13_1 Geothermal - GSHPs

Recap:

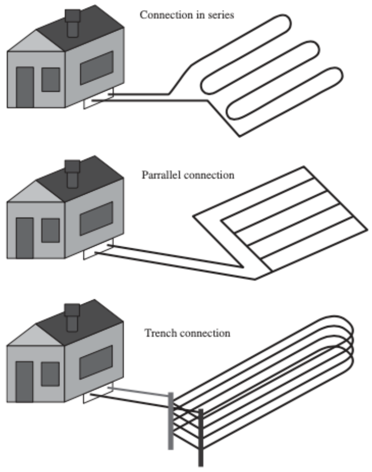
1. Direct Use:
   a. Utilizes low heat/quality resource to fill a significant need
   b. Potentially utilizes the ~50% rejected heat in the Sankey diagram

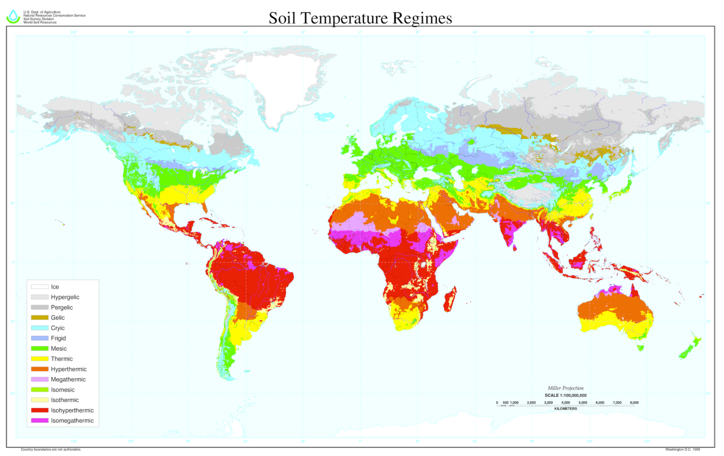
Movies: (Great Lakes SedHeat Network): https://igws.indiana.edu/glsn/speakers
(Jerrod): https://personal.ems.psu.edu/~fkd/courses/eme_497/videos/3_v_anthonyjerrod.mp4
(Shreya): https://drive.google.com/file/d/1MuDdfXslJnNfs1g8YcSj5PVVkrDgfZ5v/view

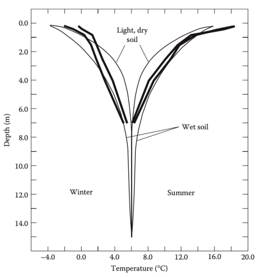
Resources: WG11 & MR 5+6

Motivation:

1. Motivation [10%] Provide context for the topic. *Use of relevant public domain videos* are a useful method for this. Why is this particular topic or sub-topic important in the broad view of geothermal energy engineering?







Soil Map: https://www.nrcs.usda.gov/wps/portal/nrcs/detail/soils/use/worldsoils/?cid=nrcs142p2_054019

Utilize low quality heat without the penalty of conversion to electricity
Distributed power opportunity for off-grid and remote use
Broadly geographically available (in US) due to climatic zonations

Scientific Questions:

2. Scientific Questions to be Answered/Outline [10%] What questions arise from the motivation. What are the sub-topical areas that address these scientific questions.

GSHPs

1. Mechanisms of heat flow in the shallow earth?
2. Mechanism of utilizing low quality heat -> high(er) quality heat?
3. Rate-limiting processes?

1. Mechanisms of heat flow in the shallow earth?

RECOVER HEAT FROM SHALLOW SUBSURFACE

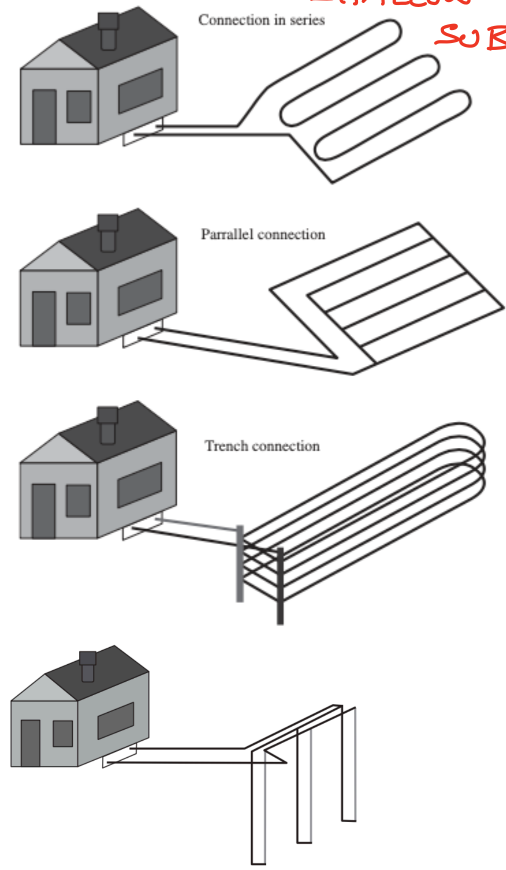
TABLE 11.2
Thermal Conductivity (W/m-K) and the Constant Pressure Heat Capacity (C_p [J/mole-K]) of Some Common Materials at 25°C

?

Material	k_{th}	C_p	Q	m³
Quartz[a]	6.5	44.5	1960	128.5
Alkali feldspar[a]	2.34	203	2000	130
Calcite[a]	2.99	82	2103	120
Kaolinite[a]	0.2	240	2408	105
Water	0.61	75.3	4181	60

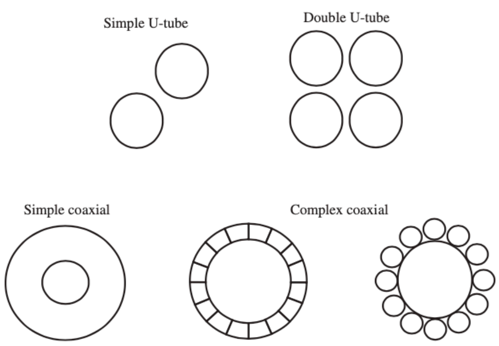
Source: [a] C_p computed from Helgeson, H.C. et al., *American Journal of Science*, 278-A, 229, 1978.

Note: The amount of heat, Q (kJ/m³-K), that must be supplied or removed, per cubic meter of material, for 1°C of temperature increase or decrease at about 25°C is shown. The number of cubic meters of each material needed to supply 7 kW of heat is shown in the column m³ (see text for details).



