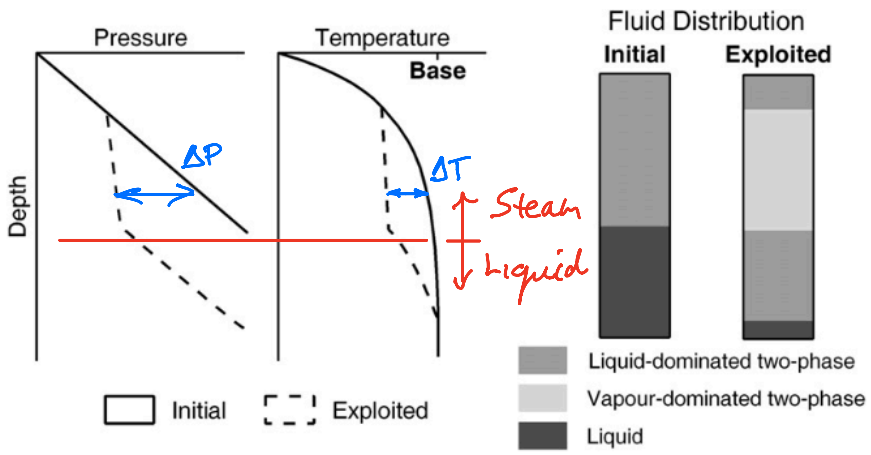


FIGURE 2.9 Fluid distribution in natural and exploited states of a liquid-dominated reservoir.

Relative importance of Δp and ΔT.

$$h = u + p/\rho$$

$$u \rightarrow \cancel{\rho} c T / \cancel{\rho}$$

Say $\Delta T = 100^\circ C$

$\Delta p = 10\ MPa \rightarrow 0$

$$u = c\Delta T = 4.1 \times 10^3\ J/kgK \times 10^2\ K$$

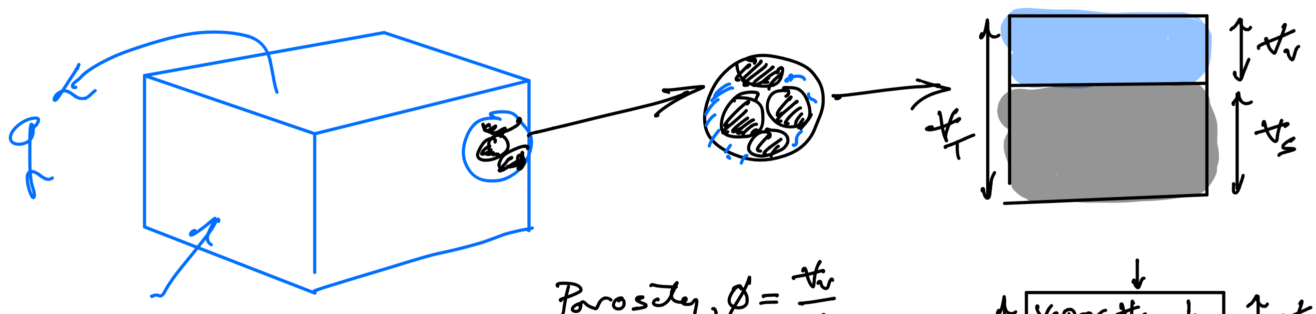
$$= 4 \times 10^5\ J/kg$$

$$\Delta p/\rho = 10^7\ \frac{N}{m^2} \times \frac{1\ m^3}{10^3\ kg} = 10^4\ J/kg.$$

Relative importance of P & T

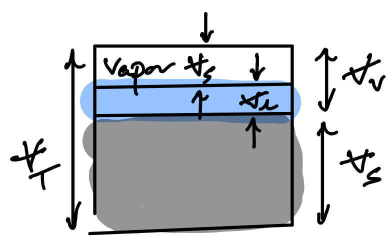
1. Heat is regenerable by resupply.
2. Pressure ~ $\frac{1}{40}^{th}$ of thermal energy

SIMPLE BOX MODELS



Porosity, $\phi = \frac{V_v}{V_T}$

Saturation, $S = \frac{V_l}{V_v}$



q = volumetric production rate m^3/s

W = mass rate = $q\rho$ = kg/s

MASS BALANCE : $\forall \frac{d}{dt}(\phi\rho) = -W$

HEAT BALANCE : $\forall \frac{d}{dt}[(1-\phi)\rho_m U_m + \phi\rho U] = -WH$

Assuming <u>isothermal</u> depletion then

$$\forall [\rho \frac{\partial \phi}{\partial t} + \phi \frac{\partial \rho}{\partial t}] = -W$$

