3_1 Thermodynamics and Geothermal Systems

Recap:

Heat supplied to the crust (~70km deep) by convection - but conduction dominates in the crust Hydrothermal regimes concentrated in - destructive, constructive and hot-spot areas

EGS low-grade power ubiquitous

Recovery of thermal energy - requires a heat-transfer mechanism
Generating power requires an energy conversion mechanism - flash to steam

Movies:

Allam Cycle: https://www.youtube.com/watch?v=vFcbev1TkoU Resources: WG3

Thermodynamics: https://en.wikipedia.org/wiki/Laws_of_thermodynamics

General understanding of heat flow and thermodynamics, follows:

2. THERMAL ENERGY

2.1 HEAT & TEMPERATURE

specific heat

a) Absent phase change: $\Delta Q = c\ \Delta T$

ΔQ = Heat ; ΔT = Temperature

SI units: °C or K

Unit of thermal energy = 1 calorie ~ 4.2 J

(1 calorie ≡ 1 gram H_2O ↑ 1°C)

b) With phase change: $\Delta Q = L$

Latent heat

Heat is absorbed but no ΔT.

Latent heat of Fusion (ice → water)

Latent heat of Evaporation (water → gas)

COEFFICIENTS FOR WATER

c (specific heat) $= 4.186\ J/g\,°C = 4.2\ kJ/kg\,°C$

L_{fusion} (latent heat) $= 334\ kJ/kg$

L_{evap} (latent heat) $= 2260\ kJ/kg$

How much heat needed to melt ice versus heating water?

$$\left.\begin{array}{l} \Delta Q = c\,\Delta T \\ \Delta Q = L \end{array}\right\} \quad L = c\,\Delta T \longrightarrow \Delta T = L/c \sim 80°C$$

How much heat needed to evaporate water versus melt ice?

$$\frac{\Delta Q_{evap}}{\Delta Q_{melt}} = \frac{L_{evap}}{L_{fusion}} = \frac{2260}{334} \sim 7$$

2.2 HEAT TRANSFER

Three modes: Conduction ; Convection ; Radiation.

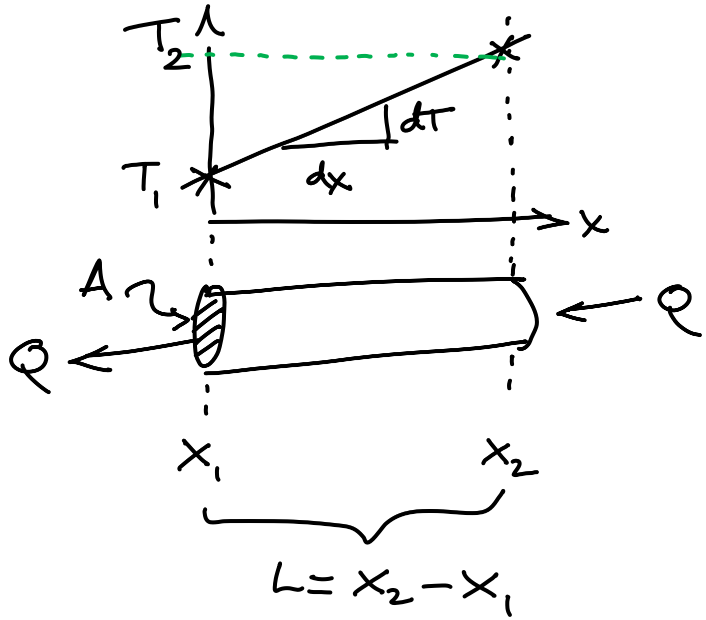
CONDUCTION:

$$Q = Ak\frac{(T_1 - T_2)}{L} \doteq -Ak\frac{\partial T}{\partial x}$$

k = thermal conductivity
(Fourier's Law)

Requires that flow is at a 'steady state'

Flux is a vectoral quantity, hence $Q = -k\frac{\partial T}{\partial x}$



Typical k Magnitudes ($W m^{-1} K^{-1}$)

Steel: ~30 – 50

Water: 0.6

Rock: ~1 – 3

Styrofoam: ~0.03

List of thermal conductivities - Wikipedia, the free encyclopedia

CONDUCTION (Cont'd)

Non-steady behavior (transient)

$$\frac{\partial}{\partial t}(\rho c T) + \frac{\partial}{\partial x} q_{Thermal} = 0$$

$$q_{thermal} = -k \frac{\partial T}{\partial x}$$

$$\rho c \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} = 0$$

Dimensional Analysis:

$$\rho c \frac{T}{t} \frac{1}{T_o} \doteq k \frac{T}{x^2} \frac{1}{T_o} \Rightarrow 1 \sim \frac{k}{\rho c} \frac{t}{x^2}$$

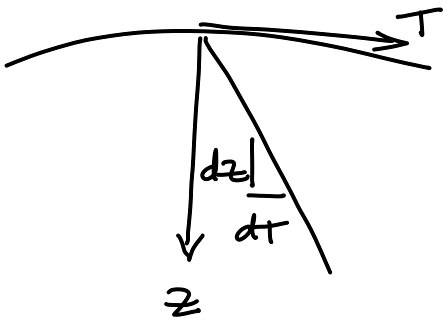
$$\sim \kappa \frac{t}{x^2}$$

Thermal diffusivity (m^2/s)

KELVIN'S PROBLEM / RUTHERFORD'S SOLUTION

How much heat is generated within the earth from radioactive decay?