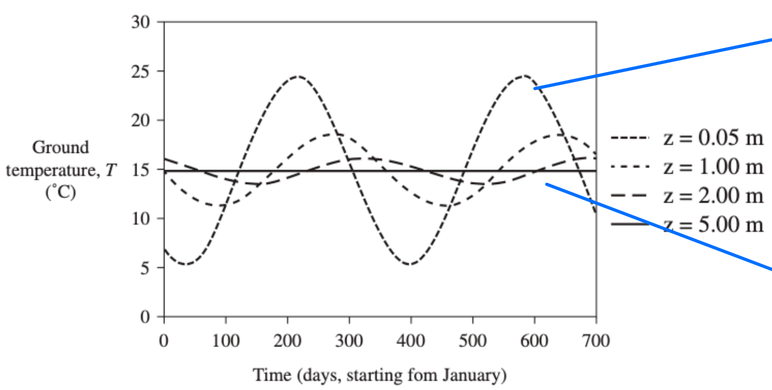
$$Q = \rho C_p \Delta T V$$

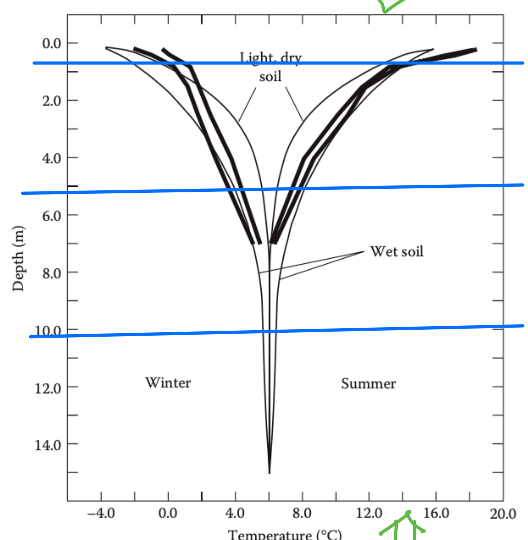
Note: Use J/kg.K



This defines amount of heat supplied but not the RATE.

INSOLATION 200 – 1500 W/m²





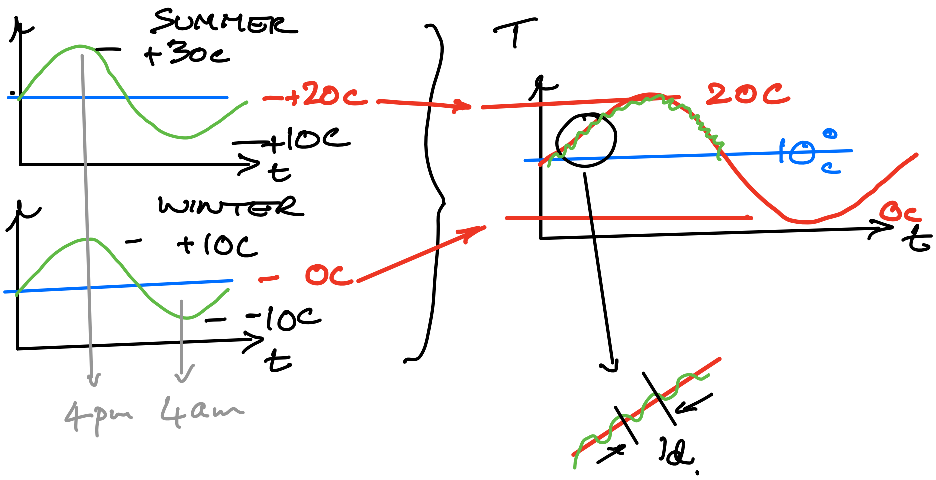
GEOTHERMAL FLUX ≐ 87 mW/m²

TEMPERATURE CHANGES IN SHALLOW SUBSURFACE

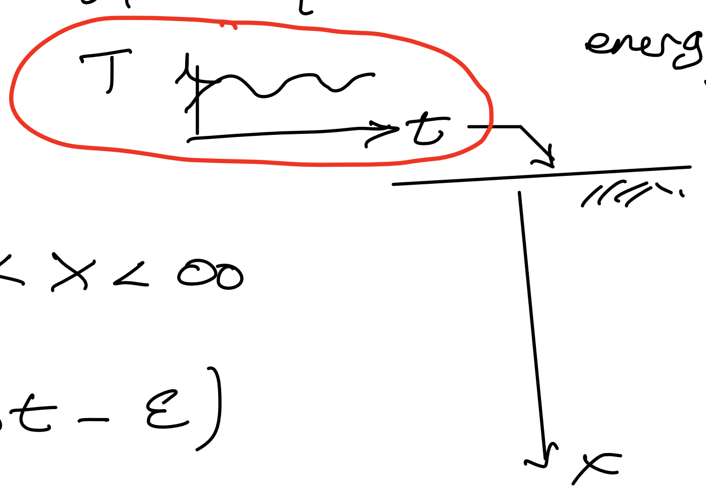
$200\ W/m^2 \gg 87\ mW/m^2$ $\therefore$ Insolation dominates

Insolation — Geothermal — sets deep temperature as average

DIURNAL (AIR) ANNUAL

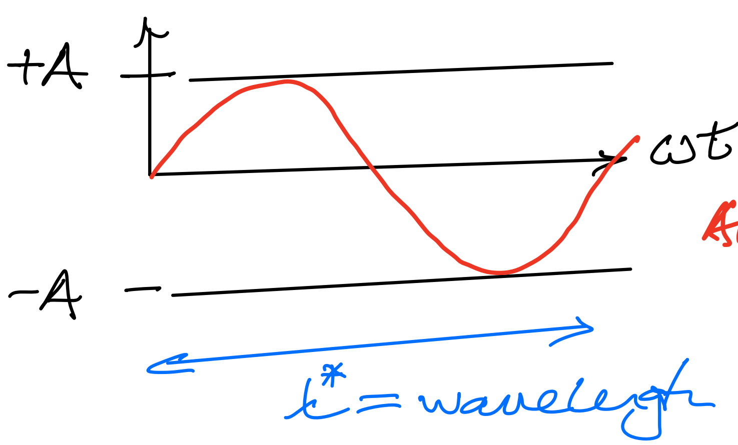


Solve: $\rho c \dfrac{\partial T}{\partial x} = \lambda \dfrac{\partial^2 T}{\partial x^2}$ 1. Fourier's Law 2. Conservation of energy