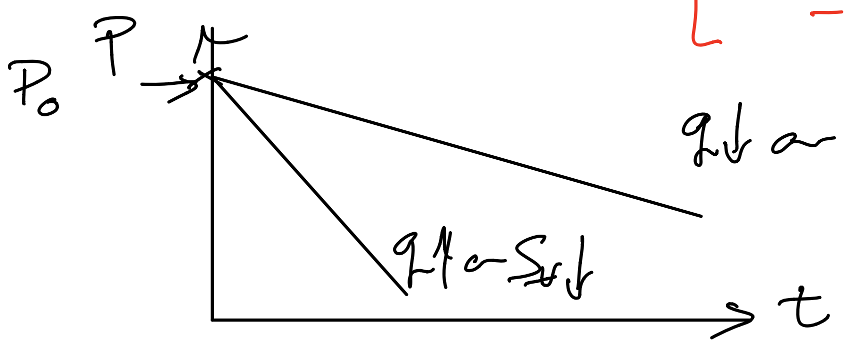
$$\forall [\rho \frac{\partial \phi}{\partial P}\frac{\partial P}{\partial t} + \phi \frac{\partial \rho}{\partial P}\frac{\partial P}{\partial t}] = -W$$

$$\forall \rho [\frac{\partial \phi}{\partial p} + \phi \frac{1}{\rho}\frac{\partial \rho}{\partial p}] \frac{\partial P}{\partial t} = -W$$

$$\frac{\partial \phi}{\partial P} = \frac{\partial \forall_v}{\partial p}\frac{1}{\forall_T} = C_m \qquad \phi . c$$

$$\rho \underline{\forall [C_m + \phi C]} \frac{\partial P}{\partial t} = -q\rho = -W$$

$$S_\forall = \forall [C_m + \phi C] \left\{ \begin{array}{l} C_m \sim 1/10\,GPa \\ c - \text{water} \sim 1/2\,GPa \\ \quad - \text{steam} \sim 1/\text{pressure} \end{array} \right.$$



NOTE: If steam reservoir then $C_{steam} >> C_m$ or C_{liquid}

then use ideal gas law

$$S_v = V\phi c = V\phi/p = \frac{V\phi}{\rho R T}$$

Thus - pressure drop modulated as:

$$\frac{dP}{dt} = -\frac{q}{S_V} = -\frac{M}{S_M}$$

$$S_V = V[c_m + \phi c]$$

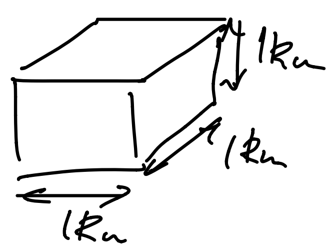
$$S_M = \rho S_V = \rho V[c_m + \phi c]$$

Definition of S_V

$$S_V = q\frac{dt}{dp} = \frac{V_{fluid}}{dp}$$

$$S_M = \frac{\text{Mass of fluid}}{dp.}$$

$dp = 10\,MPa$ (1000 m of water)



$$M_f = \rho V c\, dp = \underset{kg/m^3}{1000} \cdot \underset{m^3}{\cancel{10^9}} \cdot \frac{1}{\cancel{10^9}} \cdot 10^7$$

$$= \underline{\underline{10^{10}\ kg}}$$

Total mass of 1km cube of water is $10^9 \times 1000$ kg/m

$\Rightarrow 10^{12}$ kg $\therefore \frac{1}{100}$th.

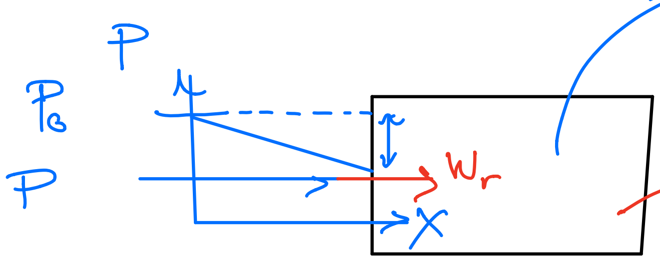
Convert pore space — water $\rightarrow$ steam.

then Mass water $= 10^{12} \times \cancel{\phi}$ 20%

$\sim 10^{11}$ kg.

Compressibility provides small release from storage!!

