Extended Finite Element Method
X-FEM

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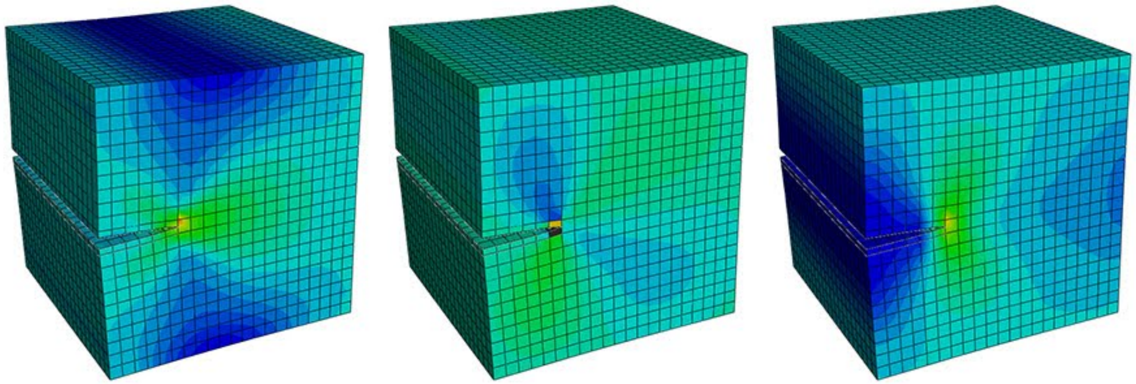
Introduction



Introduction

➢Finite Element Method (FEM)

- A numerical method for solving problems of engineering and mathematical physics.
- Application area: structural analysis, heat transfer, fluid flow, mass transport , and electromagnetic potential.
- Cracks can only propagate along the element rather than natural path.

➢Extended Finite Element Method (XFEM)

- A numerical technique based on the generalized finite element method (GFEM) and the partition of unity method (PUM).
- Developed in 1999 by Ted and collaborators.
- Powerful for discontinuous problems in mechanics, such as: crack growth, complex fluid, interface and so on.
- Independent of the internal geometry and physical interfaces, such that meshing and re-meshing difficulties in discontinuous problems can be overcome.

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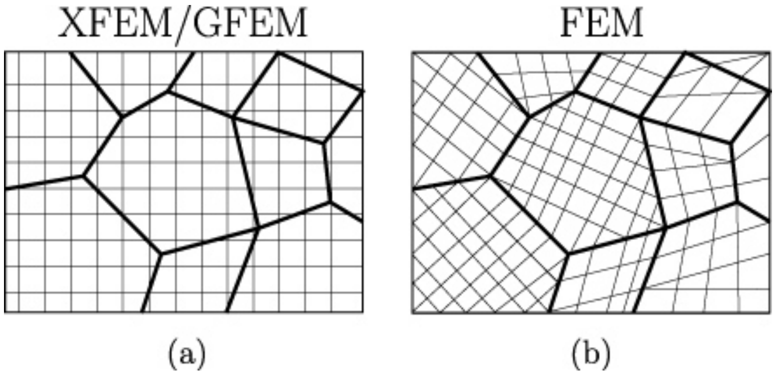
Introduction

➢XFEM vs. FEM

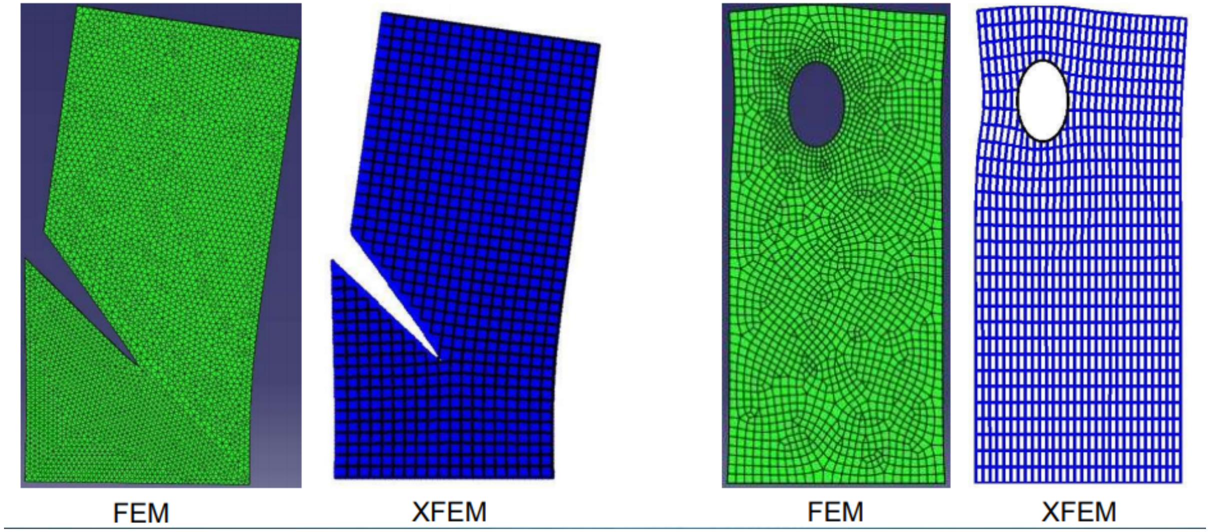
Allows simulation of initiation and propagation of a crack along an arbitrary path without the requirement of remeshing.



Introduction

➢XFEM vs. FEM

XFEM achieves this by locally enriching the FE approximation with local partitions of unity enrichment functions.



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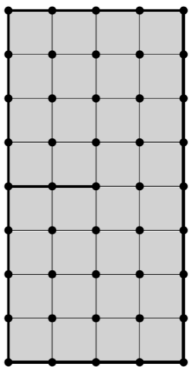
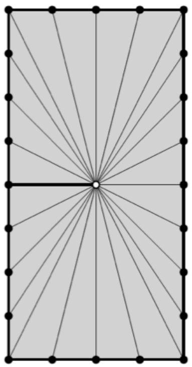
Introduction

➢Conceptual comparison of other methods



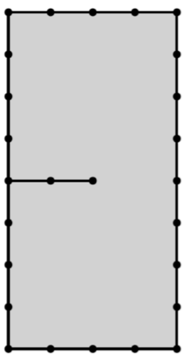
FEM:

- High amount of DOF
- Crack surface discretized
- Discretization error in all directions



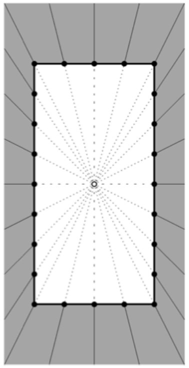
SBFEM bounded domain:

- Introduction of a scaling center
- Discretization on boundary only
- Crack surface not discretized
- Analytical solution in radial direction



BEM:

- Discretization on boundary only
- Crack surface discretized
- non-symmetric dense matrices



SBFEM unbounded domain:

- Introduction of a scaling center
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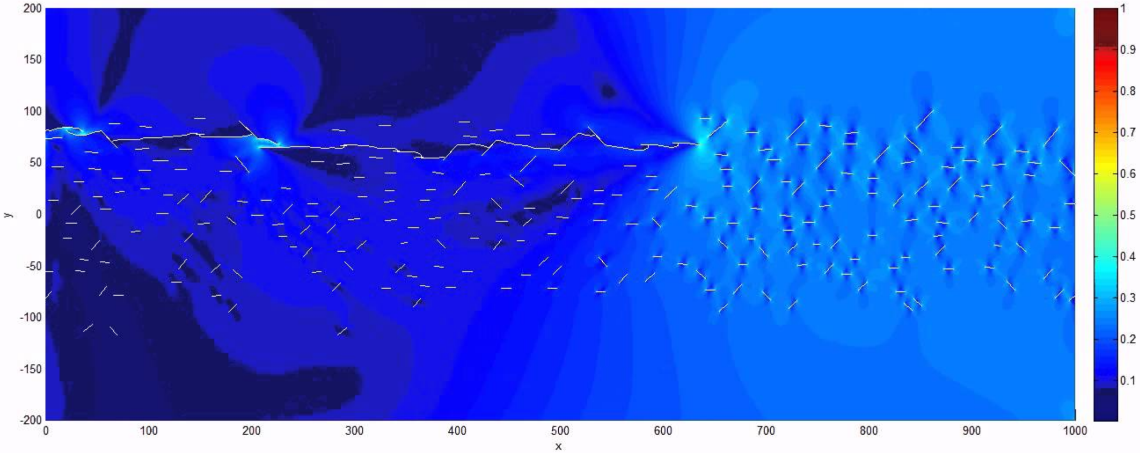
Introduction

➤Scope of application of XFEM

propagation of a few hundred cracks in a brittle material by the extended finite element method.



Source: https://www.youtube.com/watch?v=fuikZx71MhU

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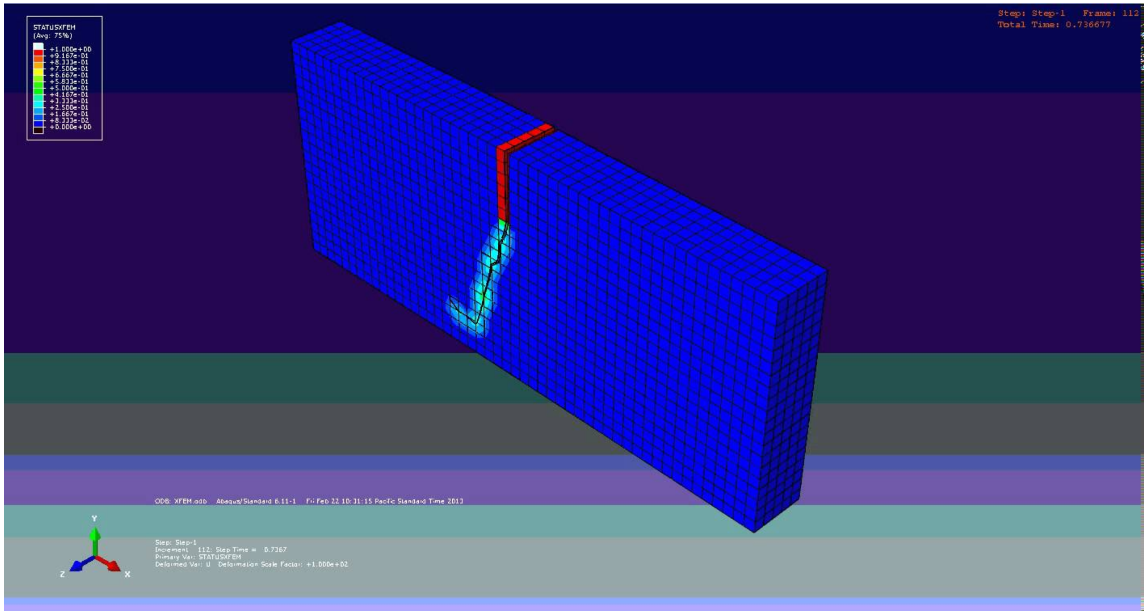
Introduction

➢Scope of application of XFEM

Crack growth problem



Source: https://www.youtube.com/watch?v=G6lFUN3jwok

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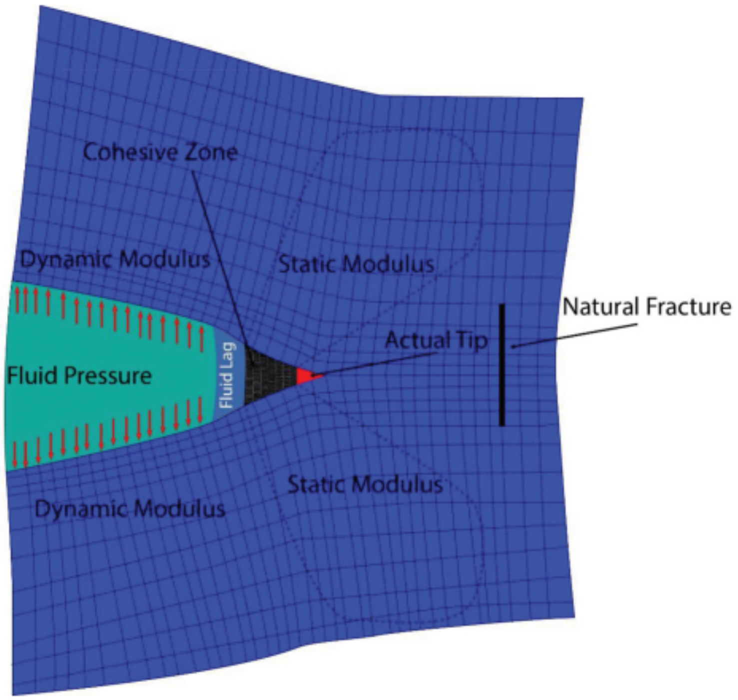
Introduction

➢Scope of application of XFEM

- Assuming the cohesive zone within the propagated hydraulic fracture, three distinct zones will contribute in the fracturing stage, which are the fully opened zone, partially damaged zone and non-damaged zone.