Assume this is the only heat source:



$$Q = -A\,k\,\frac{\partial T}{\partial z} \quad \begin{cases} A = 4\pi r^2 \\ k = 1\ \mathrm{W\,m^{-1}K^{-1}} \\ \frac{\partial T}{\partial z} \sim 30^\circ/\mathrm{km} \end{cases}$$

$$q = -k\frac{\partial T}{\partial z} \cong 30\ \mathrm{mW/m^2} \quad (\text{Solar} \equiv 1.4\ \mathrm{kW\,m^{-2}})$$

$\times 10^6$

How long does it take for one quantum of heat to traverse the crust $\sim$ 60 km thick. (A change at the surface)

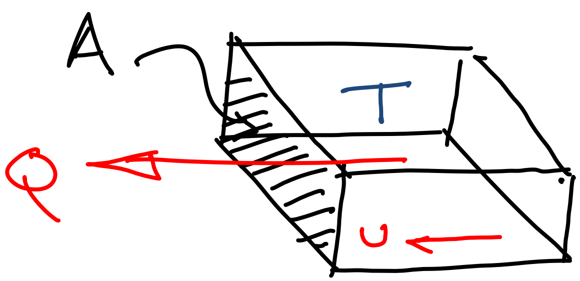
$$\frac{\kappa t}{x^2} \sim 1 \Rightarrow t \sim \frac{x^2}{\kappa} \sim \frac{(60\times 10^3)^2\,\mathrm{m}^2}{30\ \mathrm{m^2/y}} \sim 120\times 10^6\ \mathrm{y}$$

CONVECTION

Heat carried by bulk motion of fluid.

$$Q = A(\rho u)(cT)$$

Flow velocity



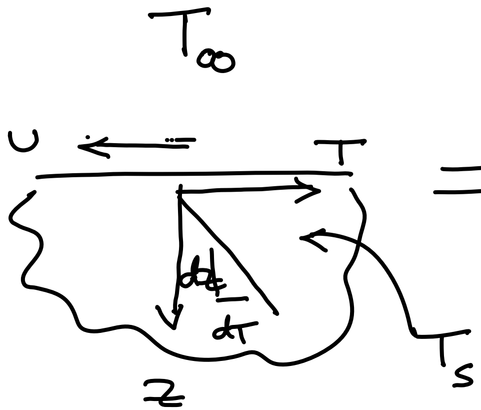
$$q = -k\frac{\partial T}{\partial x}$$

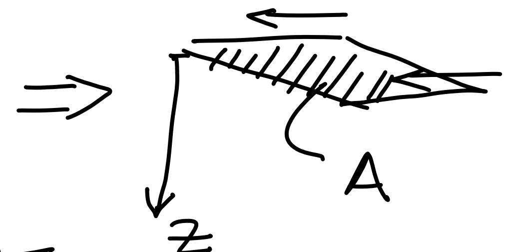
Forced convection:

1. Driven by flow
2. More rapid than conduction since provides fresh cool fluid to elevate thermal gradients $\rightarrow q = -k\,\partial T/\partial x$.

CONVECTION (Cont'd)

Convection is an efficient method of heat transfer – from static body to a flowing fluid e.g. Wind chill.





$$\frac{Q}{A} = N_u \, k \frac{(T_s - T_\infty)}{L}$$

L = characteristic length e.g. radius of bar or fin

$$N_u = f(P_r, R_e) \begin{cases} P_r = \dfrac{c\mu}{k} \\ R_e = \dfrac{\rho u L}{\mu} \end{cases}$$

RADIATION

Transport by EM waves and also works in a vacuum (space).

— Lucky for us.

Power (P_e) per unit area per unit time is:

$$P_e = \varepsilon \sigma T^4$$

ε = emissivity $(0 < \varepsilon < 1)$

$\sigma = 5.67 \times 10^{-8}\ W.\,m^{-2}\,K^{-4}$

(Stefan – Boltzmann coeff)