TRANSIENT RESPONSE



q (m^3/s)

W (kg/s) $= q\rho$

$\sim \Delta p$ in reservoir of V

Reservoir — Δp due to withdrawal

Mass change is $W = S_M \Delta p$ in dt (1)

Balance recharge with withdrawal: $(W - W_r)\Delta t$ (2)

Conservation of mass for (1) and (2)

$$S_M \frac{dP}{dt} + (W - W_r) = 0$$

Assume steady state when $W = W_r \rightarrow \frac{dP}{dt} = 0$

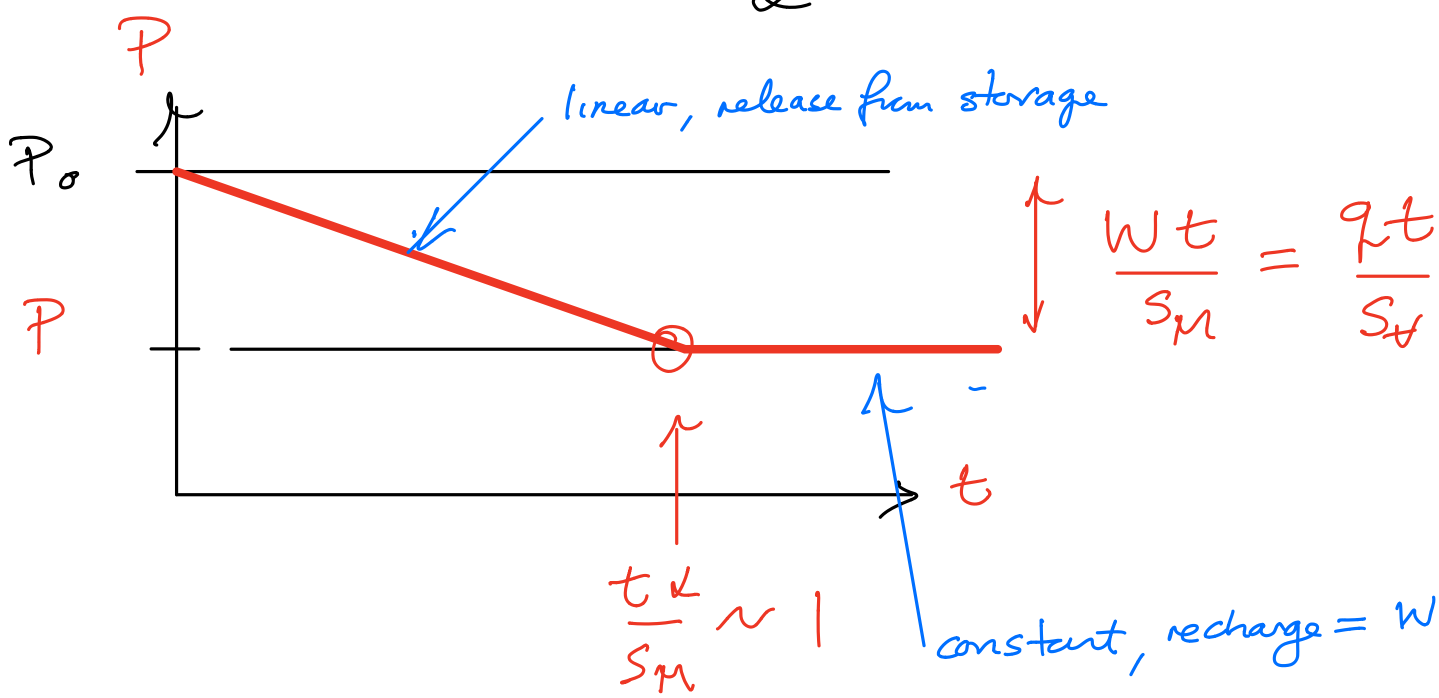
Set $W_r = \alpha (P_0 - P)$

Thus:

$$W = \alpha (P_0 - P) - S_M \frac{dP}{dt}$$

Solve:

$$P_0 - P = \frac{W}{\alpha}\left(1 - e^{-t\alpha/S_M}\right)$$



Energy recovery, $E = W c \Delta T$

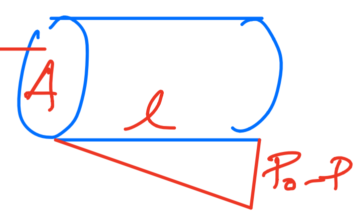
Parameterization

S_M — straightforward — $n, \rho, \forall, c$.