- The fully opened zone is the section that fully separates the upper and lower parts of the crack from the fluid flow. The partially damaged zone or process zone is located around the crack tip where the total stress acts to this zone lower than the critical stress



Source: https://www.intechopen.com/books/fracture-mechanics-properties-patterns-and-behaviours/analysis-of-interaction-between-hydraulic-and-natural-fractures

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Introduction

➢Advantages of Extended Finite Element Method (XFEM)

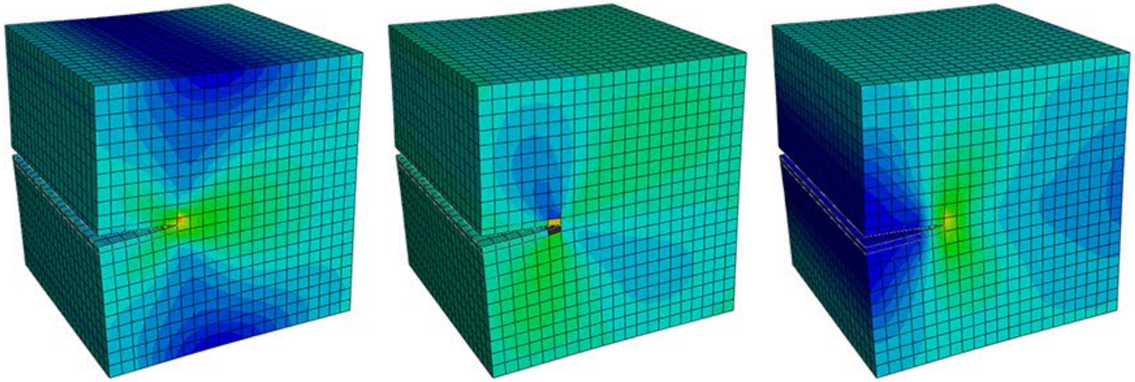
- Ease difficulties in solving problems with localized features that are not efficiently resolved by mesh refinement.
- Allows simulation of initiation and propagation of a crack along an arbitrary path without the requirement of remeshing.
- Save computational cost significantly.
- Convenient to implement in commercial software and with parallel computing.
- The description for the discontinuous field is entirely independent of mesh.
- Enriched elements with additional degrees of freedom at crack surface and crack tips.
- Not only simulate cracks, but also heterogeneous materials with voids and inclusions.

Introduction

➢ Shortcomings of Extended Finite Element Method (XFEM)

- Useful for linear elastic materials.
- Time stepping needs to be small enough to capture crack propagation.
- Hard to localize the initial fracture.

Historical Perspective



Historical Perspective

◆Belytschko and Black (1999)

- In order to solve the problem of the presence of crack, they use the minimal remeshing finite element method.
- Adding discontinuous enrichment function.
- This method allows the crack to be arbitrarily aligned within the mesh.

◆Moës et al. (1999) and Dolbow (1999)

- An improvement of a new technique for modelling cracks in the finite element framework is presented.
- eXtended Finite Element method (XFEM).
- This technique allows the entire crack to be represented independently of the mesh, and so remeshing is not necessary to model crack growth.

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Historical Perspective

◆Sukumar et al. (2000)

- The computational efficiency of algorithm in 2D, which is expected to carry over to 3D.
- Adding discontinuous enrichment function.
- The ability to exactly impose essential boundary conditions on the boundaries of convex and non-convex domains.

◆Stolarska et al. (2001)

- A new method for level set update is proposed, in the context of crack propagation modeling with the extended finite element method (X-FEM) and level set.

◆Wagner et al. (2001, 2003)

- Apply XFEM to the simulation of particulate flows.

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Historical Perspective

◆Chessa et al. (2003)

- Presenting a finite element method for axisymmetric two phase flow problem.

◆Fries et al. (2006)

- Improve X-FEM and it is proposed for arbitrary discontinuities, without the need for a mesh that aligns with the interfaces, and without introducing additional unknowns as in the XFEM.

◆Wagner et al. (2001, 2003)

- Apply XFEM to the simulation of particulate flows.

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Historical Perspective

◆Latest Improvement

- To eliminate the linear dependence and the ill-conditioning issues of the standard and the corrected XFEMs;
- To get rid of extra degree of freedom in crack tip enrichment to facilitate optimal mass lumping in dynamic analyses;
- To be interpolating at enriched nodes to enable direct essential/contact boundary treatments.

XFEM – General Principles

- X-FEM is a numerical modelling technique that involves local enrichment of approximation spaces based on the **Partitioning Of Unity Concept**.
- The enriched approximation can be written as

$$u(x) = \sum_{i=1}^{N} N_i(x)\bar{u}_i + \quad \textbf{enrichment terms}$$

- **Enrichment terms are added to increase the accuracy of the displacement function – u(x)**

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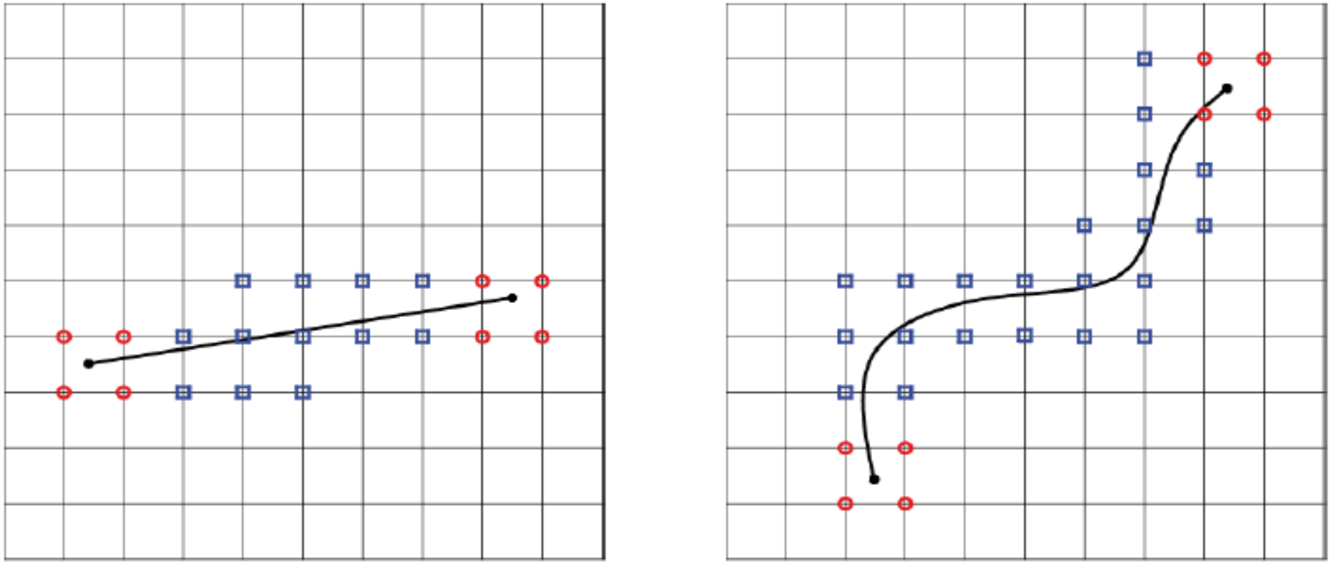
Example of Nodal Enrichment

- **enrichment is done at the nodes where discontinuities exist.**



uniform mesh grid where nodes are enriched **source: (Chatzi, 2018)**

Enrichment of Approximation Space

- Enrichment is of two types **: –**

  Intrinsic Enrichment – **basis vector is enriched**

  Extrinsic Enrichment – **approximation space is enriched**

Intrinsic Enrichment

- Here, the approximation space is enhanced by **including new basis functions into the standard approximation space** to capture the discontinuity.

- The new basis function becomes $\hat{N}_i(x) = [N^{std}(x), N^{enr}(x)]$

- Our approximation field is therefore $u(x) = \sum_{i=1}^{P} \hat{N}_i(x)\bar{a}_i$

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Extrinsic Enrichment

- Enrichment functions are added to the standard approximation. Therefore the enhanced solution field in the X-FEM becomes

$$u(x) = \sum_{i=1}^{N} N_i(x)\bar{u}_i + \sum_{j=1}^{M} N_j(x)\Psi(x)\bar{a}_j$$

For unit discontinuities

$$u(x) = \sum_{i=1}^{N} N_i(x)\bar{u}_i + \sum_{k=1}^{P} \sum_{j=1}^{M} N_j(x)\Psi_k(x)\bar{a}_k$$

For multiple discontinuities

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Enrichment Functions

- Some functions used for enrichment include but not limited to –

Level Set Function	–	for modelling weak discontinuities
Signed Distance Function	–	for modelling weak discontinuities
Heaviside Function	–	for modelling strong discontinuities

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Level Set Function (LSF)

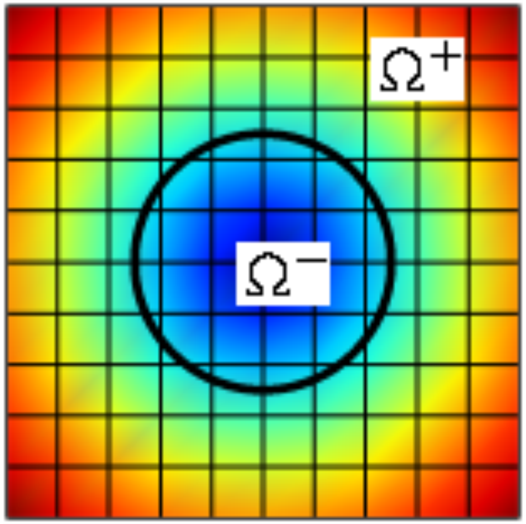
- LSF is a scalar function within a domain whose zero level is interpreted as a discontinuity.
- For a 2D domain with circular discontinuity of radius r around (0,0) .
- The discontinuity defined by level set function is given as

$$\phi(x, y) = \sqrt{x^2 + y^2} - r$$

which is a) zero on the circle (i.e. discontinuity)