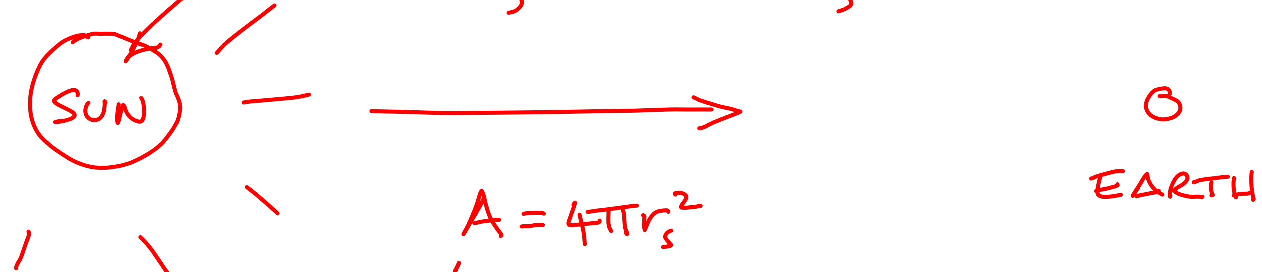
$$\left.\begin{array}{ll} \text{Emissivity:} & P_e = \varepsilon \sigma T^4 \\ \text{Absorptivity:} & P_a = \varepsilon \sigma T_0^4 \end{array}\right\} \quad \underline{\text{Net } P = P_e - P_a = \varepsilon \sigma (T^4 - T_0^4)}$$

RADIATION (Cont'd)

A 'black body' absorbs all radiation

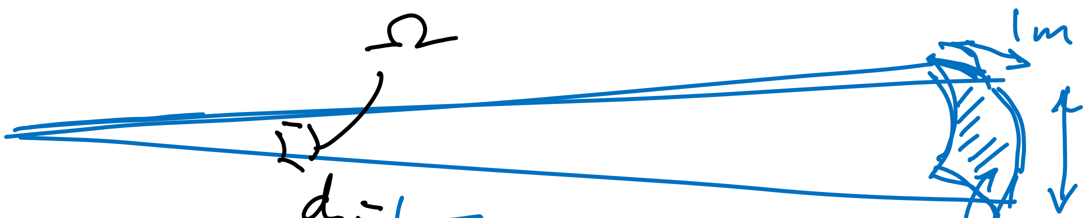
Heat output of the sun.

$T = 5800K$; $r = 7\times10^{8}m$; $\varepsilon = 1$



$A = 4\pi r_s^2$

$P_s = \sigma \varepsilon T^4 A = \underline{\underline{4\times10^{26} W}}$ (total from sun)



$d = 1.5\times10^{11} m$

$A = 1m^2$

$$\frac{1\times1m}{4\pi d^2} = \Omega$$

= Fraction of Sun's surface area.

Thermal energy from sun on 1 m^2 of Earth.

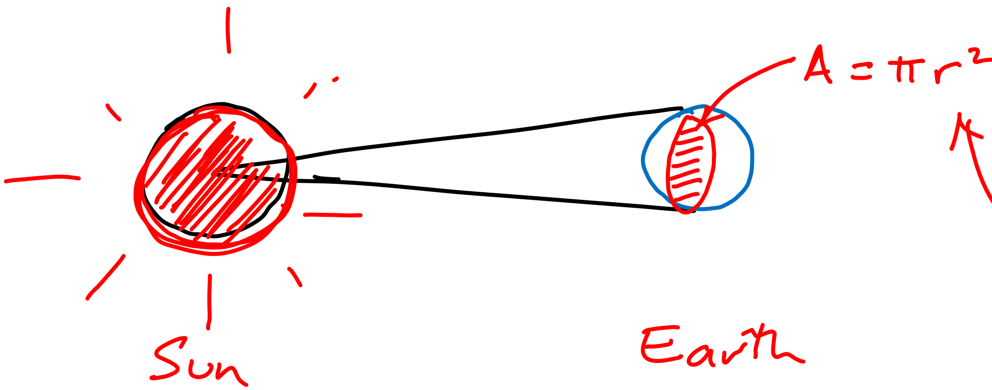
$$P_e = P_s \times \text{Fraction of sun}$$

$$= P_s \, \Omega$$

$$= 4 \times 10^{26} W \times 3.5 \times 10^{-24}$$

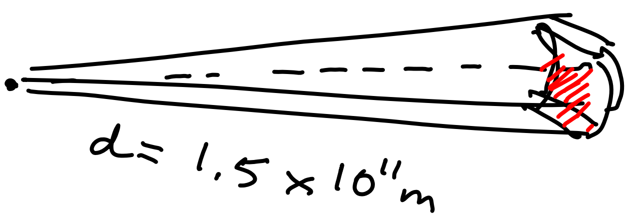
$$\sim 1.4 \ kW/m^2$$

How much radiative energy does the Earth receive?



$r = 6380$ km

Note: that @ large distance subtended angle is v. small. (Ω)



Fraction of radiation is

$$\Omega = \frac{1}{4\pi d^2}$$

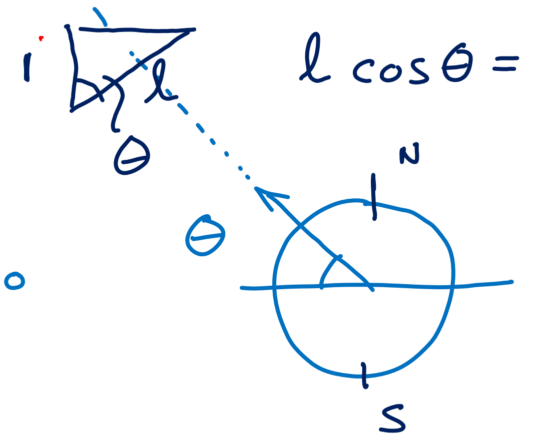
$$P_e = P_s \, \Omega \, A_e$$

$$= 4 \times 10^6 \text{ W} \times 3.5 \times 10^{-24} \times A_e$$

$$\boxed{P_e = 180 \times 10^6 \text{ GW}}$$

Worldwide fossil fuel combustion = 10 TW

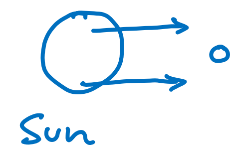
How does the incoming solar radiation change as a function of:



$l \cos\theta = 1$

a) Latitude:

Radius of sun $= 7 \times 10^{8}$ m
Radius of earth $= 6.4 \times 10^{6}$ m

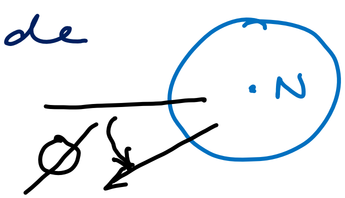


Incident radiation varies as $\frac{1}{l} = \cos\theta$

Note: θ relative to equatorial plane (not equator)

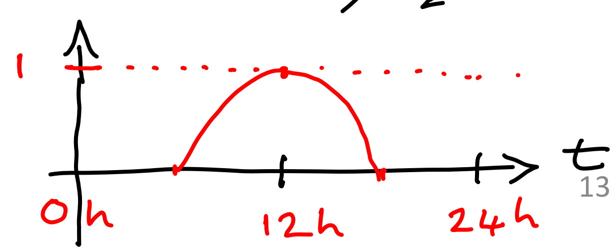
b) Longitude: (Represents time of day.)

Similar argument where ϕ is longitude relative to noon hour:



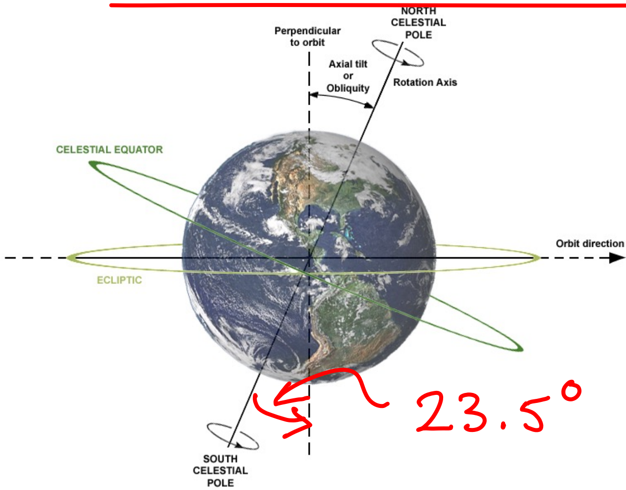
$$P_e = P_{s\,max}\,\Omega \cos\phi$$

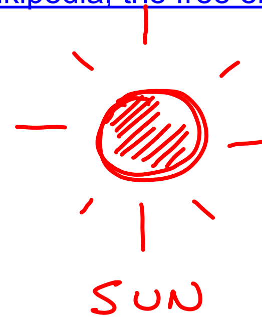
$$= P_{s\,max}\,\Omega \,\frac{(noon - t)}{24} \times 2\pi$$

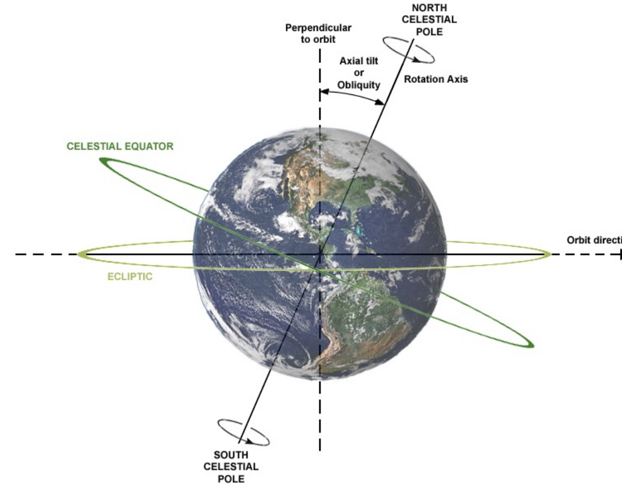


VARIATION OF RADIATION WITH LOCATION & SEASON

Axial tilt - Wikipedia, the free encyclopedia







JUNE 21 (SOLSTICE)

MAR 21

SEPT 21 (EQUINOX)

DECEMBER 21 (SOLSTICE)