α — steady state — $\alpha = W/(P_0 - P)$

$$\alpha = \frac{W}{(P_0 - P)} = \frac{q\rho}{(P_0 - P)} = \frac{\rho \cdot A k/\mu \cancel{(dP)}}{l \cdot \cancel{(P_0 - P)}}$$

q



3. Distributed parameter models - radial flow

3.3. PRESSURE TRANSIENT MODELS

The simplest model is a vertical well, circular in cross section, that fully penetrates a uniform horizontal aquifer of infinite radial extent that is sealed above and below, as shown in Figure 3.1. There is no spatial variation of rock properties (especially permeability). The only spatial variations of pressure (and temperature and saturation, if relevant) that need to be considered are those pertaining to radial distance from the well. The fluid in the aquifer is in vertical equilibrium with depth at all times, so there are no effects due to gravity.

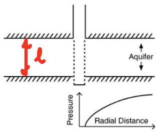


FIGURE 3.1 Basic well model for pressure transients.

3.3.1. Single-Phase Aquifer Fluid

Darcy's law in the radial (axial) form is:

$$v_r = -\frac{k}{\mu}\frac{\partial P}{\partial r} \tag{3.21}$$

In similar form, the conservation of mass equation is:

$$\varphi\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) = 0 \tag{3.22}$$

or

$$\varphi c p\frac{\partial P}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\rho\frac{k}{\mu}r\frac{\partial P}{\partial r}\right) \tag{3.23}$$

It is assumed that the compressibility is constant and that changes in the viscosity may be ignored in comparison with changes in pressure. This gives:

$$\frac{\varphi\mu c}{k}\frac{\partial P}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) \tag{3.24}$$

This is the diffusivity equation, with the hydraulic diffusivity:

$$\kappa = \frac{k}{\varphi\mu c} \tag{3.25}$$

Many solutions of this equation are available from the literature on heat conduction, which obeys the same equation.

If a well begins withdrawal at time $t = 0$, at a constant rate q or $W = \rho q$, the pressure in the aquifer is given as a function of radial distance and time by:

$$\Delta P = P - P_o = -\frac{q\mu}{4\pi kh}E_1\left(\frac{\varphi\mu c r^2}{4kt}\right) = -\frac{W\nu}{4\pi kh}E_1\left(\frac{\varphi\mu c r^2}{4kt}\right) \tag{3.26}$$

where

$$E_1(x) = \int_x^\infty \frac{1}{y}e^{-y}dy \tag{3.27}$$

$E_1(x)$ is tabulated by Abramowitz and Stegun (1965). (The function $E_1(x)$ is denoted by $-Ei(-x)$ in the petroleum literature.) For small values of the argument x, or long time:

$$E_1(x) \sim -\ln(x) - \gamma \tag{3.28}$$

where $\gamma = 0.5772$ is Euler's constant. Using this asymptotic for Eq. (3.26) gives:

$$P - P_o = -\frac{W\nu}{4\pi kh}\left\{\ln(t) + \ln\left(\frac{4k}{\varphi\mu c r^2}\right) - \gamma\right\} \tag{3.29}$$

The pressure at any point changes linearly with the logarithm of the time. In Eq. (3.26) the parameters of the aquifer and fluid enter in two groups:

$$\frac{\mu}{4\pi kh}$$

and

$$\frac{\varphi\mu c}{k} = \frac{\varphi ch}{(kh/\mu)} \tag{3.30}$$

By suitable observation of the pressure change, it may be possible to fit an observed history to theory and identify the two parameter groups kh/μ, the transmissivity or mobility-thickness, and φch, the storativity. (k/μ is called the mobility.) If the fluid viscosity μ is known, the permeability-thickness kh can be identified. Thus, in principle, two parameter groups are identifiable. One, the storativity, measures the aquifer's capacity to store fluid, and the other, the transmissivity, measures its ability to transmit fluid.