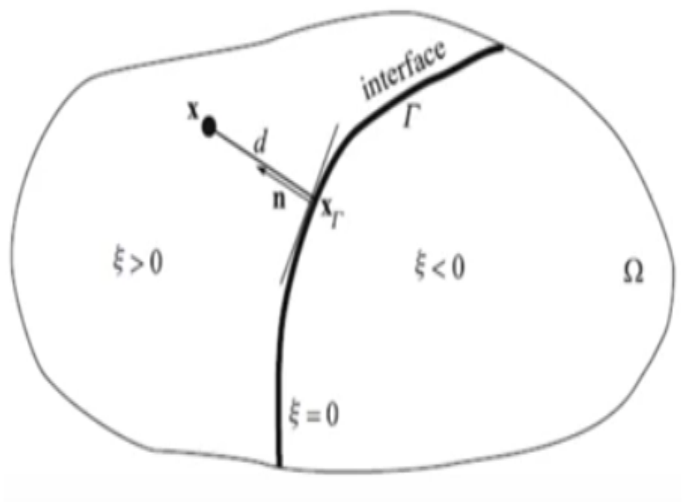
b) negative within the discontinuity

c) positive outside the discontinuity



Level Set function **(Fries, 2018)**

Signed Distance Function



Signed distance function

The signed distance function $\xi(x)$, is given as

$$\xi(x) = ||x - x_\Gamma|| \, \text{sign}(n.(x - x_\Gamma))$$

Which results in a solution where –

$\xi(x) > 0$ if x lies on the same side as the normal vector, n

$\xi(x) = 0$ if x lies on interface, Γ

$\xi(x) < 0$ if x lies opposite the direction of n

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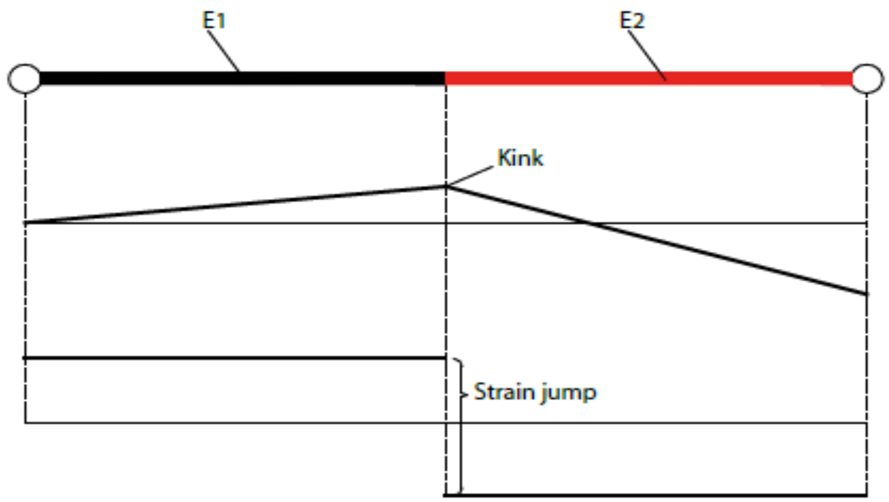
Signed Distance Function

- Weak discontinuities (e.g. difference in strain) results in kinks in the displacements (jumps in strains) as for example, biomaterial problems as shown

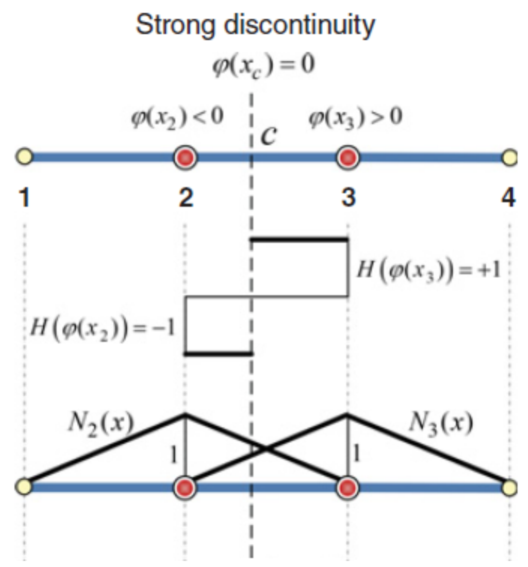


Bimaterial Bar **(Chatzi, 2018)**

Heaviside Function

- The Heaviside function, however, is used to model strong discontinuities – such as cracks or faults – where there is a jump in the displacement field.
- Heaviside function, **H(x)**, is given as follows –

$$H(x) = \begin{cases} -1 \ \textbf{if} \ \xi(x) < 0 \\ +1 \ \textbf{if} \ \xi(x) > 0 \end{cases}$$



Cracked bar **(Khoei, 2015)**

Crack Tip Enrichment

- The crack tip enrichment function (F_α) expressed in polar coordinates is given as –

$$F_\alpha = \{\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2}\}$$

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Modelling the Crack

- The displacement field upon encountering a crack is equal to the sum of the following individual displacement fields

♣**Classical field**

♣**Crack split**

♣**Crack tip**

Which is mathematically expressed as –

$$u(x) = N^{std}(x)\bar{u} + N^{Hev}(x)\bar{d} + N^{tip}(x)\bar{d}$$

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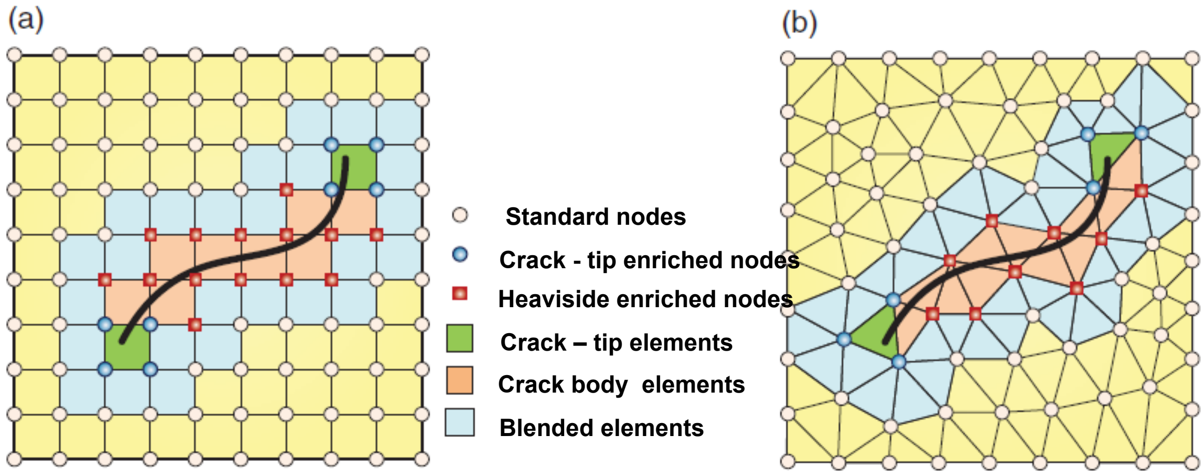
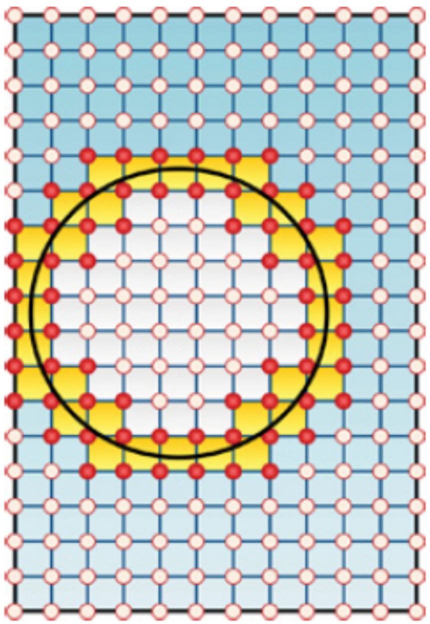
Modelling the Crack

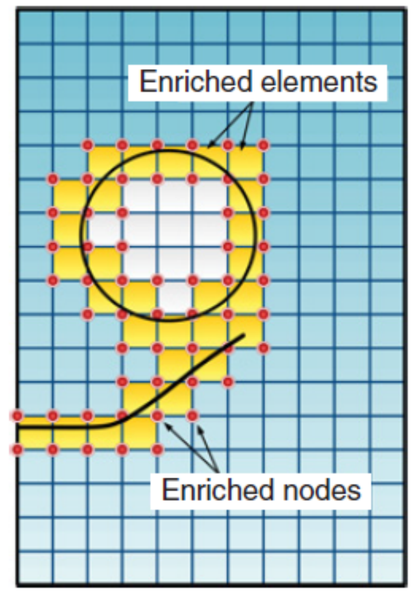


The X-FEM modeling of a cracked body with the Heaviside and crack tip enrichment functions: (a) Quadrilateral mesh; (b) Triangular mesh **(Khoei, 2015)**

Modelling Discontinuities



(a) A body with bimaterial interface



(b)A body with bimaterial interface and a crack **(Khoei, 15)**

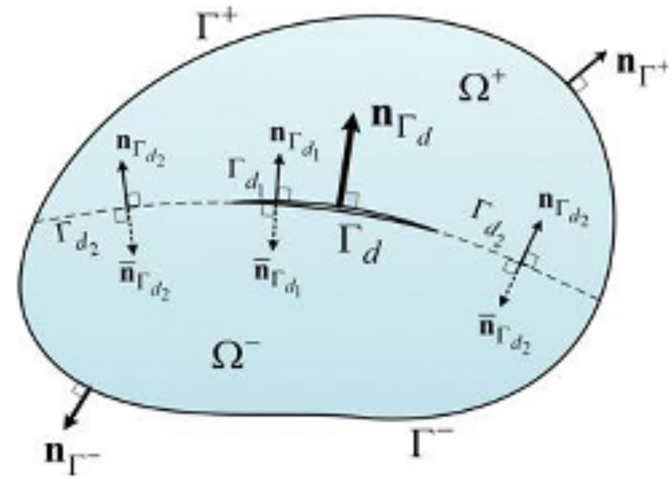
Governing Equation

Equilibrium Equation

- $\nabla \cdot \sigma + b = 0$

Boundary Conditions

- Displacement: $u = u_i$ on Γ_u
- Traction: $\sigma \cdot n_\Gamma = t$ on Γ_t
- Internal or Crack: $\sigma \cdot n_{\Gamma d} = t$ $on\, \Gamma_d$



- Γ_u : displacement boundary
- Γ_t : traction boundary
- Γ_d : internal boundary
- σ: stress tensor
- b: body force
- t: external traction

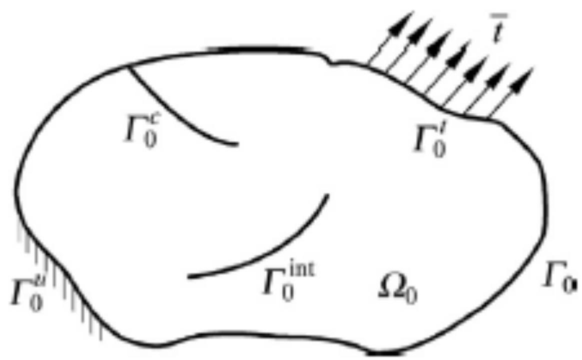
Governing Equation

Conservation of Energy

- Conservation of Momentum: $\partial P_{ji} / \partial X_j + \rho_0 b_i = \rho_0 u_i$
- Kinematics Equation by Green Strain: $E_{ij} = 1/2 \, (F_{ki} \cdot F_{kj} - \delta_{ij})$

Boundary Conditions

- $u_i = u_i$ on Γ_0^t
- $n_j^0 P_{ji} = t_i^0$ $on \, \Gamma_0^t$
- $u_i(0) = u_{i0}$
- $P_{ij}(0) = P_{ij0}$



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Governing Equation

Displacement Field

- $u(X) = \sum_{i} N_i(X) u_i + \sum_{j} \Psi(X) a_j$
- $\delta u = \sum_{i} N_i(X) \delta u_i + \sum_{j} \Psi(X) \delta a_j$

Weak Form (from Conservation of Momentum)