UNIVERSITY PARK = 41°N

$P_{e\,max} \simeq 1.4\ kWm^{-1} \cos\theta$

$\theta_{summer} = 41° - 24° = 17°N$ $\sim 1.33\ kWm^{-1}$

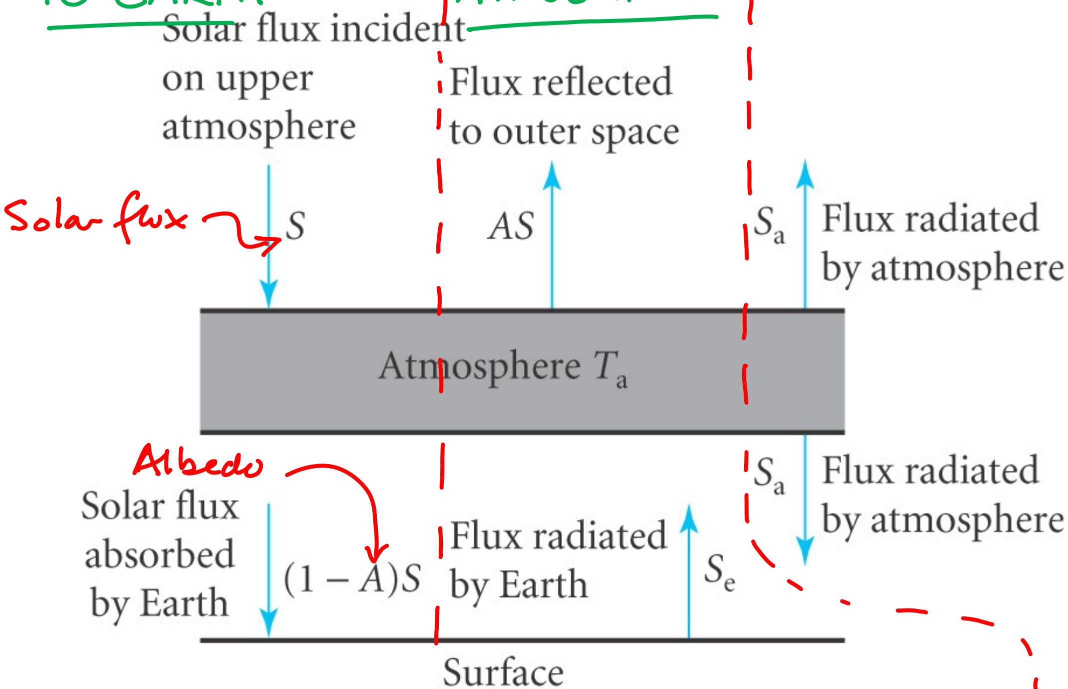
$\theta_{winter} = 41° + 24° = 65°N$ $\sim 0.6\ kWm^{-1}$

But average radiation (annual) is for $\theta = 41°$ and 12h/day

$[P_{e\,av} \sim 1.06\ kWm^{-1}]$ (i.e. 50%)

GREENHOUSE EFFECT

TO EARTH | ATMOSPHERE



TO EARTH	ATMOSPHERE
INCOMING RADIATION	INCOMING RADIATION
$(1-A)S\pi R^2$	$(1-A)S\pi R^2$
EARTH AS BLACK BODY	BLACK BODY (ATMOS.)
$4\pi R^2 \sigma T^4$	$4\pi R^2 \sigma T_a^4$
EQUATE	EQUATE
$T = 255\,K$ (−18C)	$T_a \sim 255\,K$

ATMOSPHERE AS BLACK BODY

SUN ATMOSPHERE

$$(1-A)S\pi R^2 + 4\pi R^2 \sigma T_a$$

$$4\pi R^2 \sigma T_E$$

(EARTH)