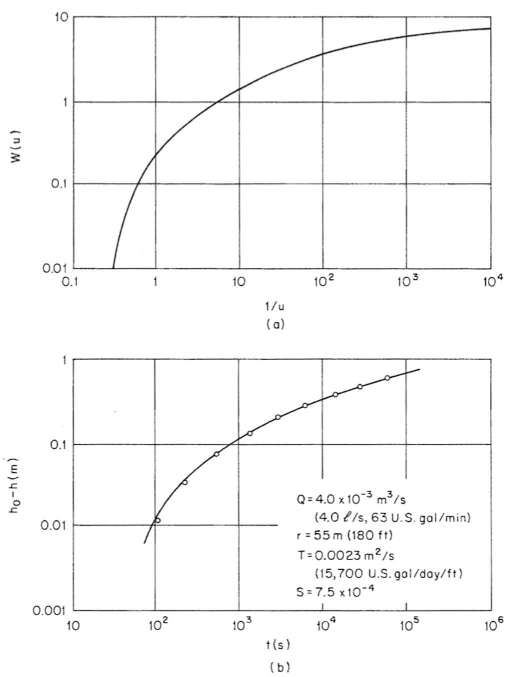
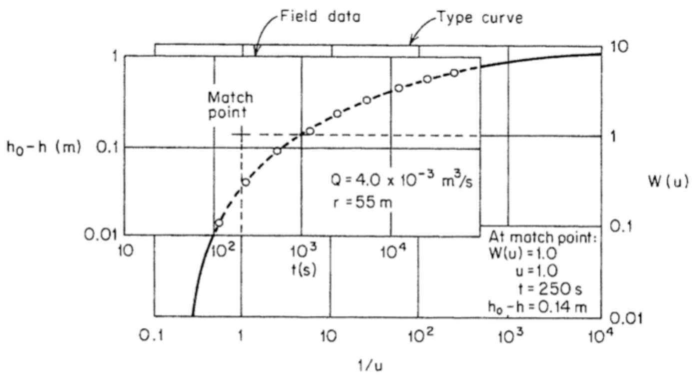


Figure 8.5 (a) Theoretical curve of $W(u)$ versus $1/u$. (b) Calculated curve of $h_0 - h$ versus t.



$$T = \frac{QW(u)}{4\pi(h_0-h)} = \frac{(4.0\times10^{-3})(1.0)}{(4.0)(3.14)(0.14)} = 0.0023\ \text{m}^2/\text{s}\ \ (15{,}700\ \text{U.S. gal/day/ft})$$

$$S = \frac{4uTt}{r^2} = \frac{(4.0)(1.0)(0.0023)(250)}{(55.0)^2} = 7.5\times10^{-4}$$

Figure 8.22 Determination of T and S from $h_0 - h$ versus t data using the log-log curve-matching procedure and the $W(u)$ versus $1/u$-type curve.

Conversions: $T = Kl = \frac{k}{\mu}.\rho g.l$

$(h_o - h) = (P_o - P)/\gamma$

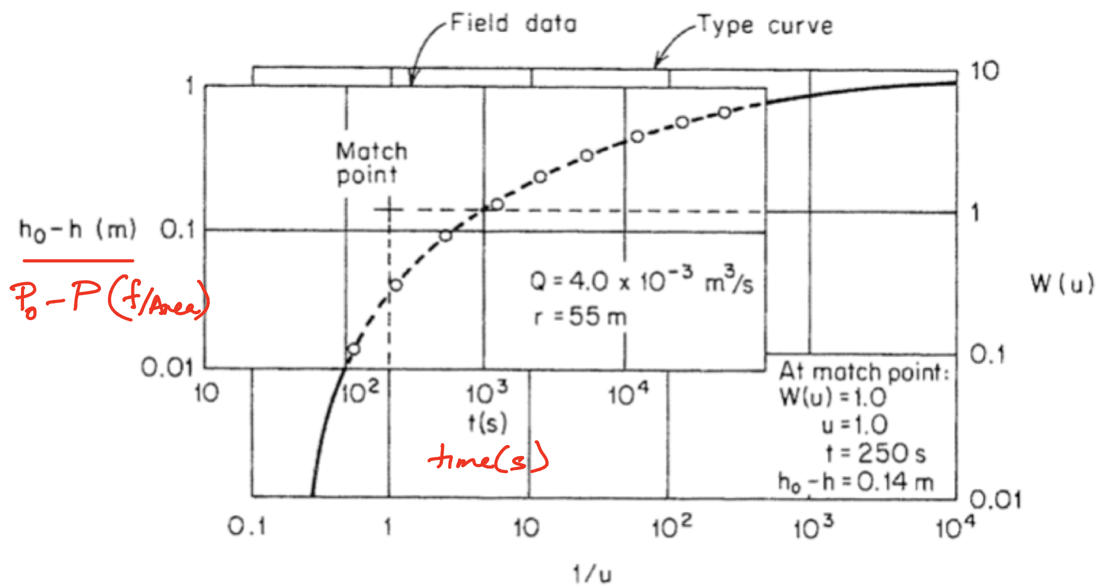
$Q = q$

$S = S_s.l = \rho g c l.$

Plot $(P_o - P)$-vs-time

Find matchpoints: $\frac{1}{u} = W(u) = 1$

$P_o - P$ and t.