- $\int \delta u_i \left(\partial P_{ji} / \partial X_j + \rho_0 b_i - \rho_0 u_i \right) d\Omega_0 = 0$

Expanding the Derivative of the Product of 1st Term

- $\int \delta u_i \, \partial P_{ji} / \partial X_j \, d\Omega_0 = \int \partial / \partial X_j \left(\delta u_i P_{ji} \right) d\Omega_0 - \int \partial(\delta u_i) / \partial X_j \, P_{ji} \, d\Omega_0$
- $\int \partial / \partial X_j \left(\delta u_i P_{ji} \right) d\Omega_0 \cong \int \delta u_i \, t_i^0 \, d\Gamma_0$

Governing Equation

Assumptions

- No contact and friction between crack (discontinuous) surfaces
- $\int \left(\frac{\partial (\delta u_i)}{\partial X_j} P_{ji} - \delta u_i \rho_0 b_i + \delta u_i \rho_0 u_i \right) d\Omega_0 - \int \delta u_i t_i^0 \, d\Gamma_0 = 0$

Or More Generally

- $W_{int} = W_{ext}$
- $\int P_{ij} \, \delta \varepsilon \, d\Omega_0 = \int b \delta u \, d\Omega_0 + \int t \cdot \delta u \, d\Gamma_0$

Governing Equation

2D Discretization: $\mathrm{KU}=F$

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{ua} \\ \mathbf{K}_{au} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{a}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{F}_a \end{Bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \int_\Omega (\mathbf{B}^{std})^T \mathbf{D}\,\mathbf{B}^{std} d\Omega & \int_\Omega (\mathbf{B}^{std})^T \mathbf{D}\,\mathbf{B}^{enr} d\Omega \\ \int_\Omega (\mathbf{B}^{enr})^T \mathbf{D}\,\mathbf{B}^{std} d\Omega & \int_\Omega (\mathbf{B}^{enr})^T \mathbf{D}\,\mathbf{B}^{enr} d\Omega \end{bmatrix}$$

$$\mathbf{F} = \begin{Bmatrix} \int_{\Gamma_t} (\mathbf{N}^{std})^T \bar{\mathbf{t}}\, d\Gamma + \int_\Omega (\mathbf{N}^{std})^T \mathbf{b}\, d\Omega \\ \int_{\Gamma_t} (\mathbf{N}^{enr})^T \bar{\mathbf{t}}\, d\Gamma + \int_\Omega (\mathbf{N}^{enr})^T \mathbf{b}\, d\Omega \end{Bmatrix}$$

$$\mathbf{B}_i^{std} = \begin{bmatrix} \partial N_i/\partial x & 0 \\ 0 & \partial N_i/\partial y \\ \partial N_i/\partial y & \partial N_i/\partial x \end{bmatrix} \qquad \mathbf{B}_j^{enr} = \begin{bmatrix} \partial\left[N_j(\boldsymbol{x})\left(\psi(\boldsymbol{x})-\psi(\boldsymbol{x}_j)\right)\right]/\partial x & 0 \\ 0 & \partial\left[N_j(\boldsymbol{x})\left(\psi(\boldsymbol{x})-\psi(\boldsymbol{x}_j)\right)\right]/\partial y \\ \partial\left[N_j(\boldsymbol{x})\left(\psi(\boldsymbol{x})-\psi(\boldsymbol{x}_j)\right)\right]/\partial y & \partial\left[N_j(\boldsymbol{x})\left(\psi(\boldsymbol{x})-\psi(\boldsymbol{x}_j)\right)\right]/\partial x \end{bmatrix}$$

$$\mathbf{N}_i^{std} = \begin{bmatrix} N_i(\boldsymbol{x}) & 0 \\ 0 & N_i(\boldsymbol{x}) \end{bmatrix}$$

$$\mathbf{N}_j^{enr} = \begin{bmatrix} N_j(\boldsymbol{x})\left(\psi(\boldsymbol{x})-\psi(\boldsymbol{x}_j)\right) & 0 \\ 0 & N_j(\boldsymbol{x})\left(\psi(\boldsymbol{x})-\psi(\boldsymbol{x}_j)\right) \end{bmatrix}$$

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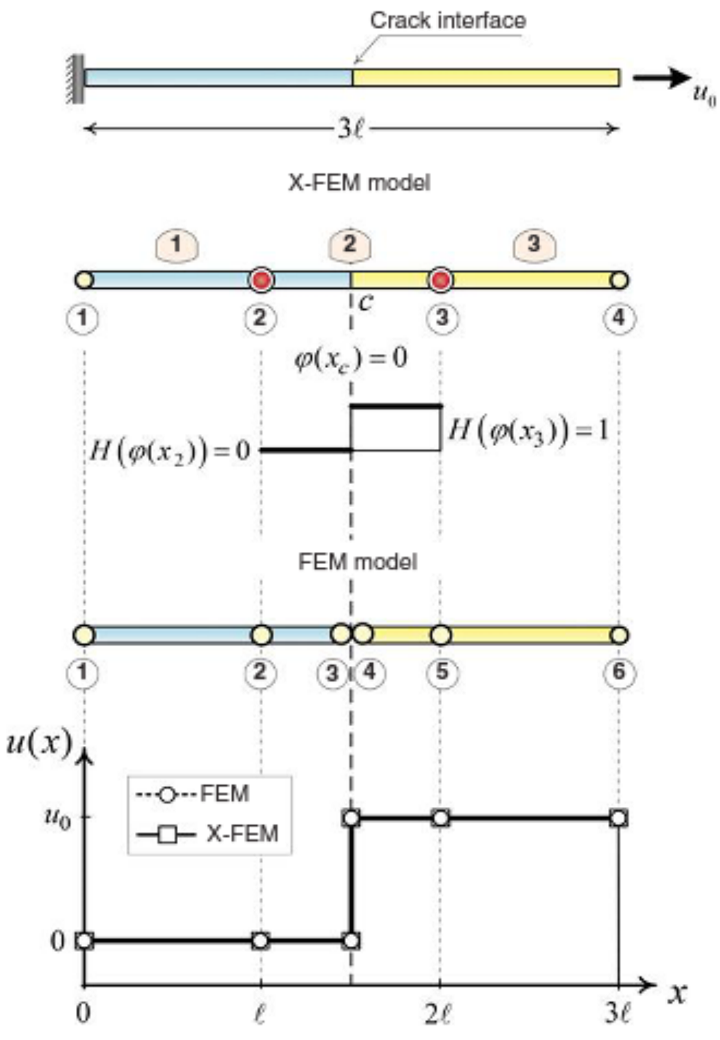
Hand Calculation

1D Bar Example

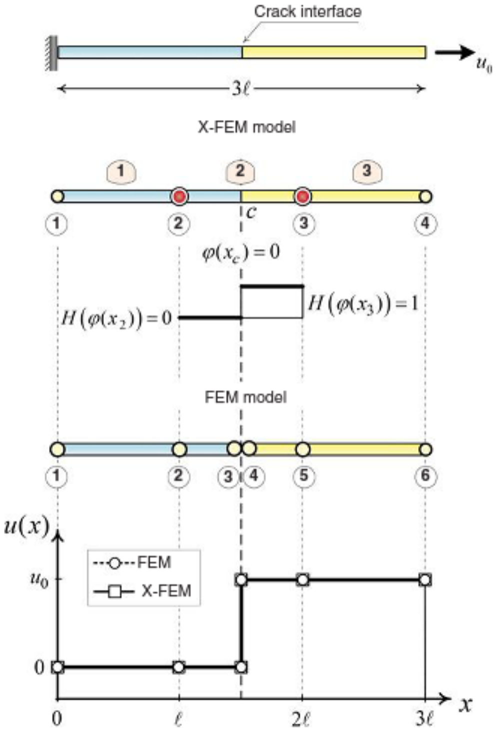


Hand Calculation

$$^{(1)}\mathbf{N}^{std}_{(x)} = \left\langle 1-\frac{x}{\ell} \;\; \frac{x}{\ell} \right\rangle, \quad ^{(1)}\mathbf{N}^{enr}_{(x)} = \frac{x}{\ell}(H(\varphi)-H(\varphi_2))$$

$$^{(1)}\mathbf{B}^{std}_{(x)} = \left\langle -\frac{1}{\ell} \;\; \frac{1}{\ell} \right\rangle, \quad ^{(1)}\mathbf{B}^{enr}_{(x)} = \frac{1}{\ell}(H(\varphi)-H(\varphi_2))$$

$$^{(1)}\mathbf{K} = \begin{bmatrix} \int_\ell (\mathbf{B}^{std})^T AE\, \mathbf{B}^{std} dx & \int_\ell (\mathbf{B}^{std})^T AE\, \mathbf{B}^{enr} dx \\ \int_\ell (\mathbf{B}^{enr})^T AE\, \mathbf{B}^{std} dx & \int_\ell (\mathbf{B}^{enr})^T AE\, \mathbf{B}^{enr} dx \end{bmatrix} = \frac{AE}{\ell}\begin{bmatrix} +1 & -1 & 0 \\ -1 & +1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ a_2 \end{matrix}$$

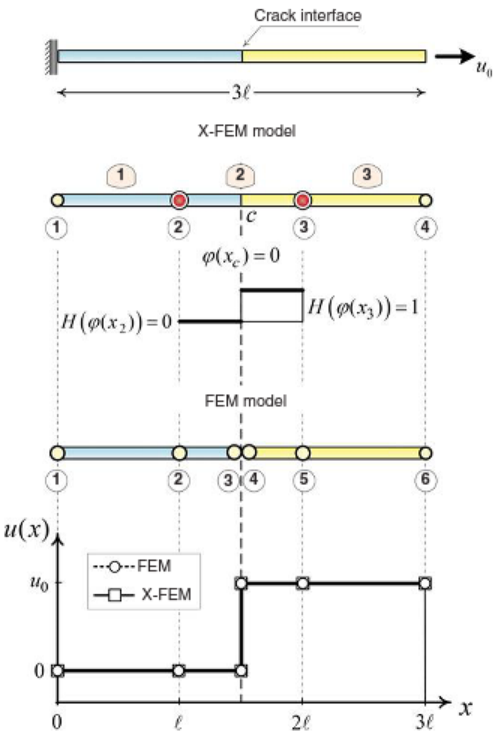


Hand Calcualtion

$$^{(2)}\mathbf{N}^{std}_{(x)} = \left\langle 1-\frac{x}{\ell} \quad \frac{x}{\ell} \right\rangle, \quad ^{(2)}\mathbf{N}^{enr}_{(x)} = \left\langle \left(1-\frac{x}{\ell}\right)(H(\varphi)-H(\varphi_2)) \quad \frac{x}{\ell}(H(\varphi)-H(\varphi_3)) \right\rangle$$

$$^{(2)}\mathbf{B}^{std}_{(x)} = \left\langle -\frac{1}{\ell} \quad \frac{1}{\ell} \right\rangle, \quad ^{(2)}\mathbf{B}^{enr}_{(x)} = \left\langle -\frac{1}{\ell}(H(\varphi)-H(\varphi_2)) \quad \frac{1}{\ell}(H(\varphi)-H(\varphi_3)) \right\rangle$$

$$^{(2)}\mathbf{K} = \begin{bmatrix} \int_\ell (\mathbf{B}^{std})^T AE\,\mathbf{B}^{std} dx & \int_\ell (\mathbf{B}^{std})^T AE\,\mathbf{B}^{enr} dx \\ \int_\ell (\mathbf{B}^{enr})^T AE\,\mathbf{B}^{std} dx & \int_\ell (\mathbf{B}^{enr})^T AE\,\mathbf{B}^{enr} dx \end{bmatrix} = \frac{AE}{\ell} \begin{bmatrix} 1 & -1 & 0.5 & 0.5 \\ -1 & +1 & -0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0 \\ 0.5 & -0.5 & 0 & 0.5 \end{bmatrix} \begin{matrix} u_2 \\ u_3 \\ a_2 \\ a_3 \end{matrix}$$

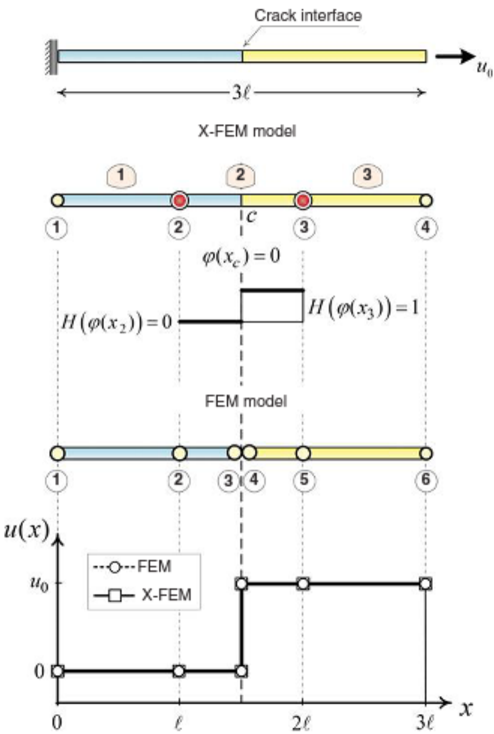


Hand Calcualtion

$$ {}^{(3)}\mathbf{N}^{std}_{(x)} = \left\langle 1-\frac{x}{\ell} \quad \frac{x}{\ell} \right\rangle, \quad {}^{(3)}\mathbf{N}^{enr}_{(x)} = \left(1-\frac{x}{\ell}\right)\left(H(\varphi)-H(\varphi_3)\right) $$

$$ {}^{(3)}\mathbf{B}^{std}_{(x)} = \left\langle -\frac{1}{\ell} \quad \frac{1}{\ell} \right\rangle, \quad {}^{(3)}\mathbf{B}^{enr}_{(x)} = -\frac{1}{\ell}\left(H(\varphi)-H(\varphi_3)\right) $$

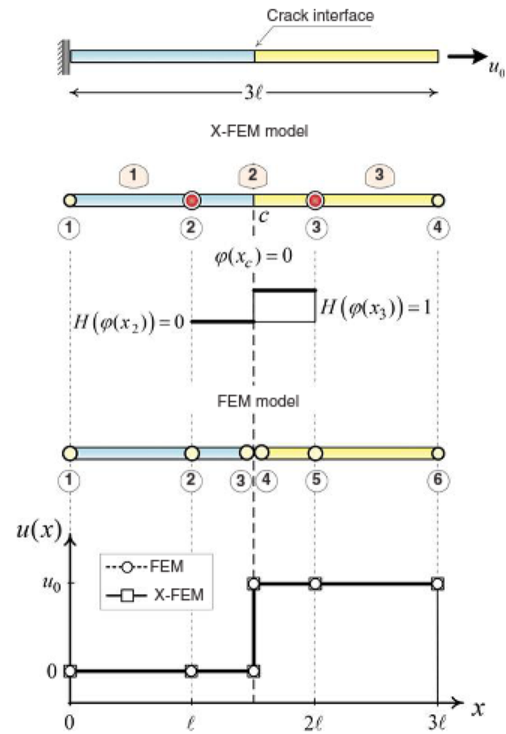
$$ {}^{(3)}\mathbf{K} = \begin{bmatrix} \int_\ell (\mathbf{B}^{std})^T AE\,\mathbf{B}^{std}dx & \int_\ell (\mathbf{B}^{std})^T AE\,\mathbf{B}^{enr}dx \\ \int_\ell (\mathbf{B}^{enr})^T AE\,\mathbf{B}^{std}dx & \int_\ell (\mathbf{B}^{enr})^T AE\,\mathbf{B}^{enr}dx \end{bmatrix} = \frac{AE}{\ell}\begin{bmatrix} +1 & -1 & 0 \\ -1 & +1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} u_3 \\ u_4 \\ a_3 \end{matrix} $$



Hand Calculation

$$\mathbf{K}^{\mathrm{XFEM}} = \frac{AE}{\ell}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0.5 & 0.5 \\ 0 & -1 & 2 & -1 & -0.5 & -0.5 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0.5 & -0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & -0.5 & 0 & 0 & 0.5 \end{bmatrix}\begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ a_2 \\ a_3 \end{matrix}$$

$$\begin{pmatrix} u_1 & u_2 & u_3 \\ u_4 & a_2 & a_3 \end{pmatrix} = \begin{pmatrix} u_0 & u_0 & u_0 & u_0 \end{pmatrix}$$



$$\mathbf{K}^{\mathrm{FEM}} = \frac{AE}{\ell}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & -2 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{matrix}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ u_0 \\ u_0 \\ u_0 \end{pmatrix}$$

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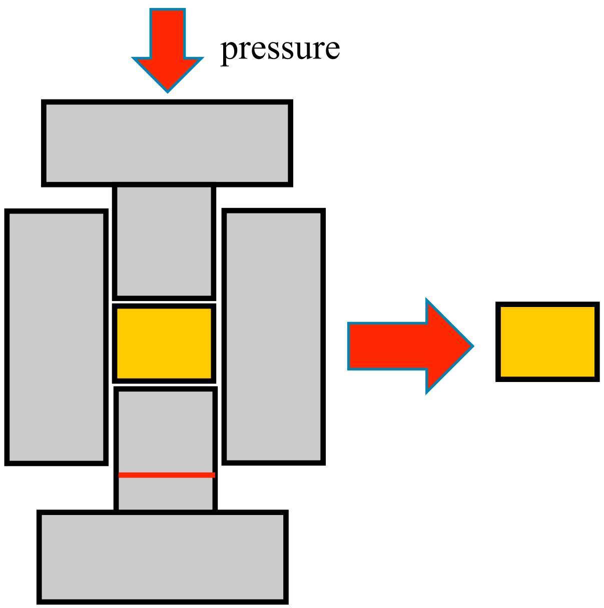
Numerical Example

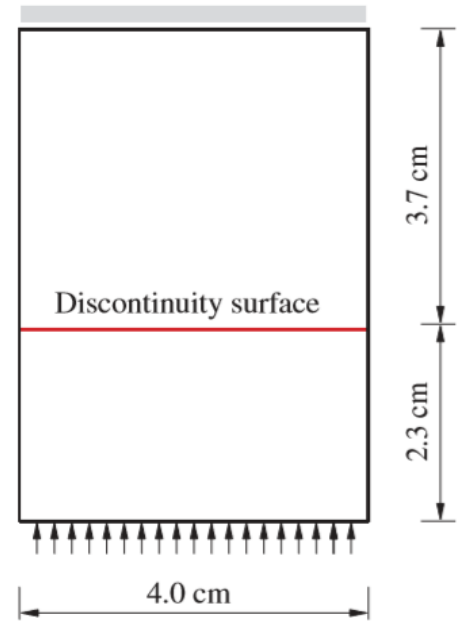
1. Die-Pressing with a Horizontal Material Interface



Simple Configuration of Die-pressing

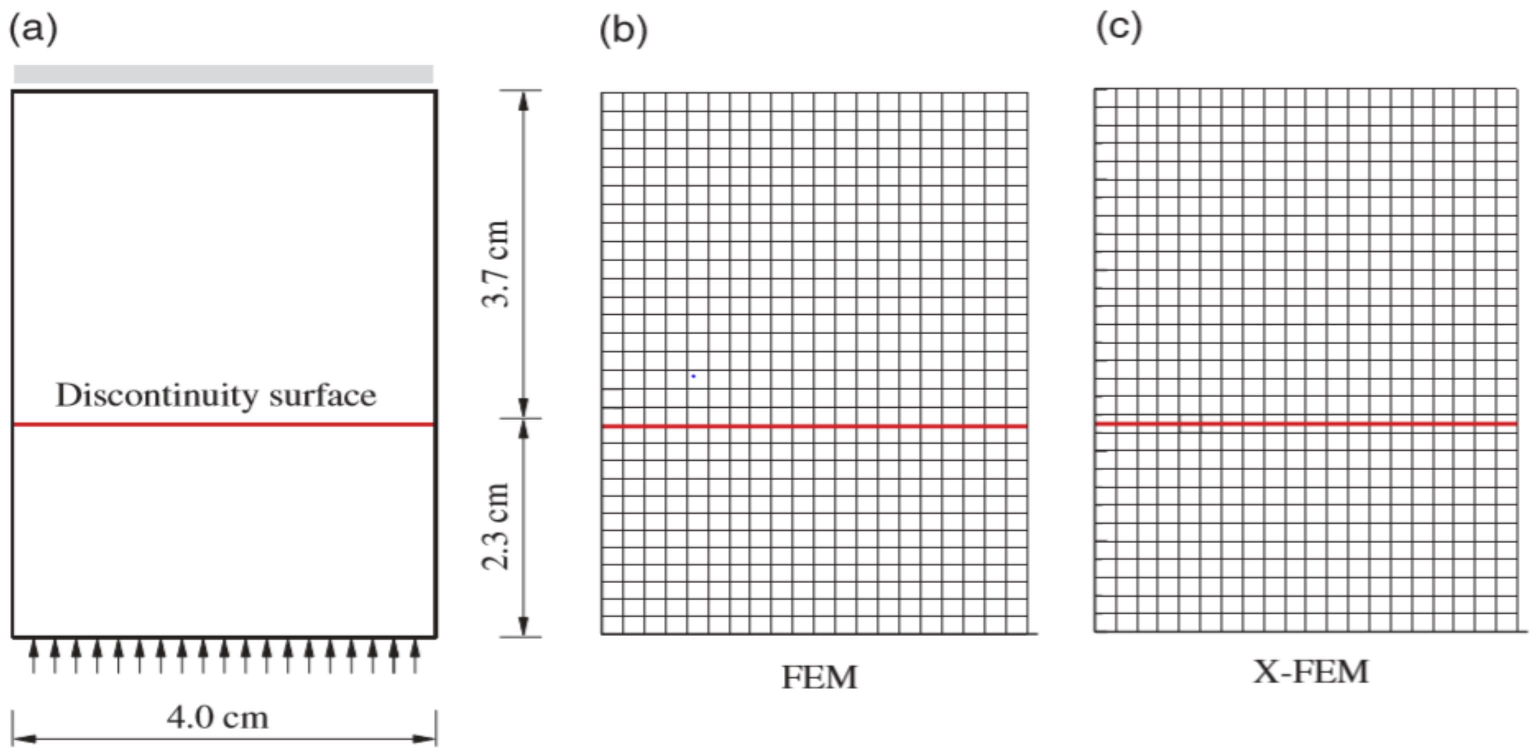
Upper part:
Young's modulus: 2.1×10^5 kg/cm^2
Poisson ratio: 0.35

Lower part:
Young's modulus: 2.1×10^6 kg/cm^2
Poisson ratio: 0.35



Sketch of Die-Pressing with a Horizontal Material Interface (Khoei 2014)

A free-die pressing with horizontal material interface is restrained at the top of edge. A uniform compaction is imposed at the bottom up to 1.3 cm height reduction.