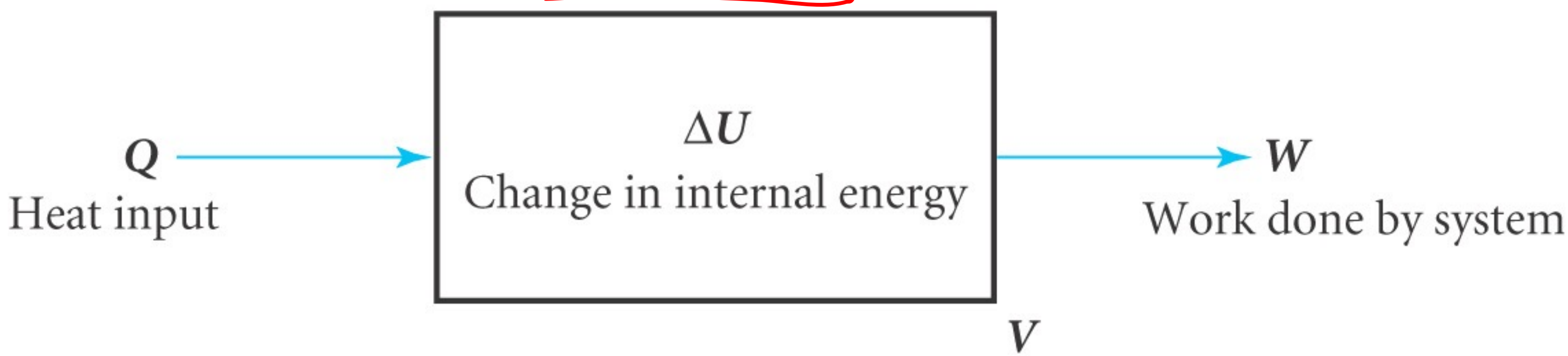
Solve

$$T_E \sim 303K \quad (30°c)$$

2.3 FIRST LAW OF THERMODYNAMICS

HEAT   WORK DONE   INTERNAL ENERGY

$$\boxed{Q - W = \Delta U}$$



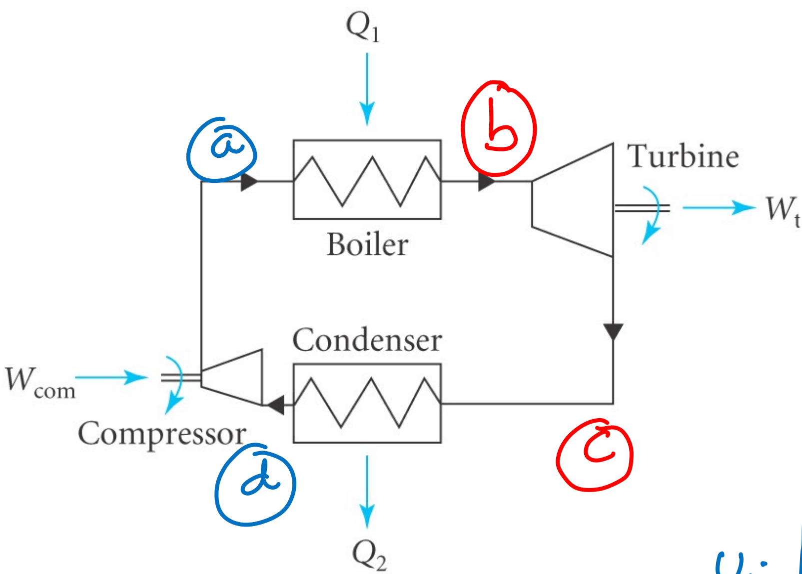
STATEMENT THAT:

ENERGY IS NEITHER CREATED OR DESTROYED

BUT MAY BE <u>REDISTRIBUTED</u>

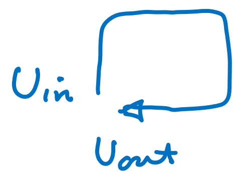
CONVENTION (POSITIVE): $\begin{cases} Q - \text{heat flows } \underline{\text{into}} \text{ system } \& \\ W - \text{work done } \underline{\text{by}} \text{ system.} \end{cases}$

2.4 CLOSED CYCLE FOR A STEAM PLANT

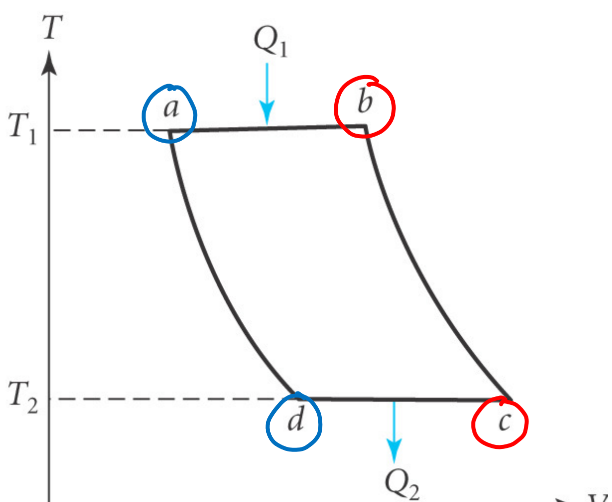


Typical circuit for power conversion.

Work input @ compressor (W_{com})
Heat input @ boiler (Q_1)
Work output @ turbine (W_t)
Heat output @ condenser (Q_2)



$\Delta U = 0$ since working fluid begins circuit with the same int. energy it ends



FIRST LAW: $(Q_1 - Q_2) - (W_t - W_{com}) = 0$

EFFICIENCY, $\eta = \dfrac{\text{net work output}}{\text{heat input}}$

$$\eta = \frac{(W_t - W_{com})}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \doteq 1 - \frac{T_2}{T_1}$$

2.5 USEFUL THERMODYNAMIC QUANTITIES

T = temperature
P = pressure
$v = \frac{1}{\rho}$ = specific volume
u = specific internal energy
h = specific enthalpy
s = specific entropy

six key quantities but they are interdependent — any two define system state

"specific" = per unit mass

ENTHALPY: $h = u + pv$

DESCRIBES:

1. Heat transfer at constant pressure: (e.g. boilers and condensers)

via $\Delta h = \Delta u + \cancel{\Delta pv}$ zero since $pv \equiv$ constant

$Q - \cancel{W}^{\,0} = \Delta u \quad \therefore \quad Q = \Delta h$

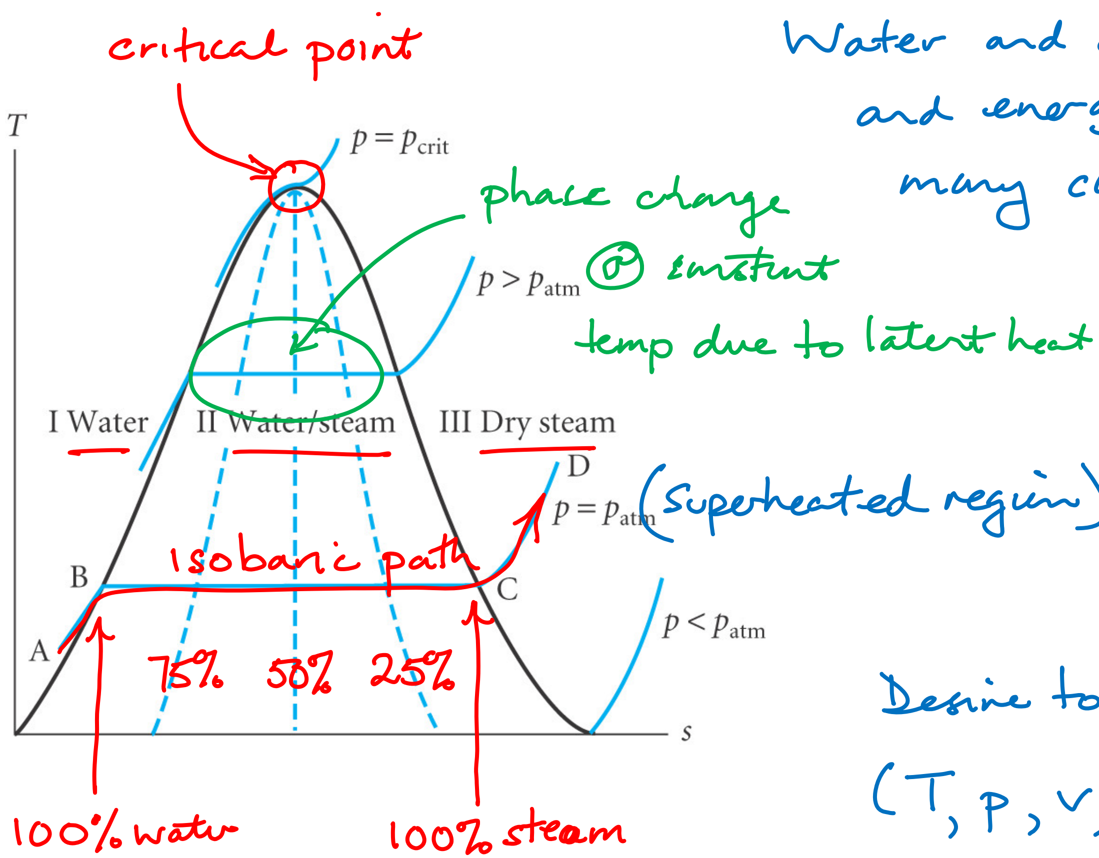
2. Adiabatic (no heat change) compression of gases: (e.g. compressors & turbines)

$\cancel{Q}^{\,0} - W = \Delta u \quad \therefore \quad W = -\Delta h$

ENTROPY (degree of disorder)

Change in entropy $\Delta S = \dfrac{\text{Reversible heat supplied}}{\text{System temperature}} = \dfrac{\Delta Q_{rev}}{T}$

2.6 THERMAL PROPERTIES OF WATER & STEAM



Water and other fluids are heat and energy transfer media in many conversion processes.

Desire to define fluid properties (T, p, v, u, h, s) throughout all regions I, II, and III.

In this figure: $v, u, h = f(T, s, p)$

SCALING STEAM PROPERTIES IN THE TWO-PHASE REGION II

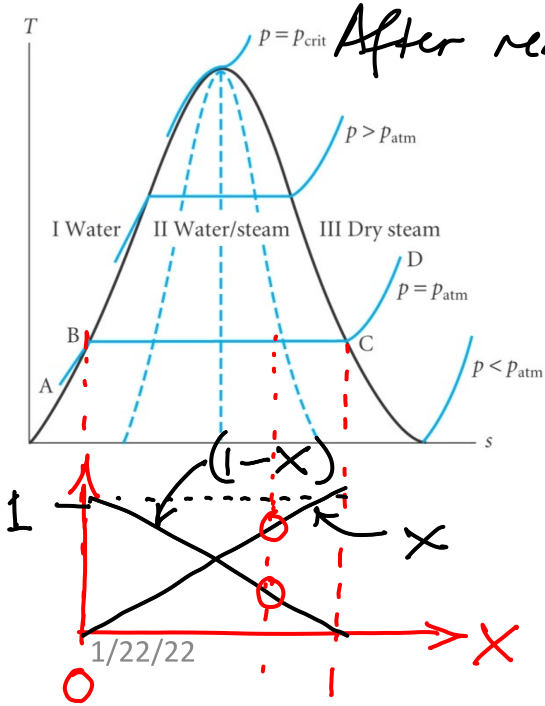
TOTAL MASS: $m = m_f + m_g$

TOTAL VOLUME: $\forall = \forall_f + \forall_g = \dfrac{m_f}{\rho_f} + \dfrac{m_g}{\rho_g} = m_f v_f + m_g v_g$