ICs: $T = 0 \quad 0 < X < \infty$

BCs: $T = A\cos(\omega t - \varepsilon)$



Amplitude $= A \doteq$ K or C

Ang. Frequency $= \omega \doteq \dfrac{1}{sec}$

$n =$ frequency $= \dfrac{\omega}{2\pi}$

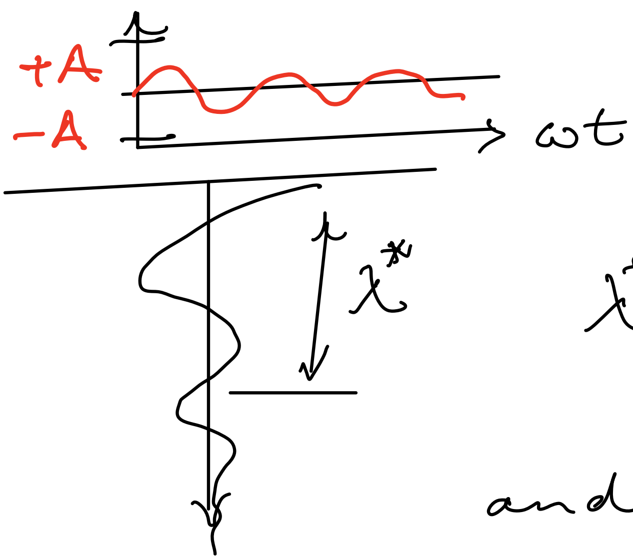
SOLUTION (dynamic steady state)

$$T = A e^{-kx} \cos(\omega t - kx - \varepsilon)$$

with $k = \left(\dfrac{\omega}{2K}\right)^{1/2}$ = wave number = $\dfrac{1}{m}$

$K = \dfrac{\lambda}{\rho c}$ = thermal diffusivity m^2/s

Wavelength (in space not time ωt)



$$\lambda^* = \frac{2\pi}{k} = \frac{4\pi K}{n}$$

and n = frequency = $\dfrac{\omega}{2\pi}$

Set $kx = 2\pi$ (i.e. one wavelength)

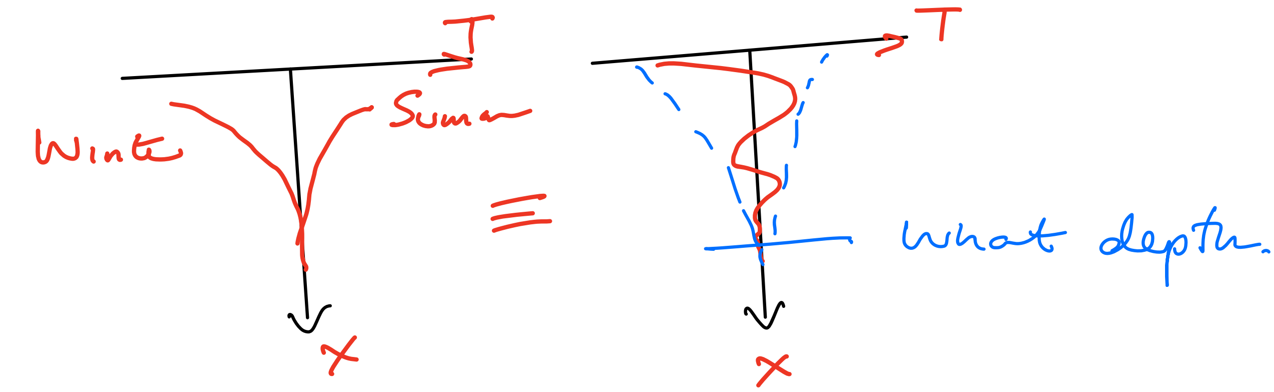
Then $2\pi = kx = \left(\dfrac{\omega}{2K}\right)^{1/2} x$ & $\underline{\underline{\omega = 2\pi n}}$

$$2\pi = \left(\frac{2\pi n}{2K}\right)^{1/2} x$$

or $$x = (2\pi)^{1/2} \cdot \left(\frac{2K}{n}\right)^{1/2}$$

$$x = \left(\frac{4\pi K}{n}\right)^{1/2} = \lambda^*$$

Depth for no temperature change



$$T = A e^{-kx} \left(\quad \right)$$

And $e^{(-2\pi)} \approx 0.0019 \approx 0.002$

$$\approxeq 0.2\% \equiv 0$$

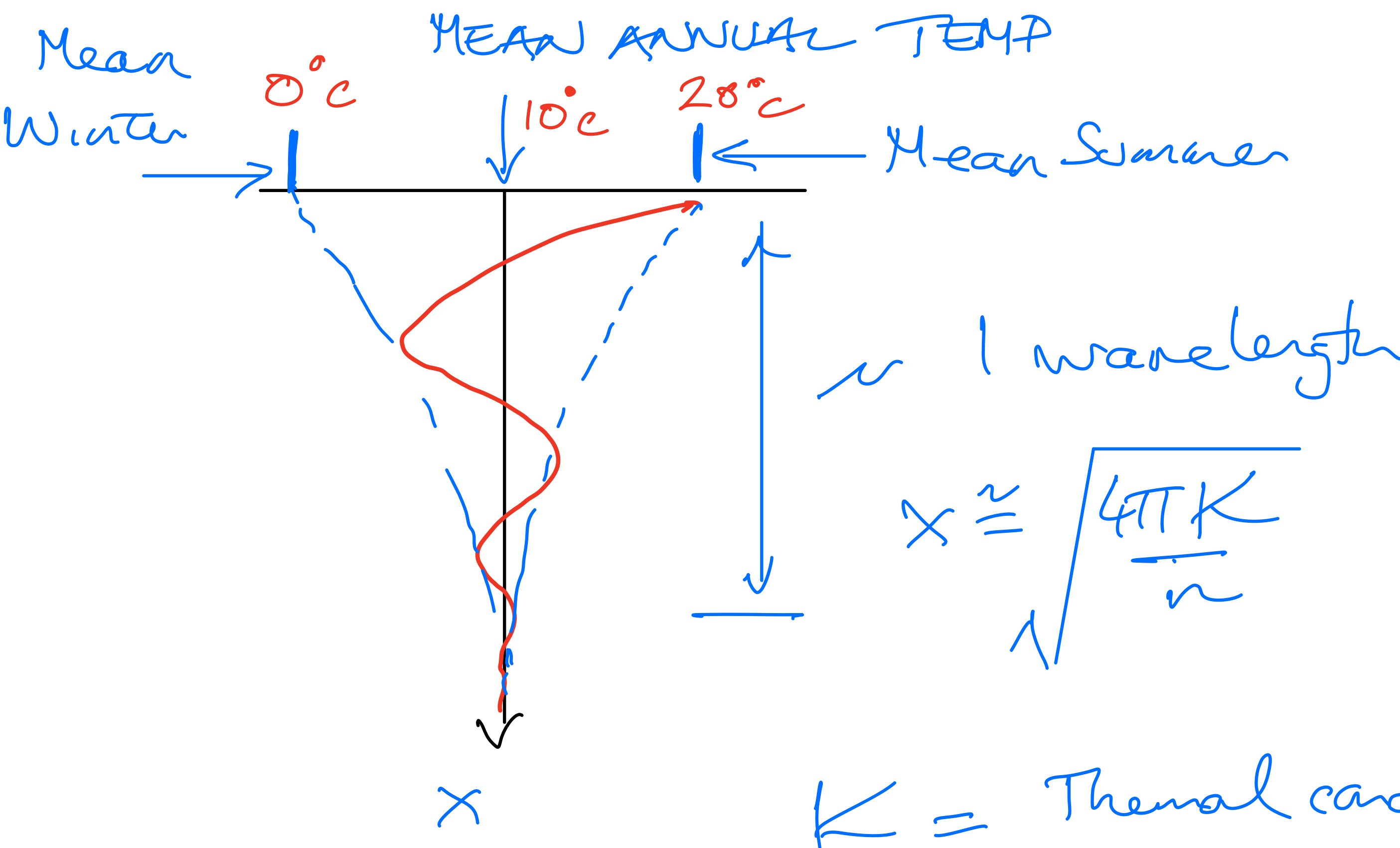
$-2\pi = -kx \quad \therefore \quad x = \left(\frac{4\pi K}{n}\right)^{1/2}$