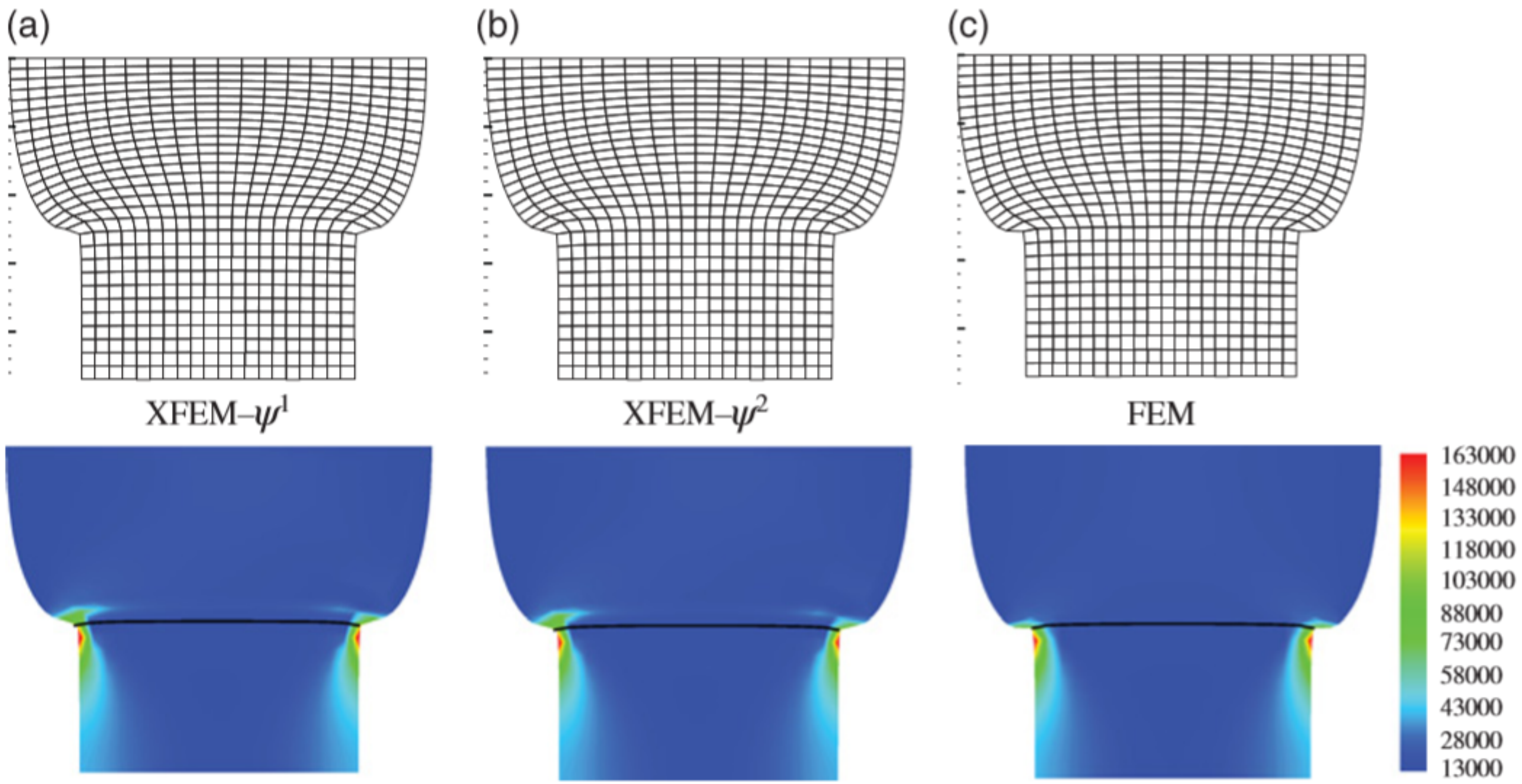


Die pressing with horizontal material interface: (a) Example definition; (b) The coarse FEM mesh; (c) The coarse X-FEM mesh. (Khoei 2014)

Both FEM and X-FEM techniques are employed.

In X-FEM analysis the interface passes through the elements. Due to the discontinuity in different material properties, different enrichment functions (ψ^1 and ψ^2) are applied.

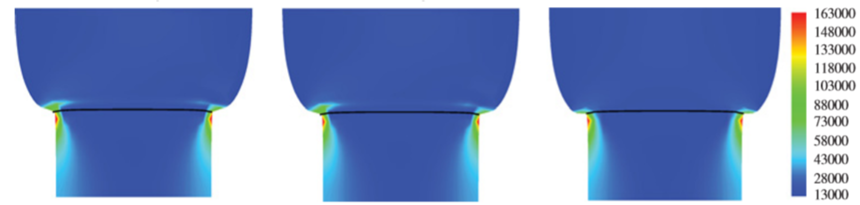
In FEM analysis, the interface is the boundary of elements.

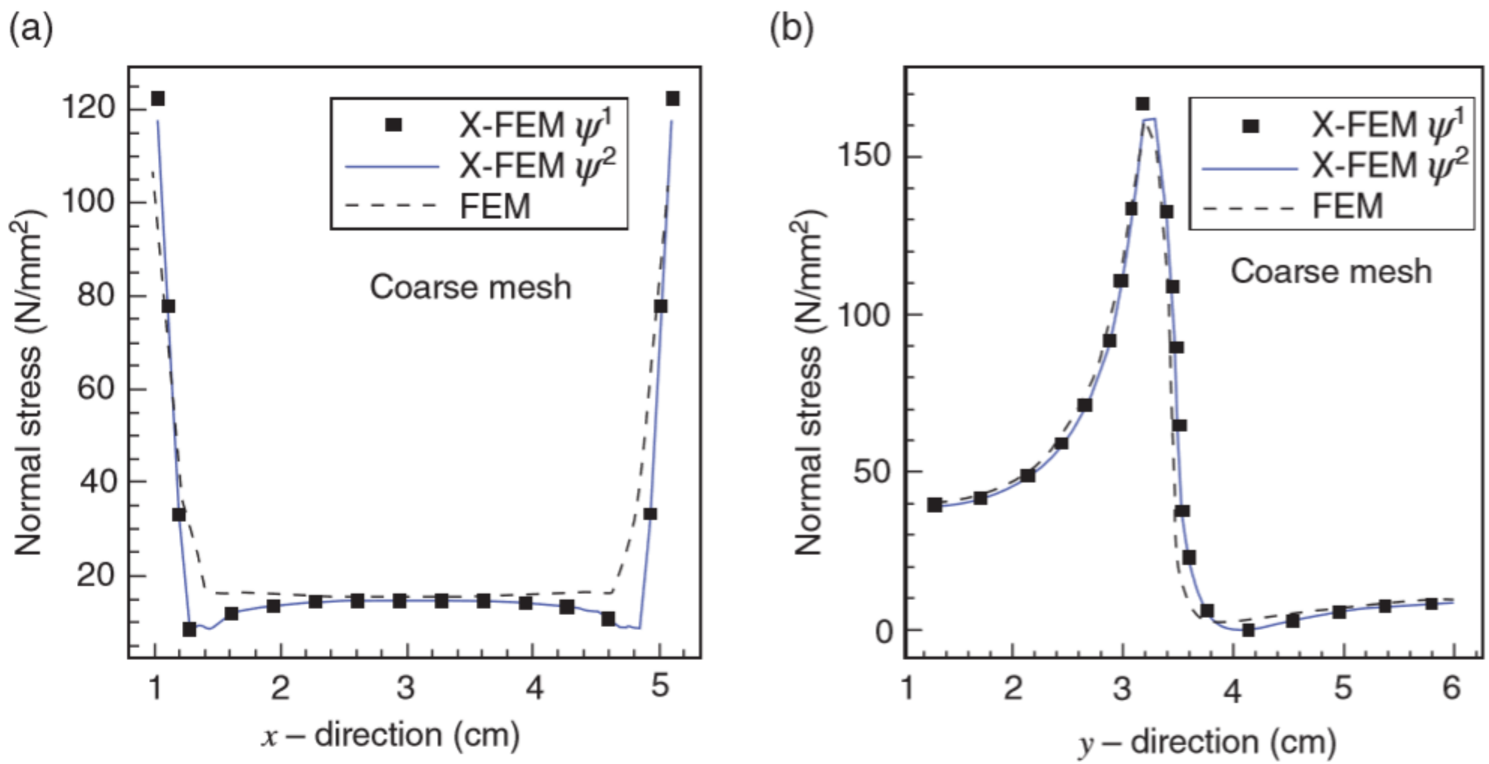


The deformed configuration of die pressing with horizontal material interface: (a) The X-FEM with enrichment function ψ^1; (b) The X-FEM with enrichment function ψ^2; (c) The coarse FEM model. (Khoei 2014)

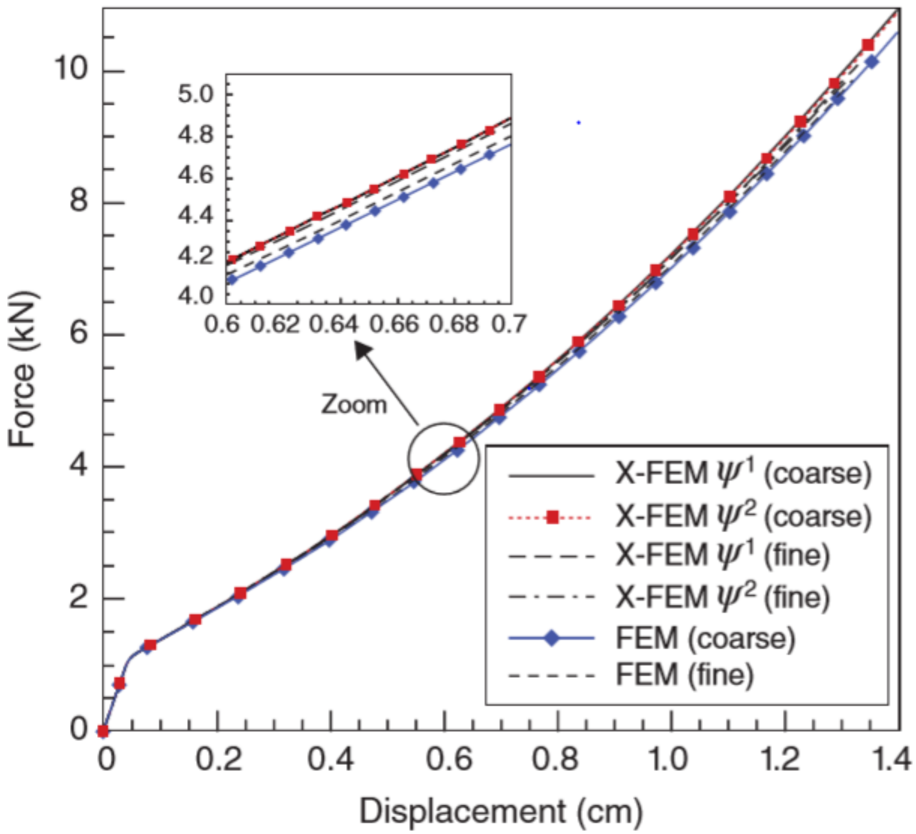
From X-FEM and FEM analysis, the deformed configuration of meshes is obtained. When the compaction deformation is 1.3 cm, the distribution curves of normal stress σ_y are along the interface.

(a) The stress distribution along the interface;
(b) The stress distribution perpendicular to the interface at the left-hand side.





Die pressing with horizontal material interface: A comparison of the normal stress σ_y between FEM and X-FEM techniques using the enrichment function. . (Khoei 2014)



The variation of reaction force with vertical displacement for a die pressing with horizontal material interface. (Khoei 2014)

Comparison of reaction force versus vertical displacements between the x-FEM and FEM approaches shows that X-FEM can model the deformation problems as well as FEM model analysis.

Also, X-FEM method can be applied to more complicated discontinuity boundary.

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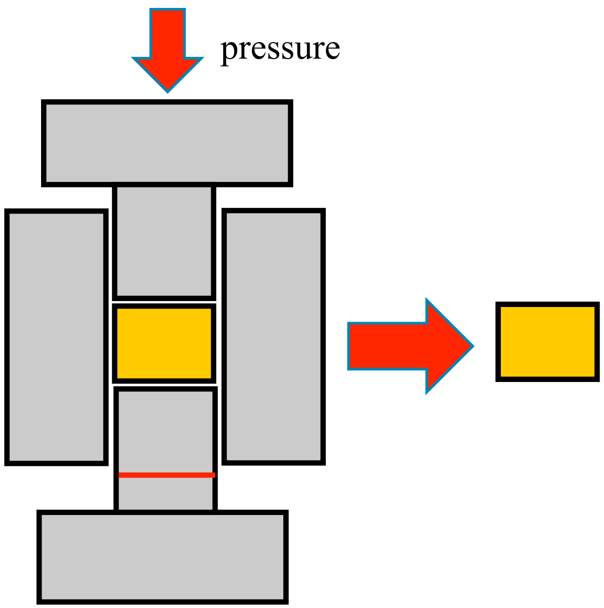
Numerical Example

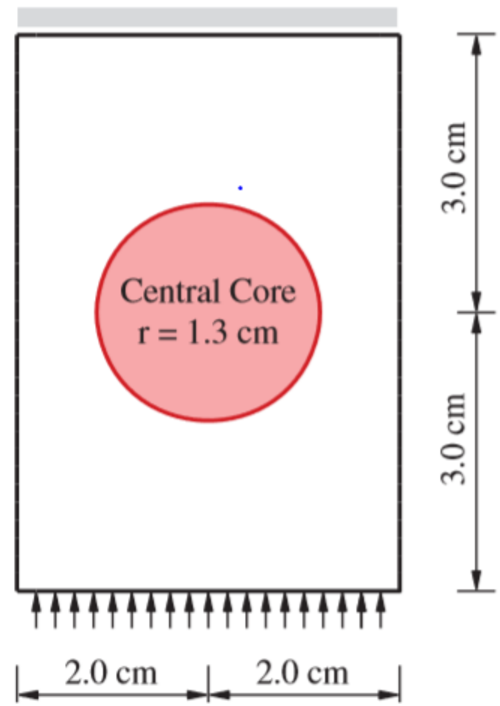
2. Die-Pressing with a Rigid Central Core



Simple Configuration of Die-pressing

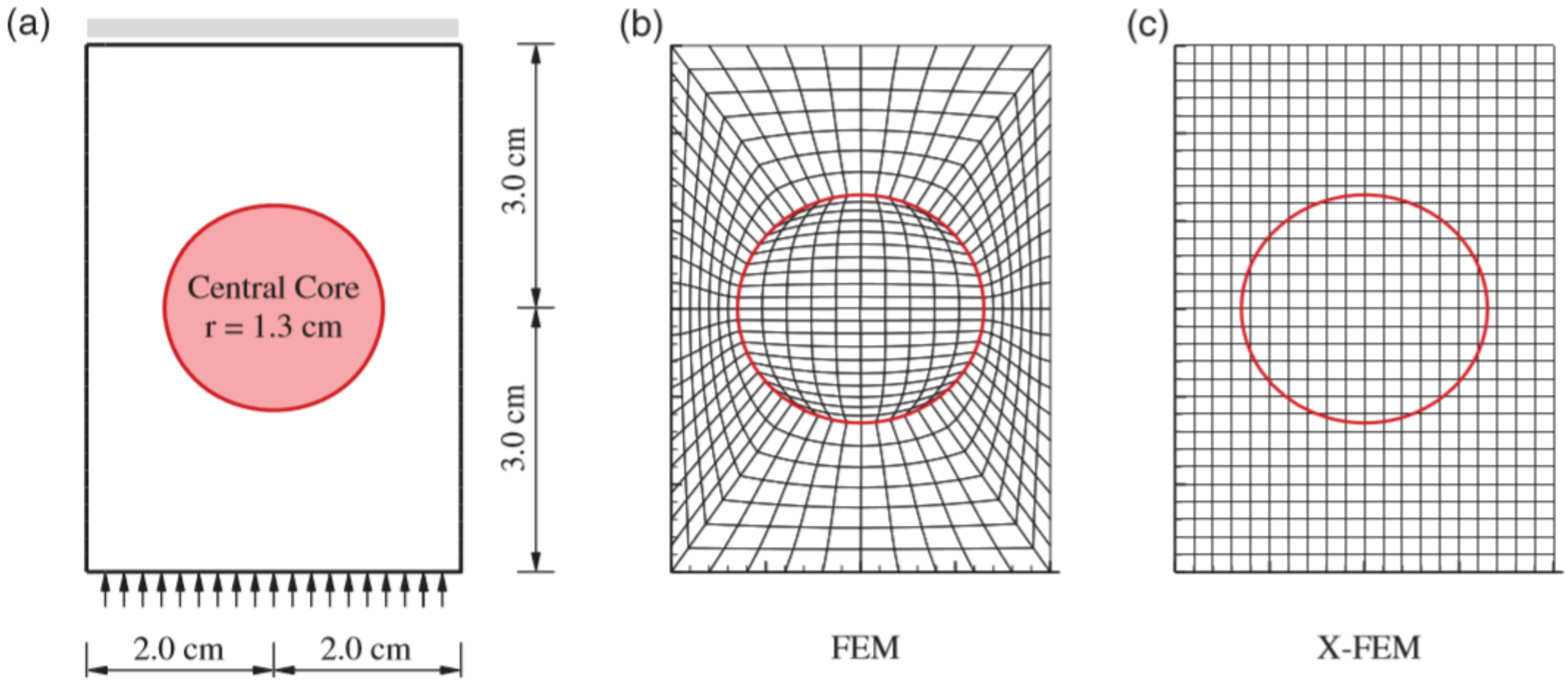
Outside part:
Young's modulus: 2.1×10^5 kg/cm^2
Poisson ratio: 0.35

Inside part:
Young's modulus: 2.1×10^6 kg/cm^2
Poisson ratio: 0.35



Sketch of Die-Pressing with a Horizontal Material Interface (Khoei 2014)

A free-die pressing with horizontal material interface is restrained at the top of edge. A uniform compaction is imposed at the bottom up to 50% volume reduction.



Die pressing with a rigid central core: (a) Example definition; (b) The coarse FEM mesh; (c) The coarse X-FEM mesh. (Khoei 2014)

Both FEM and X-FEM techniques are employed.

Due to the discontinuity in different material properties, different enrichment functions (ψ^1 and ψ^2) are applied in the X-FEM analysis.

In FEM analysis, the interface is the boundary of elements. Mesh size is modified in each calculation step.

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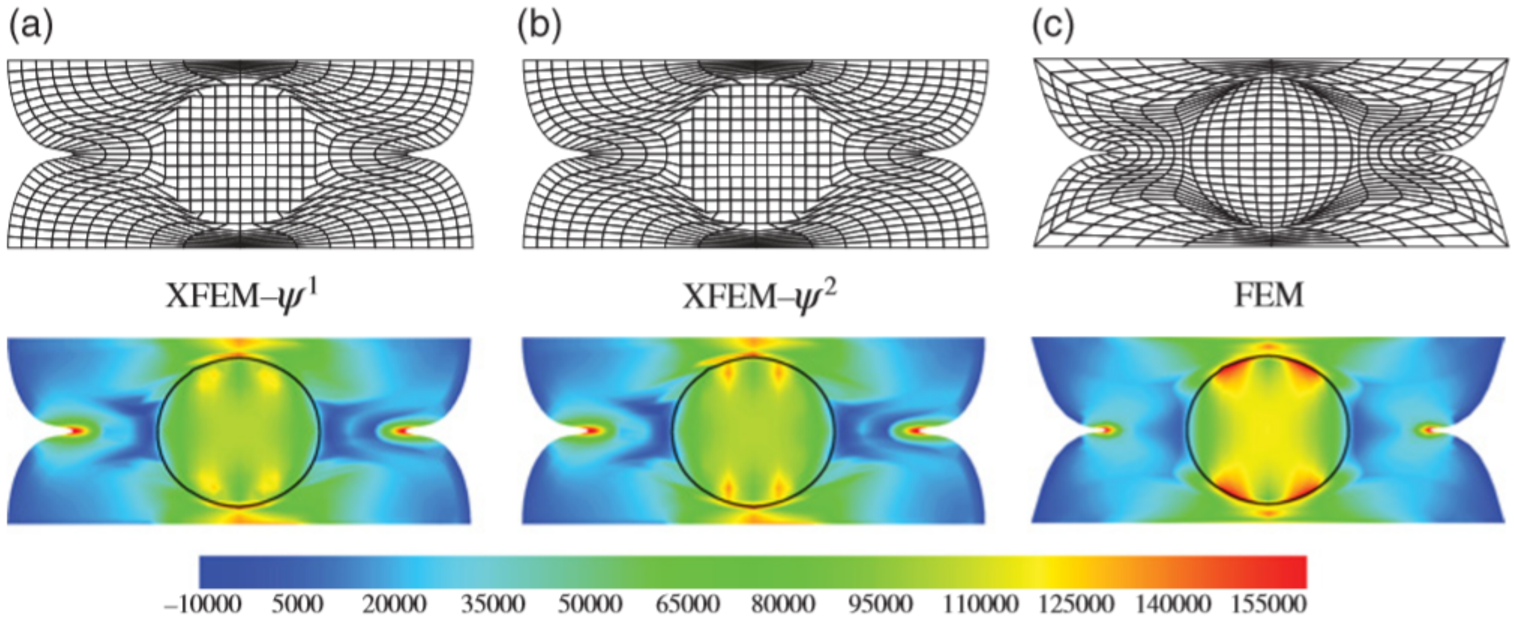
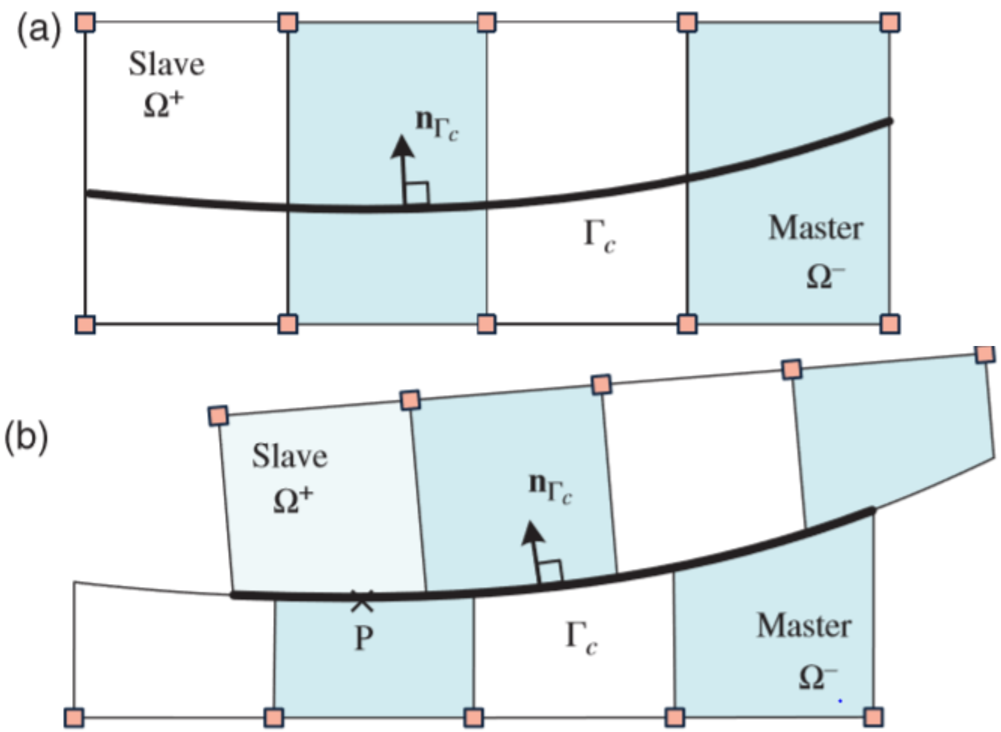


The deformed configuration of die pressing with a rigid core: (a) The X-FEM with enrichment function ψ^1; (b) The X-FEM with enrichment function ψ^2; (c) The coarse FEM model. (Khoei 2014)

Comparison among (a) The X-FEM with enrichment function, (b) The X-FEM with enrichment function and FEM analysis, prediction (b) is closer to FEM result. The enrichment function can be modified in order to obtain the optimal result.