Evaluate reservoir properties

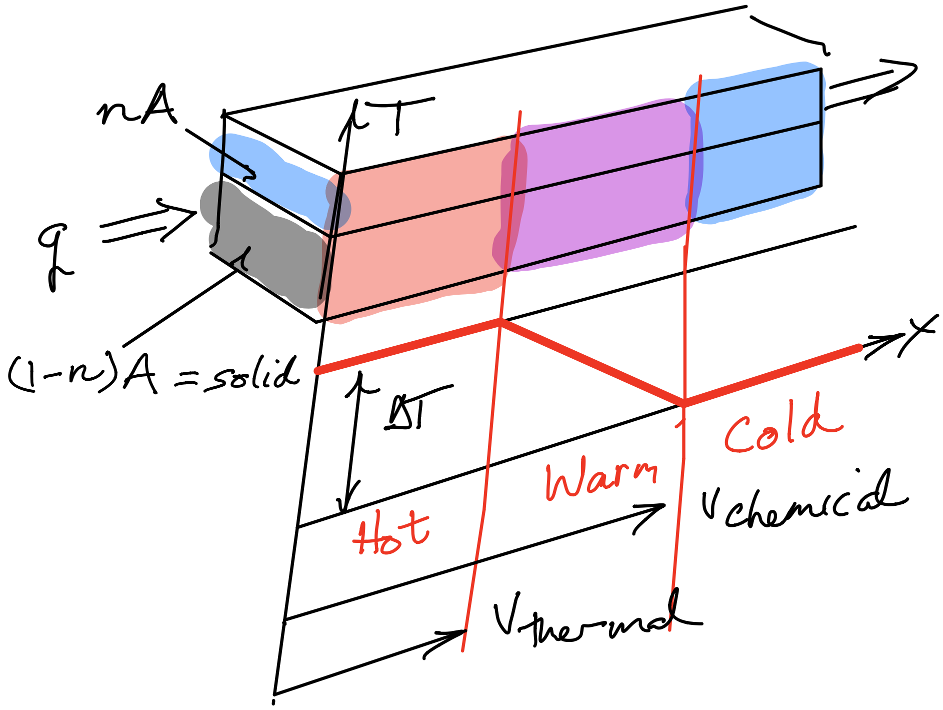
$$T = \frac{k}{\mu}\cancel{\rho g}\,\ell = \frac{q\,W(u)}{4\pi(P_o - P)}\cancel{\rho g}$$

$$\boxed{k = \frac{\mu}{\ell}\cdot\frac{q\,W(u)}{4\pi(P_o - P)}}$$

$$S = \cancel{\rho g}\,c\,\cancel{\ell} = 4ut\frac{k}{\mu}\cancel{\rho g}\frac{\cancel{\ell}}{r^2}$$

$$\boxed{c = 4ut\frac{k}{\mu}\frac{1}{r^2}}$$

4. Thermal breakthrough



Hot fluid injected
Cold reservoir
(backwards?)

$$\frac{q}{A\phi} = V_{chemical}$$

Thermal energy injected: $H_{th} = \rho c\, q\, \Delta T\, dt$

Thermal energy absorbed: $H_{ab} = (\rho c)_{reservoir}\, \Delta T\, V_{th}\, A\, dt$

Equating:

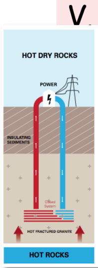
$$\rho c \left(V_{chem} \cancel{A} \phi\right) \cancel{\Delta T}\, \cancel{dt} = (\rho c)_r \cancel{\Delta T}\, V_{thermal} \cancel{A}\, \cancel{dt}$$

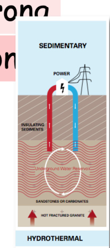
$$\frac{V_{thermal}}{V_{chemical}} = \frac{\phi \rho c}{\phi \rho c + (1-\phi)\rho_m c_m} = \left[1 + \frac{(1-\phi)}{\phi}\underbrace{\frac{\rho_m c_m}{\rho c}}_{\sim \frac{1}{2}}\right]^{-1}$$

n	Vchem/Vthermal
10%	5.5
20%	3.0
30%	2.2
100%	1

Contrasts Between EGSs & SGRs

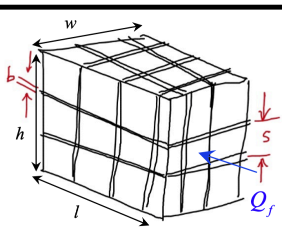
EGS (Order of Mag.)	Property	SGRs (Order of Mag)
Fractured-non-porous	General	Porous-fractured
<<1%,<1%	Porosity, n_0 -> n_{stim}	~10-30%, ~same
microD -> mD	Permeability, k_0 -> k_{stim}	>mD -> >>mD
10^6	K_f/k_{matrix}	10^6 ->1
10-100m	Heat transfer length, s	1m -> 1cm
>>100/1. >100/1	*$Heat_{solid}/Heat_{fluid}$	~10/1-2/1, same
?	Chemistry	?
V. Strong	TM Perm. Feedbacks	Less strong
oderate, late time	TC Perm. Feedbacks	Stron





$$*\frac{\text{Heat in solid}}{\text{Heat in fluid}} = \frac{\cancel{V}(1-n)\rho_R c_R \Delta T}{\cancel{V}(n)\rho_W c_W \Delta T} = \frac{(1-n)}{n}\frac{\rho_R c_R}{\rho_W c_W}$$

Thermal Drawdown EGS -vs- SGRs

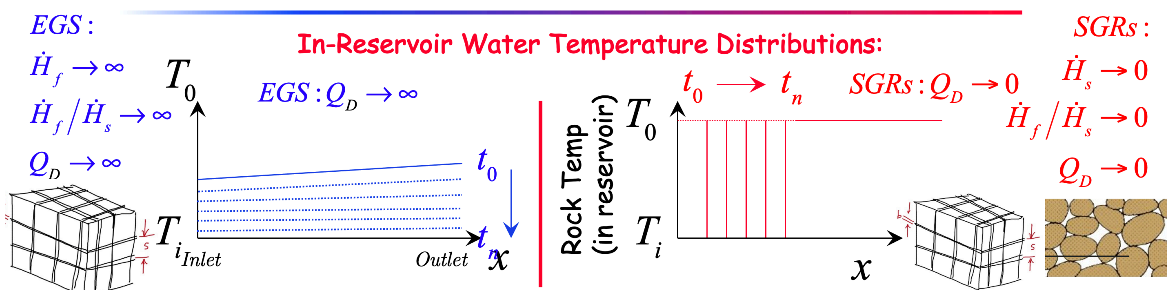


$$\dot{H}_{solid} \sim A\lambda_R \frac{dT}{dx} \sim \frac{V\lambda_R \Delta T}{s^2}$$

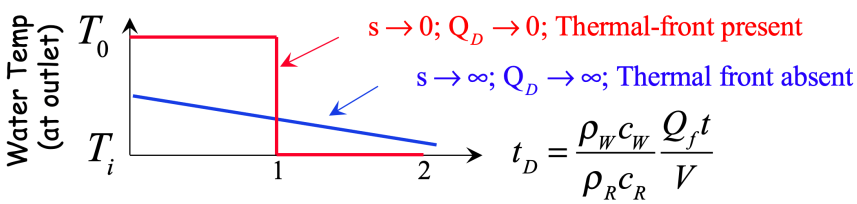
$$\dot{H}_{fluid} \sim Q_f \rho_W c_W \Delta T$$

$$\frac{\dot{H}_f}{\dot{H}_s} \sim \frac{\rho_W c_W}{\lambda_R}\frac{Q_f s^2}{V} = Q_D$$

In-Reservoir Water Temperature Distributions:



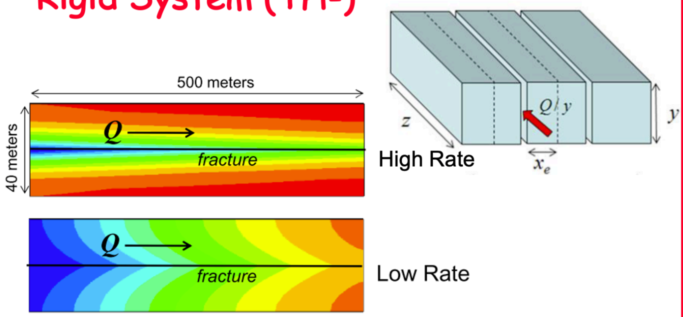
Thermal Output:

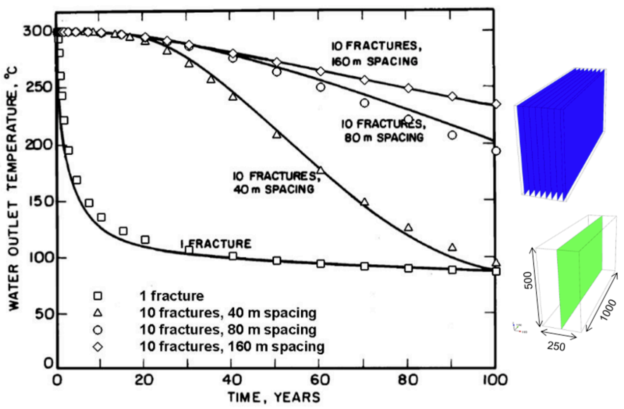


$$t_D = \frac{\rho_W c_W}{\rho_R c_R}\frac{Q_f t}{V}$$

Thermal Short-Circuiting

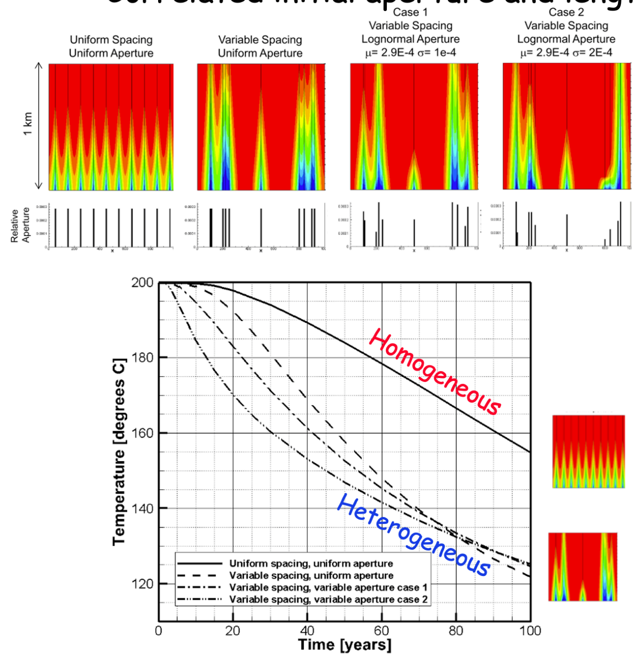
Rigid System (TH-)





Deformable System (THM)

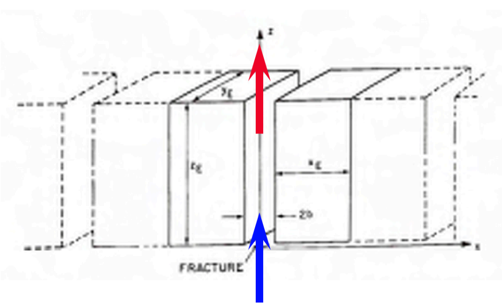
- Positive dQ->dT->dU->dQ
- Correlated initial aperture and length



[Doe, et al, 39th Stanford Geotherm. Wkshp., 2014]

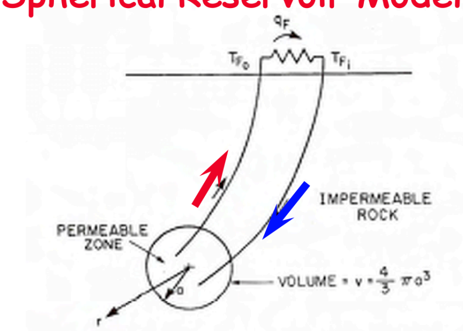
Thermal Recovery at Field Scale

Parallel Flow Model

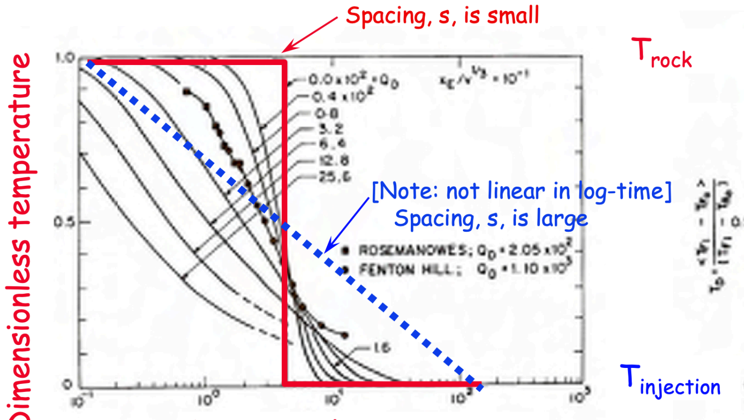


[Gringarten and Witherspoon, Geothermics,1974]

Spherical Reservoir Model



[Elsworth, JGR, 1989]



[Elsworth, JVGR, 1990]