If $K = 30\ m^2/yr$.

Wavelength	Freq
2.7 cm	1/min
1 m	1/day
20 m	1/year

$$\boxed{\left(4\pi \frac{30}{1}\right)^{1/2} = \sqrt{360\ m^2/yr \cdot yr} \sim \underline{\underline{20\ m}}}$$

TEMPERATURE PROFILES

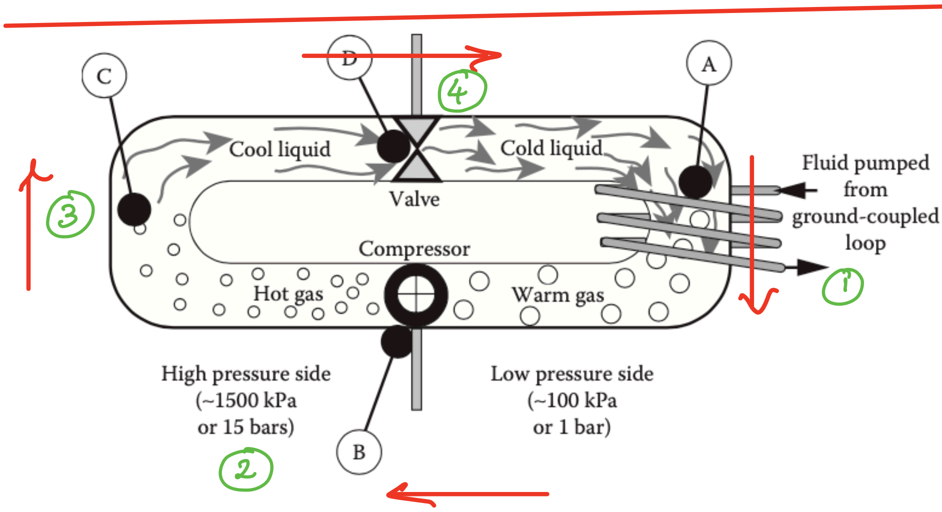


$$x \cong \sqrt{\frac{4\pi K}{\omega}}$$

$$K = \frac{\text{Thermal cond}}{\rho c}$$

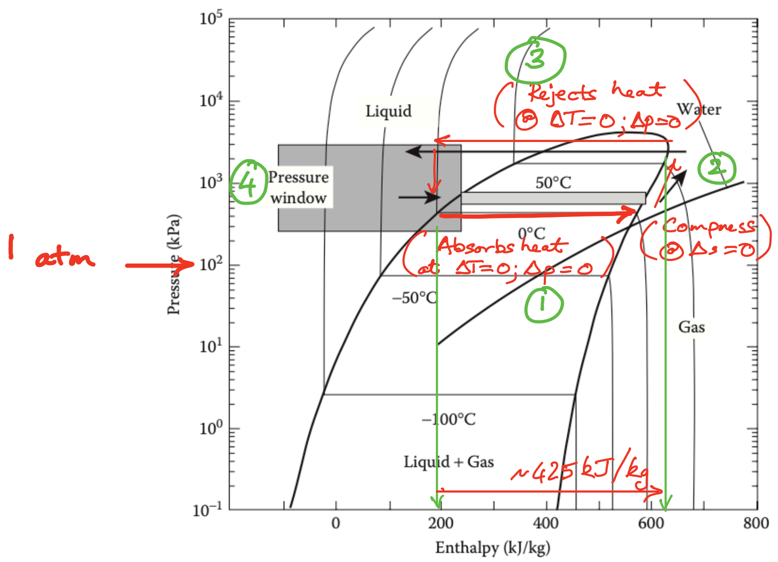
2. Mechanisms of utilizing low quality heat -> high(er) quality heat?

THERMODYNAMICS OF HEAT PUMPS



COIL FROM GROUND

$\Delta T \sim \pm 10°C$



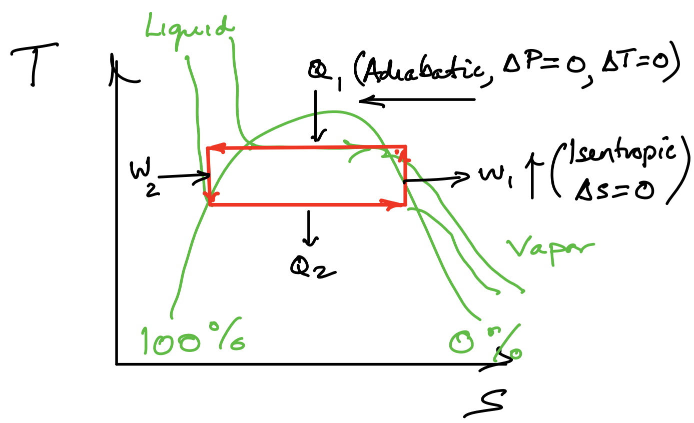


FIGURE 11.4 Enthalpy–pressure diagram for propane showing the two-phase liquid–gas region. For reference, the low-temperature limb of the liquid boundary for the water system is also shown (see Figure 3.8 for details of the water system).

CARNOT CYCLE – Same as deep geothermal.

PRINCIPLE – Heat gained leg 1 >> Energy of compression leg 2

Allows $+\Delta H$ in leg 1 (from ground – low quality)

↳ Rejected to building leg 3

↳ @ energy cost of Δp in 2

TUNE 2-PHASE FLUID ⟶ 0 – 50°C BP

or for PA, 0 – 20°C or ~10°C.

i.e. Capable of liquid → vapor @ 10°C

and @ $p > atm$

FOR PROPANE $\Delta H \sim 425$ kJ/kg (of propane)

Geothermal water $C = 4.18$ kJ/kg.K.

TABLE 11.1
Thermodynamic Properties of Some Compounds Potentially Useful as Refrigerants

Name	Formula	Molecular Weight (g/mol)	Density (kg/m³)	Melting Temperature (°C)	Boiling Temperature (°C)	Heat of Vaporization (kJ/kg)	Constant Pressure Heat Capacity (kJ/kg-K)
R134a	$H_2FC\text{-}CF_3$	102.03	1206	−101	−26.6	215.9	0.853
Propane	C_3H_8	44.096	582	−187.7	−42.1	425.31	1.701
Isopentane	C_5H_{12}	72.15	626	−160	28	344.4	2.288

EFFICIENCY: For 1 kg of geothermal water/s and a 1.5 kW compressor

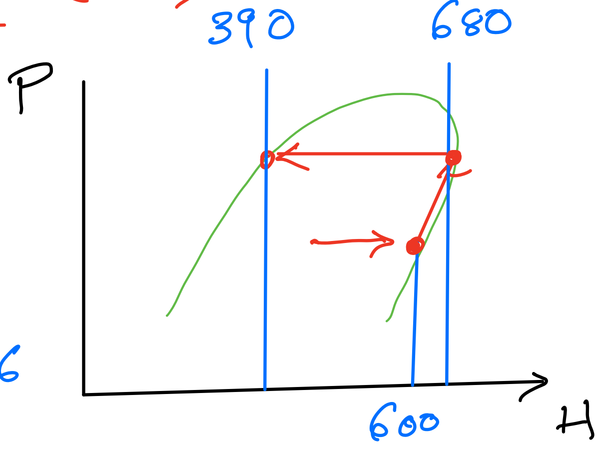
$$\frac{E_{total}}{E_{consumed}} = \frac{4180\ \text{J/s} + 0.8 \times 1500\ \text{J/s}}{1500\ \text{J/s}}$$

$$\sim 3.6$$

COEFFICIENT OF PERFORMANCE (COP)

$$COP = \frac{\text{Delivered heat}}{\text{Comp. elec. demand}}$$

$$= \frac{680 - 390\ \text{kJ/kg}}{680 - 600\ \text{kJ/kg}} \simeq 3.6$$



Note – assumes 100% efficency of compressor (not 0.8).

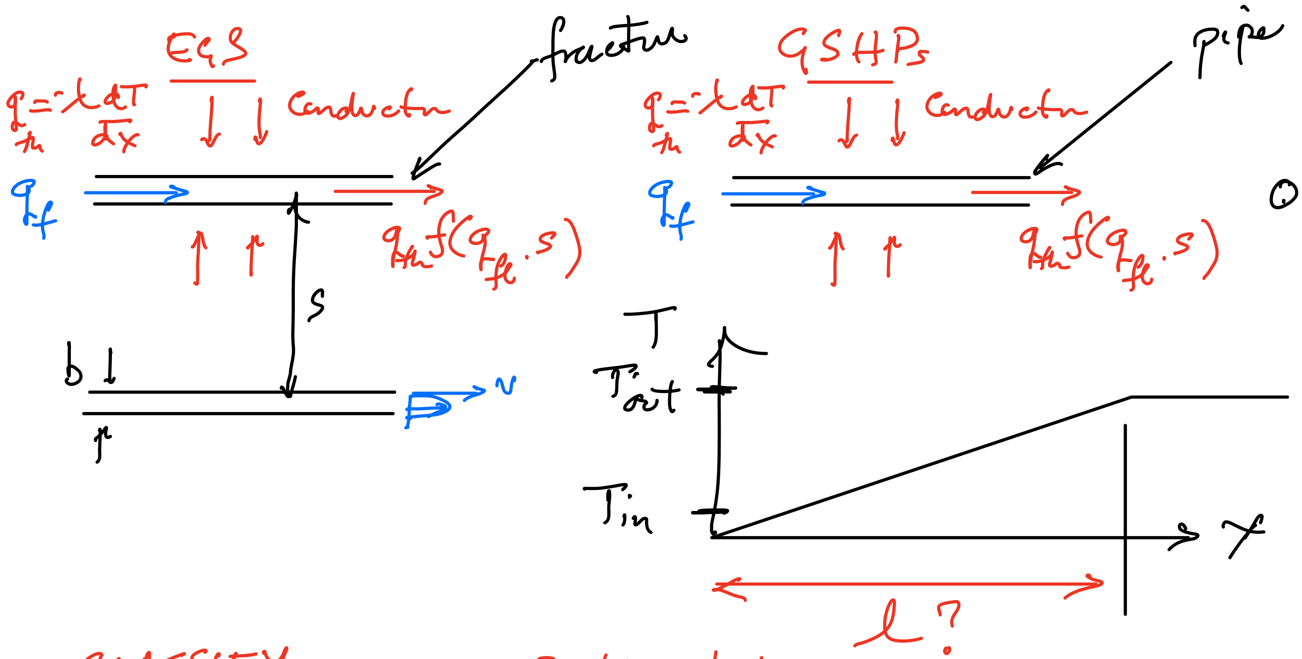
GSHPs

1. Mechanisms of heat flow in the shallow earth?
2. Mechanism of utilizing low quality heat -> high(er) quality heat?
3. Rate-limiting processes?

HEAT FLOW IN GEOTHERMAL (WATER) SYSTEM.

Similar to EGS – Rate Limited

Engineered – Engineered (constructed).



GLASSLEY

C_H = Building heating load ~ 12kW (not elec load of 1kW).

Heating loop:

$$L_H(\mathrm{m}) = \frac{\{(C_H) \times [(\mathrm{COP}-1)/\mathrm{COP}] \times [R_P + (R_S \times F_H)]\}}{(T_L - T_{min})} \qquad (11.4)$$

For a cooling loop, the corresponding equation is

Energy Efficiency Ratio ≡ COP cooling

$$L_C(\mathrm{m}) = \frac{\{(C_C) \times [(\mathrm{EER}+3.412)/\mathrm{EER}] \times [R_P + (R_S \times F_C)]\}}{(T_{max} - T_H)} \qquad (11.5)$$

where:

R_P is the resistance to heat flow of the pipe (which is equivalent to 1/thermal conductivity of the pipe)

R_S is the resistance to heat flow of the soil (which is equivalent to 1/thermal conductivity of the soil)

F_H (F_C) is the fraction of time the heating (cooling) system will be operating

T_L (T_H) is the minimum (maximum) soil temperature at the depth of installation

T_{min} (T_{max}) is the minimum (maximum) fluid temperature for the selected heat pump

Check units:

$COP \doteq -$

$T \doteq K$

$R \doteq 1/(W/(m.K)) \doteq \frac{m.K}{W}$

$L_H(m) \doteq \frac{\cancel{W}(-) \times m\cancel{K}/\cancel{W}}{\cancel{K}} \doteq m$ ✓

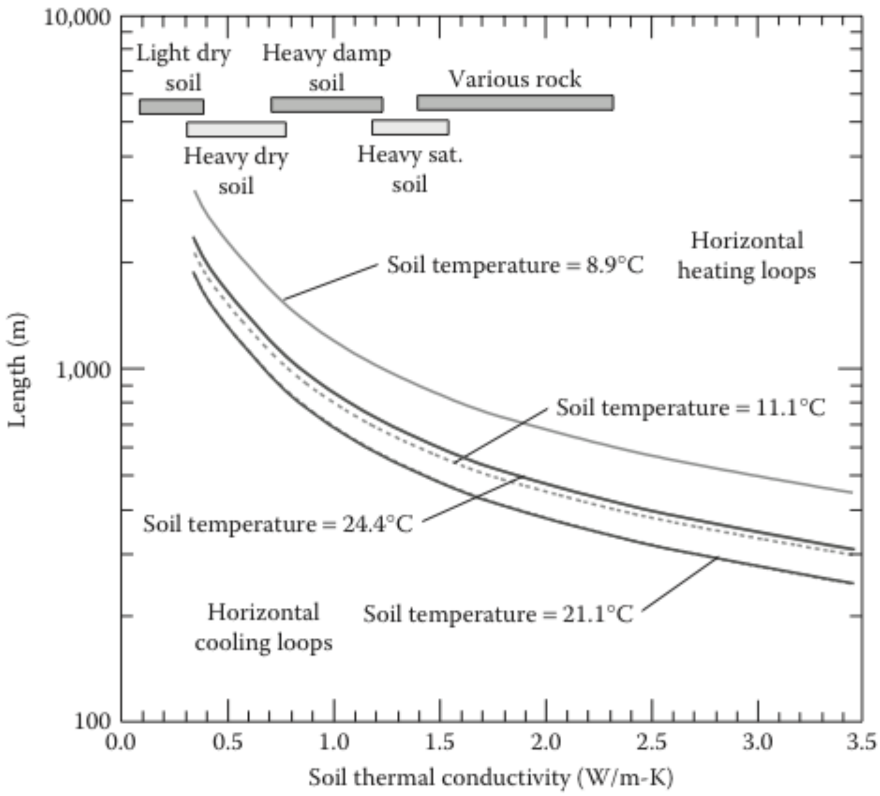
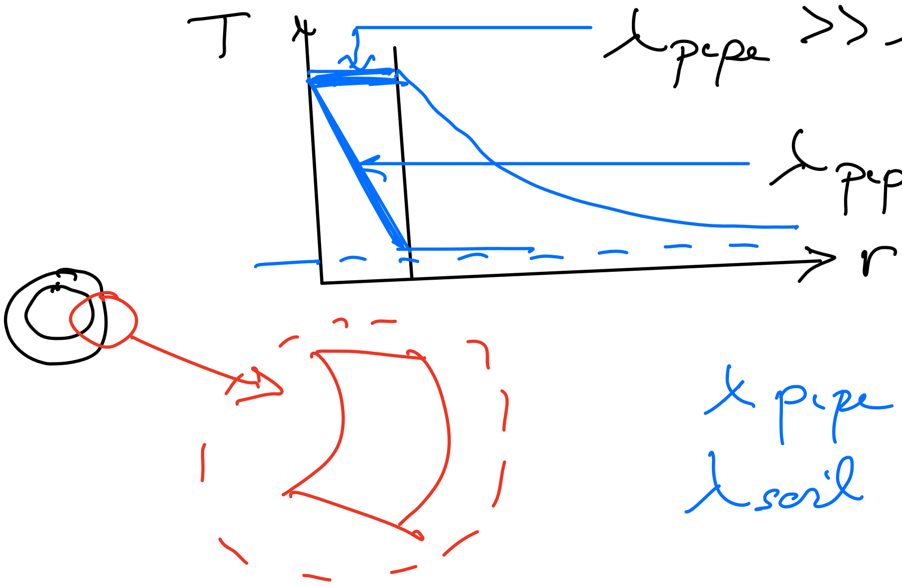


FIGURE 11.7 Computed loop length for heating and cooling, closed-loop GHP systems. In these calculations, it was assumed that the COP of the heat pump was 3.24 and the EER was 7.8. Pipe thermal conductivity was assumed to be 14.8 W/m-K, the heating and cooling run time fractions were 0.5 and 0.6, respectively, and the heat pump fluid T_{max} and T_{min} were 37.8°C and 4.4°C, respectively. For reference, the range of thermal conductivities for light, dry soil (Light dry soil); heavy, dry soil (Heavy dry soil); heavy, damp soil (Heavy damp soil); heavy, saturated soil (Heavy sat. soil); and crystalline rocks (Various rock) are also shown.

Suggests a length of ~ 1000m !!

RATIONAL BASIS FOR DESIGN



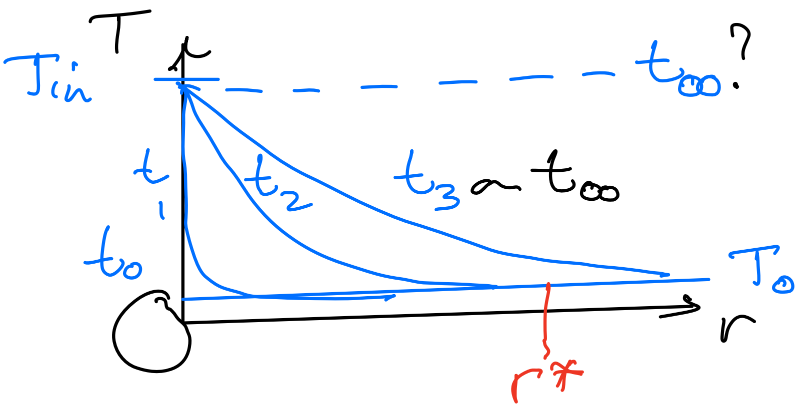
$\lambda_{pipe} >> \lambda_{soil}$ ✓

$\lambda_{pipe} << \lambda_{soil}$ ✗

$\lambda_{pipe} \sim 15$ W/mK ✓

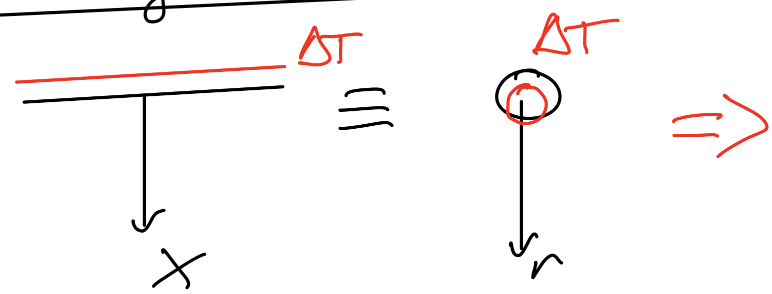
$\lambda_{soil} \sim 1$ W/mK.

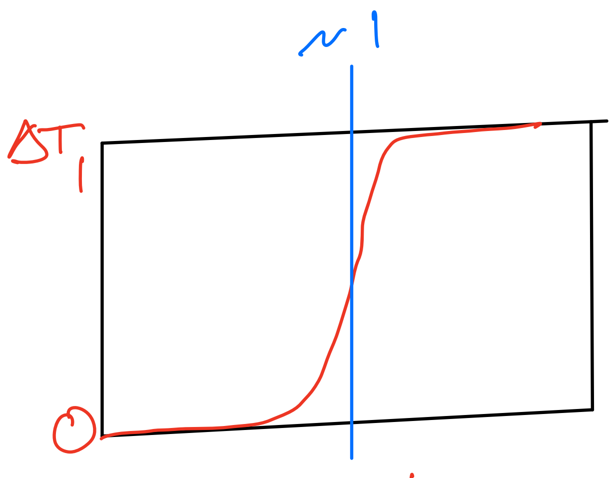
STEADY STATE OF TRANSIENT BEHAVIOR?



(to first order)

Analogous to:





$$t_D = \frac{Kt}{r^{*2}}$$

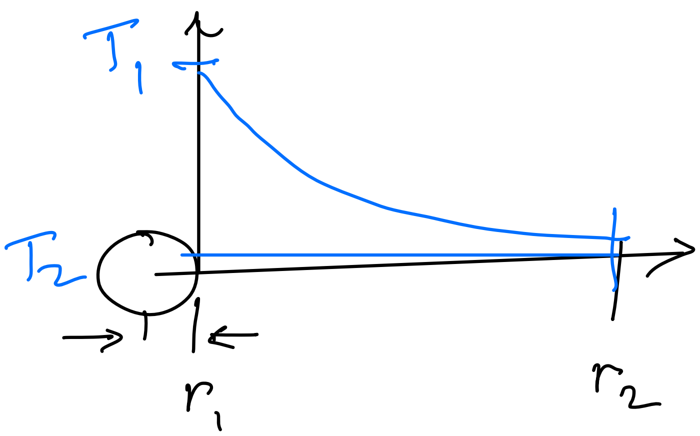
STEADY HEAT FLOW AROUND PIPE

1. What does it look like
2. What is heat flux to pipe?
3. How long does it take (more than $\frac{1}{2}$y?)

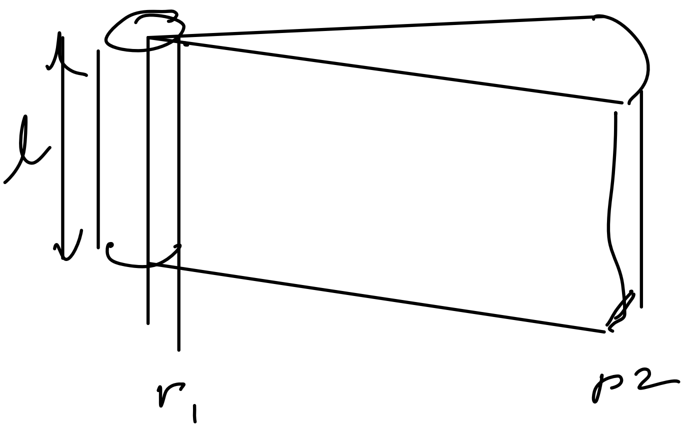
Same as water flow to well

Darcy's Law $\quad q = -K\frac{dh}{dr} \quad$ or $\quad -\frac{k}{\mu}\frac{dp}{dr}$

Fourier's Law $\quad q = -\lambda\frac{dT}{dr}$.



$$Q_f = 2\pi l \frac{k}{\mu}\frac{(P_2-P_1)}{\ln(r_2/r_1)}$$



$$Q_{th} = 2\pi l \lambda\frac{(T_2-T_1)}{\ln(r_2/r_1)}$$

This gives steady flux.

But – 1. Is steady reached?

2. If so. What is r_2?

IS STEADY FLUX REACHED?

$$t_D \sim 1 = \frac{Kt}{r_2^2} \longrightarrow r_2 \approx \sqrt{Kt}$$

$10\pi \, m^2/yr$ $\quad \frac{1}{2} yr.$

$$r_2 \sim \sqrt{16} \sim \underline{4m}$$

Steady flow to pipe if $r_2 \sim 1m$

$r_1 \sim 1cm = 10^{-2} m$

Thermal load for house? $\sim 10 kW$

(not 1 kW for elec)

$$\underline{Q = 10^4 W}$$

Soil conductivity $\sim \lambda = 1 \, W/(m.K)$

Temperature drop $\sim 10°c = \Delta T.$

$\ln(4/10^{-2}) \sim 6.0$

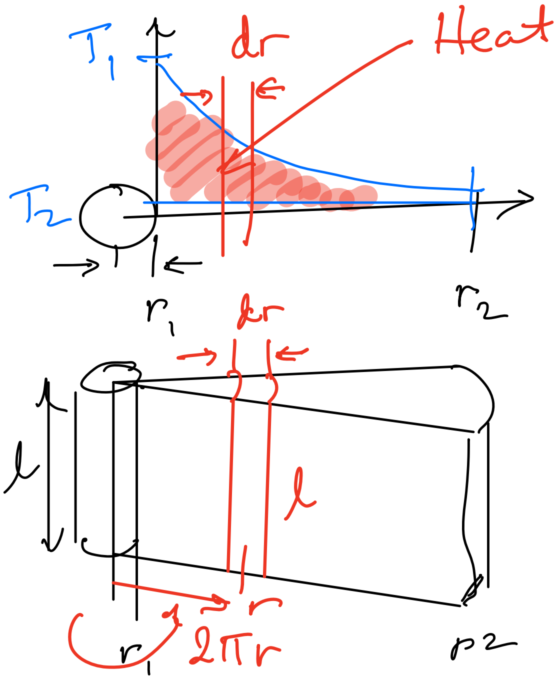
$$\boxed{Q_{th} = 2\pi l \frac{\lambda (T_2 - T_1)}{\ln(r_2/r_1)}}$$

$$l = \frac{Q \ln(r_2/r_1)}{2\pi \cdot \lambda \cdot \Delta T}$$

$$l = \frac{10^4 W \cdot \cancel{6}}{\cancel{2\pi} \; 1 \, W/(mK) \cdot 10 K}$$

$l \sim 10^3 m$ QED.

HOW MUCH HEAT RELEASED FROM STORAGE IN GETTING TO STEADY STATE?



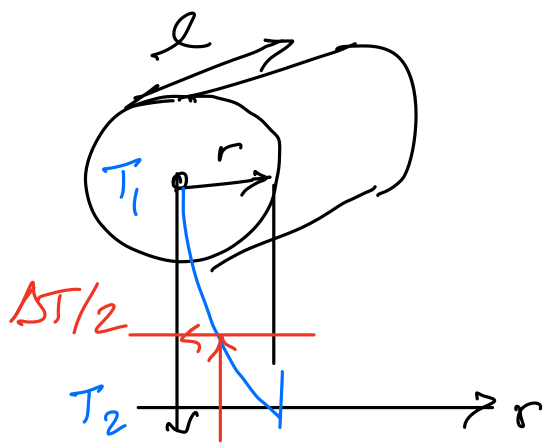
$W = \forall \rho c T$

$$W = \int \frac{2\pi r\,dr\,l}{\forall} \rho c T$$

recover fn $T = f(r)$

i.e. set Q = known fn (r_1/r_2) then determ. for r_3.

Estimate:



$10°C/2$

1000m

$$W = \forall \rho c T \doteq \pi r^2 \cdot l \cdot \rho c \cdot T$$

4m   2000kg/m³   1000J/kg·K

$$W = (50\,m^2)(1000\,m)(2000\,kg/m^3) \times (1000\,J/kg\cdot K) \times 5\,°C$$

$$W = 10^2\ 10^3\ 10^3\ 10^3 \times 5 = 5\times 10^{11}\,J \text{ in } \tfrac{1}{2}y.$$

$$= \underline{5\times 10^{11}}\,J \text{ in } \tfrac{1}{2}y$$

$$\tfrac{1}{2}y = \sim 10^5\,s \times 100\,d \cong 10^7\,s$$

$$\text{Power is } \frac{W}{1/2\,y} = \frac{5\times 10^{11}\,J}{10^7\,s} \sim 5\times 10^4\,\frac{J}{s}$$

Power from storage = 50 kW

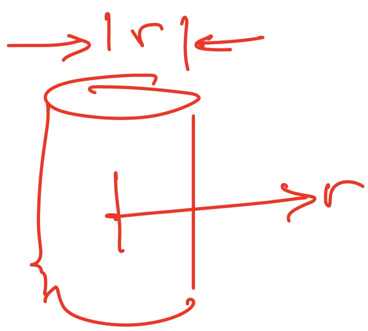
Power for steady state = $\underline{10\,kW}$

Assumed house load for steady state

<u>Conclusion</u>: Radius of cooling in soil less than 4 m. since storage supplies × 5 of steady flow

QED.

How long to cool a can of seltzer or cook a steak?



$$t_D = \frac{Kt}{r^2} \sim 1$$

$$K = \frac{\lambda}{\rho c} \sim \frac{0.5\ W/(m.K)}{10^3\ kg/m^3 \cdot 4.1\ kJ/kg \cdot K}$$

$$K \sim 0.1 \times 10^{-6}\ m^2/s$$

$$t \sim (r^2/K) \sim (2\times10^{-2}m)^2/0.1\times10^{-6}$$

$$\sim \frac{4\times10^{-4}}{0.1\times10^{-6}} \sim 40\times10^{2}\ s$$

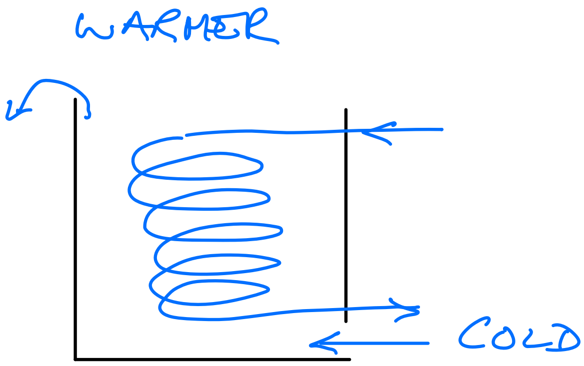
$4000s \sim 60\ \text{min} \sim 1\ \text{hr.}$

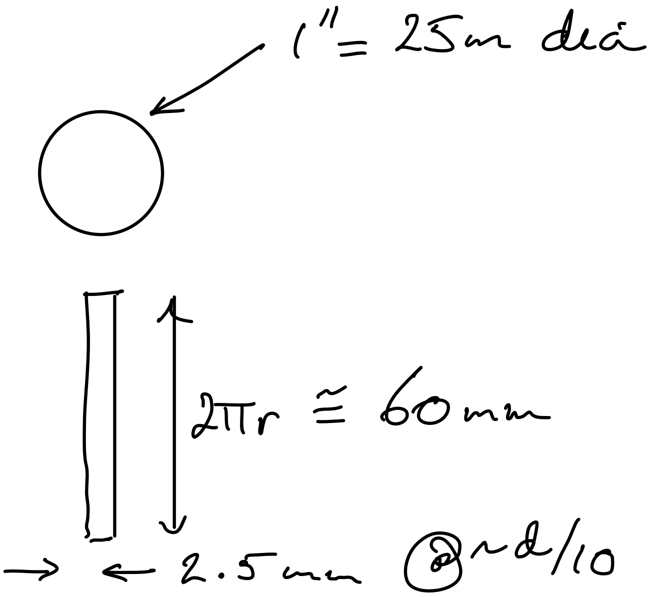
Note scaling with r^2

Cooking a steak. Double the thickness

Quadruple the cooking time

USE OF B/H HEAT EXCHANGER OR CRYPT





$$\overset{10\,kW}{\cancel{Q}} = A\,\lambda\,\frac{dT}{dy} = \frac{L \times (60\times10^{-3}\,m)\ 1\,.\,10}{2.5\times10^{-3}\,m}$$

$$\frac{10^4 .\ (2.5\times10^{-3}\,m)}{(60\times10^{-3}\,m)\times 1 \times 10} \doteq \frac{\cancel{N}.\cancel{m}}{\cancel{s}}\ \frac{\cancel{m}\ \cancel{K}\ m}{\cancel{m}\ \cancel{Nm/s}\ .\ \cancel{K}}$$

$$L \cong 10^4 \left(\frac{\overset{\sim 1/20}{\cancel{2.5}}}{\cancel{60}}\right) \frac{1}{10}$$

$$L \sim \frac{10^3}{20} \sim 50m \quad \text{QED.}$$

Volume of water needed to maintain temp

is. $10kW = 10^4 W = \dot{M}\,c\,\Delta T$

$$\dot{M} \sim 10^4\,J/s/(c\,\Delta T) \sim 10^4/(4.1\times10^3).10$$

$\dot{M} \sim 0.25 \; kg/s$

$10^5 \; secs \doteq 0.25 \times 10^5 \; kg \equiv 25 \times 10^3 \; kg/d.$