Numerical Example

3. X-FEM Modeling of large Sliding Contact Problems



Large slide along a discontinuity: (a) The initial configuration; (b) The deformed configuration after sliding. (Khoei 2014)

The sign of Heaviside function and the direction of the unit normal on the contact surface (Γc) decides the slave and master segment at each contact point.

Once the sliding begins, the slave points would belong to a different master body. A geometric shape update function should be applied in X-FEM analysis.

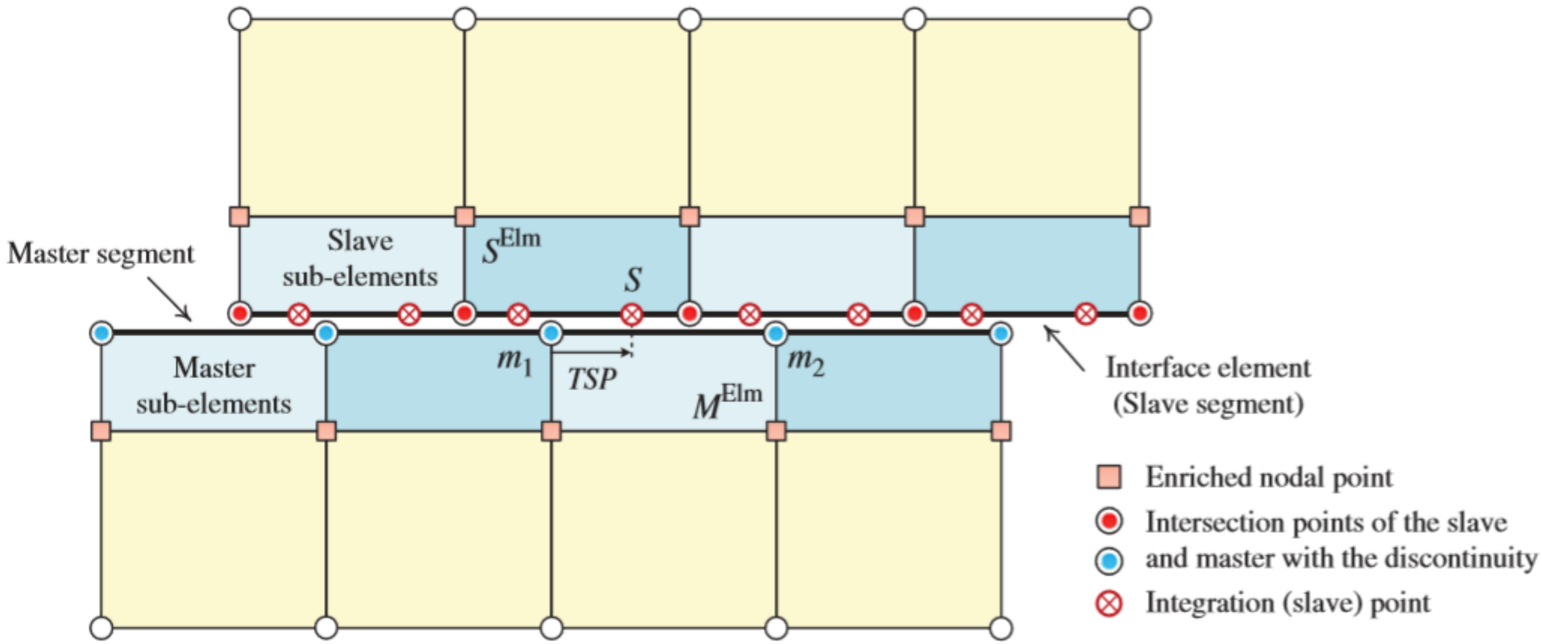


Illustration of the contact interface discretization in a large slide. (Khoei 2014)

The interface of slave segment is divided into several Interface element. The intersection points of master segment and slave body are displayed. Assign the integration (slave) point. Calculate the integration of salve point in two different reference coordinates: reference coordinate of initial slave elements and coordinate of reference point projected on the master edge.

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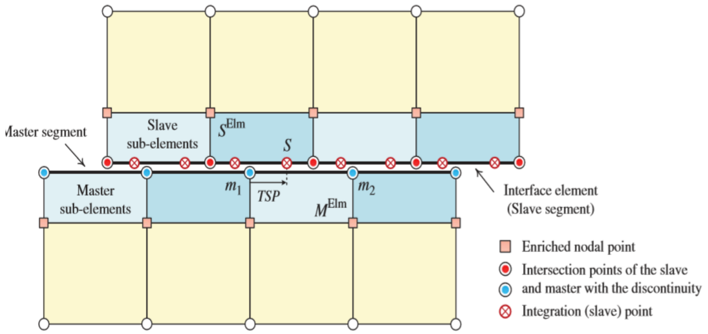
The displacement jump can be evaluated as:

$$[\mathbf{u}] = \left[\mathbf{N}_{(\xi)}^{S^{std}}\bar{\mathbf{u}}^S + \mathbf{N}_{(\xi)}^{S^{enr}}\bar{\mathbf{a}}^S\right] - \left[\mathbf{N}_{(\eta)}^{m^{std}}\bar{\mathbf{u}}^m + \mathbf{N}_{(\eta)}^{m^{enr}}\bar{\mathbf{a}}^m\right]$$

$$= \left[\mathbf{N}^{S^{std}} \quad \mathbf{N}^{S^{enr}} \quad -\mathbf{N}^{m^{std}} \quad -\mathbf{N}^{m^{enr}}\right]\begin{Bmatrix}\bar{\mathbf{u}}^S \\ \bar{\mathbf{a}}^S \\ \bar{\mathbf{u}}^m \\ \bar{\mathbf{a}}^m\end{Bmatrix} \equiv \mathbf{N}^{S-m}\bar{\mathbf{U}}^{\text{con}}$$



Contact interface discretization in a large slide (Khoei 2014)

The shape function of the slave-master at the contact surface is expressed as:

$$\mathbf{N}^{S-m} = \left[\mathbf{N}^{S^{std}} \quad \mathbf{N}^{S^{enr}} \quad -\mathbf{N}^{m^{std}} \quad -\mathbf{N}^{m^{enr}}\right]$$

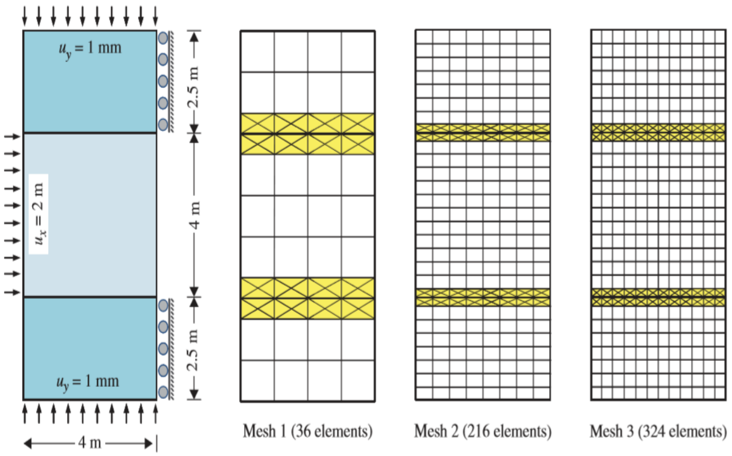
Slave point S comes into contact with the master segment (m1-m2).

ξ is the reference coordinate of slave point in the initial slave element, and η is the reference coordinate of slave point projected on master edge in the initial master element.

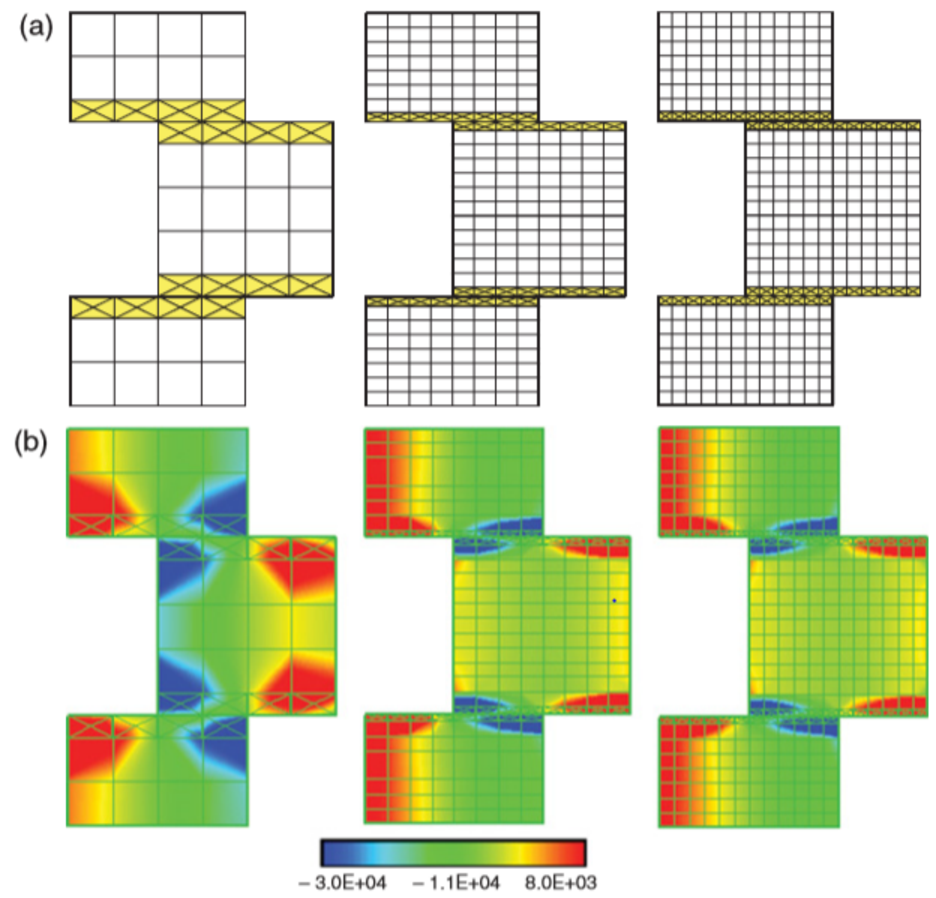
$N^{S^{std}}$ and $N^{S^{enr}}$ are the standard and enriched shape functions of slave functions of slave element. $N^{m^{std}}$ and $N^{m^{enr}}$ are the standard and enriched shape functions of master element.

Example of large slide with horizontal material interface

The large sliding contact behavior of a frictionless problem is investigated for a plate with horizontal material interfaces.
The bottom and top part is constrained. Apply the force from left. Horizontal slide would happen.



Large slide with horizontal material interface together with the initial X-FEM meshes. (Khoei 2014)



(a) The deformed configuration of X-FEM meshes; (b) The distribution of stress contours for various X-FEM meshes. (Khoei 2014)

Example application

Evolution of one crack on a aluminum panel. (OneraMNU, 2011)
https://youtu.be/toNLY59GMaY

Another example of crack propagation with X-FEM. (Stephane Brodas, 2013)
https://youtu.be/fuikZx71MhU

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THANK YOU