SPECIFIC VOLUME: $v = \dfrac{\forall}{m} = \dfrac{m_f v_f + m_g v_g}{m}$

After rearrangement: $v = (1-x)v_f + x\, v_g$

$x = \dfrac{m_g}{m}$



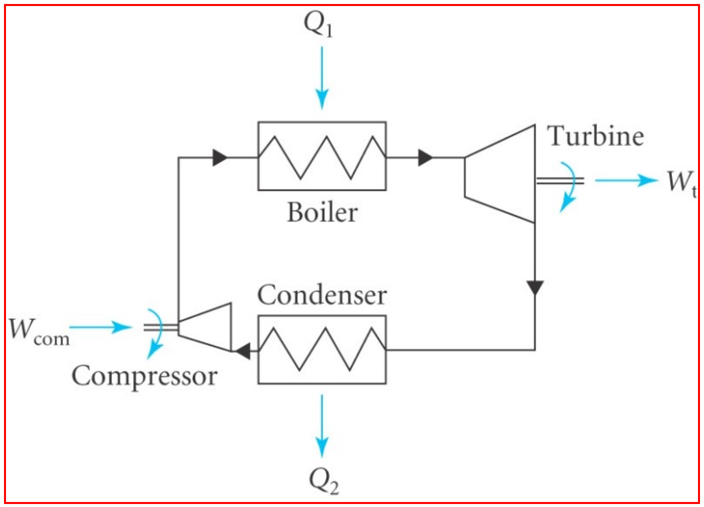
Also: $u = (1-x)u_f + x\, u_g$

$h = (1-x)h_f + x\, h_g$

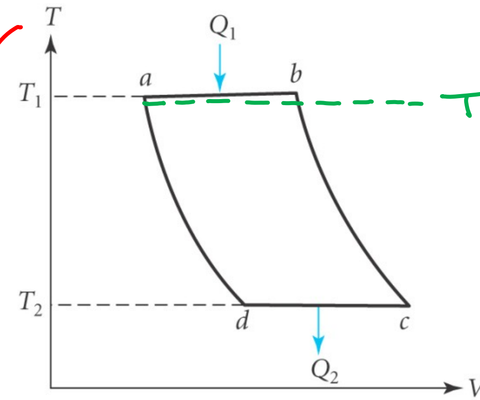
$s = (1-x)s_f + x\, s_f$

This merely scales properties proportionally to mass present of each phase

2.7 DRAWBACKS OF CARNOT CYCLE



T–V space

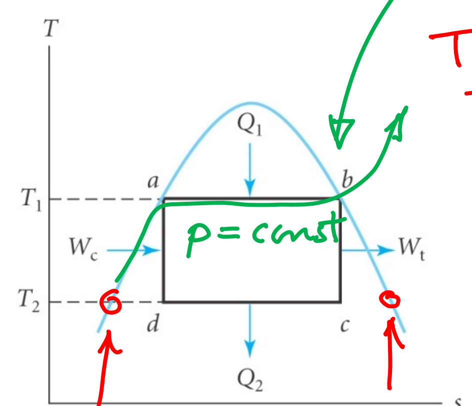


$T_1 = \text{constant}$

∴ boiler must be @ constant pressure

∴ cannot operate in dry steam region

T–s space



100% water

100% steam

∴ condenser cycle begins with some water (c) and 'cools' to some steam (d).

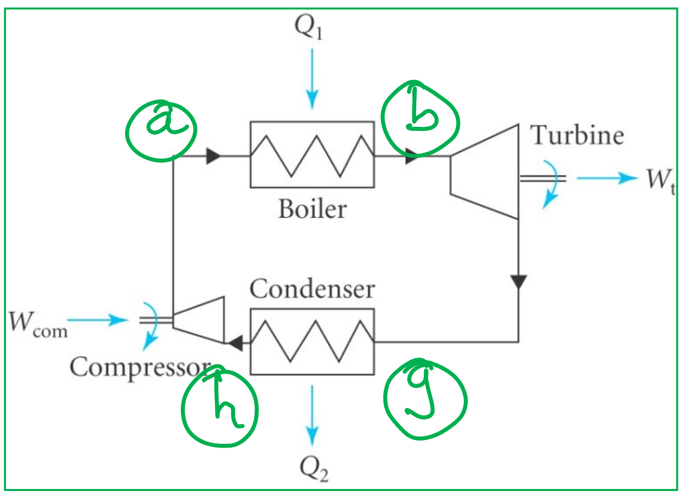
Hence less efficient than going 100% steam → 100% water

2.8 RANKINE CYCLE FOR STEAM POWER PLANTS

(2.7: Carnot efficiency, $\eta_c = 1 - \frac{T_2}{T_1}$, is max possible efficiency but (for constant T_1 reservoir → this requires isobaric boiler

which is unusual)

Rankine cycle is more common — two flavors



① Without reheat:

② With reheat: