Lattice Boltzmann Method

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Project Outline

• Introduction
• Historical Perspective
• General Principle
• Governing Equation
• Hand-Calculation Example
• Numerical Example
• Field Application
• References
Lattice Boltzmann Method is a dynamic method that simulates the macroscopic behavior of fluids by using a simple mesoscopic model. It inherited the main principles of Lattice Gas Automaton (LGA) and made improvements. From lattice gas automaton, it is possible to derive the macroscopic Navier-Stokes equations.
Introduction

**Specialty** of Lattice Boltzmann Method & **Difference** from the traditional macroscopic numerical calculation method:

1. It is based on and starts from **Non-equilibrium statistical mechanics and Discrete model**

2. It connected dynamic lattice model, whose time, space and velocity phase space are fully **discrete**, with **Boltzmann equation**.

3. The implementation of this method can describe the law of fluid motion without Solving **Navier-Stokes equations**
Introduction

Achievements of Lattice Boltzmann method from a macroscopic perspective

1. It connected macroscopic and microscopic world;

2. It connected continuous model and discrete model

3. It is an all-new perspective to understand the nature of fluids.

All in all, its successful application reflect a fundamental principle of scientific research. That is, conservation is the most fundamental law in the material world, which guides the movement and development of the material world. There are certain internal links between the macroscopic and microscopic world, which is in fact a dialectical unity.
Introduction

Compare LBM with CFD

*LBM* vs. *CFD (traditional)*

*CFD (traditional):* Computational Fluid Dynamics, including Navier-Stokes equations, Euler equation, Burnett equation.

*LBM:* Lattice-Boltzmann method
The extent of gas rarefaction refers to the ratio of the average free path of gas molecule to the characteristic length.

The Knudsen number ($k \downarrow n$) represents: $k \downarrow n = \lambda / L$

$\lambda$ is the average free path of gas molecule; $L$ is the characteristic length.

The following figure will show that different CFD equations could be applied to different ranges of $k \downarrow n$. 
Introduction

Boltzmann equation

- Euler equation
- Naiver-Stokes equations
- Burnett equation

0.001 \quad 0.01 \quad 0.1 \quad 1 \quad 10 \quad Kn \rightarrow \infty

- Continuous medium (conventional density)
- Sliding range (slightly rarefied)
- Transition range (moderately rarefied)
- Free molecule range (Highly rarefied)
# Introduction

<table>
<thead>
<tr>
<th>Navier-Stokes equation</th>
<th>Lattice Boltzmann equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho \left( \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right) = -\nabla p + \mu \nabla^2 u )</td>
<td>( \frac{\partial f}{\partial t} + e \cdot \nabla f = -\frac{1}{\tau} \left( f - f^{EQ} \right) )</td>
</tr>
</tbody>
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- second-order PDE
- need to treat the non-linear convective term \( u \cdot \nabla u \)
- need to solve Poisson equation for the pressure \( p \)

| |
|------------------------|---------------------------|
| avoids convective term, convection becomes simple advection | pressure \( p \) is obtained from equation of state |

Table 1: Comparison between Navier-Stokes equation and lattice Boltzmann equation.
Compared with traditional computational fluid dynamics methods, Lattice-Boltzmann method has the following advantages:

(1) Its **algorithm is simple**, which can simulate various complicated nonlinear macroscopic phenomena;

(2) It can handle complicated **boundary conditions**

(3) The **values of pressure** in the lattice Boltzmann method can be **directly solved by the state equation**;

(4) It is **easy to program**, and the processing before and after calculation is also very simple

(5) It is easy to process and complete the parallel tasks based LBM;

(6) It can **directly simulate connected-domain flow fields** with complex geometric boundaries, such as porous media.
Ludwig Eduard Boltzmann (February 20, 1844 – September 5, 1906) was an Austrian physicist and philosopher whose greatest achievement was in the development of statistical mechanics, which explains and predicts how the properties of atoms (such as mass, charge, and structure) determine the physical properties of matter (such as viscosity, thermal conductivity, and diffusion).
Historical Perspective

Boltzmann Equation (1872):

Describe the dynamics of an ideal gas.

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{F}{m} \frac{\partial f}{\partial v} = \frac{\partial f}{\partial t} \bigg|_{\text{collision}}
\]

Where, \( f \) represents the distribution of single-particle position and momentum.

Lattice Boltzmann Equation:

\[
f(x + \xi \delta_t, \xi, t + \delta_t) = e^{-\delta_t/\lambda} f(x, \xi, t) + \frac{1}{\lambda} e^{-\delta_t/\lambda} \times \int_0^{\delta_t} e^{\frac{t'}{\lambda}} g(x + \xi t', \xi, t + t') dt'
\]
LBM Derivation

Novier-stokes equation

Boltzmann equation

LBE

Engineer

Mathematic

Newton

physics

Lattice Gas automata
Historical Perspective

The Lattice gas model meaning:

- to establish a simple model as far as possible to be able to simulate a system consisting of a large number of particles;

- reflecting the true collision of granules, so that we can get the fluid Macro features for a long time.
Historical Perspective

J·Hardy, Y·Pomeau and O·Pazzis (1973)-HPP Model

Only Four Direction!!

U·Frish, Y·Pomeau and B·Hasslacher (1986)-FHP Model

Non-Galilean invariance!!

McNamara and Zanetti(1988)-LB Model

Still improving !!
Historical Perspective

**Lattice Gas Automata** (LGA, 1992):

- Type of cellular automation used to simulate fluid flow
- Precursor to the Lattice Boltzmann Methods

Disadvantages:

- Lack of Galilean invariance
- Statistical noise
- Difficulty in expanding the model to handle three dimensional problems
Simplicity and efficiency

- When solving compressible Navier-Stokes equations, LBM resembles a pseudo-compressible method, increasing its simplicity and extensibility through artificial compressibility.

- Similar pseudo-compression method, LBM does not involve Poisson equation

- Most of the calculations in LBM are local and more suitable for parallel

- LBM requires a lot of memory to store the distribution function, which is also the main bottleneck of LBM

- The nature of LBM is time-dependent, so calculating steady flow is not particularly efficient
Why Lattice Boltzmann method?

Geometry
- LBM is well suited for mass-conservative fluid simulation of complex boundaries (e.g. porous media)
- LBM can well realize mass-conserving mobile boundary problems and it is very attractive for soft material simulation

Thermal effect
- Thermal disturbances originate from the microscopic and average macroscopic, LBM includes them in the mesoscopic description
  - Simulation of energy conservation in LBM is not straightforward

Sound generation
  - LBM is not suitable for direct simulation of long-distance acoustic transmission under real adhesion
  - LBM does not adapt to strong compressible (e.g. ultrasonic and transonic) fluids
Multiphase flow and multicomponent flow

- Many methods for solving multiphase flow and multicomponent flow using LBM
- LBM is suitable for simulation of multi-phase flow and multi-component flow in complex boundary
  - The lattice-based method has the existence of spurious currents between fluid-fluid interface
  - The current multiphase flow and multicomponent flow methods of LBM do not make good use of the kinetics principle.
  - In the simulation of multi-phase flow and multi-component flow, the values of viscosity and density are limited.
General Principles

- Continuum (Macroscopic scale), finite difference, finite volume, finite element, etc.
- Navier-Stokes Equations

- Molecular Dynamics (Microscopic scale), Hamilton’s Equation.

- Lattice Boltzmann Method (Mesoscopic scale), Boltzmann Equation
General Principles

Lattice Gas Automata (LGA)
General Principles

https://www.youtube.com/watch?v=8gorVDJS1QA
General Principles

Lattice Gas Automata (LGA)

LGA

Single-Particle Distribution Function (Boolean Variables)

LBM

Averaged Particle Distribution (Mesoscopic Variables)
General Principles

Lattice Arrangement

\omega_i = \frac{4}{6}, \frac{1}{6}, \frac{1}{6}

\omega_i = \frac{2}{6}, \frac{1}{6} \times 4

\omega_i = \frac{1}{4}

DnQm

D3Q15
**General Principles**

**Bounce Back**

*Good For Porous Media*
Governing Equation

Boltzmann Transport Equation

\[
\frac{df}{dt} = \mathbb{M}(f)
\]

\[
\frac{\partial f}{\partial t} + \frac{\partial f}{\partial r} \cdot c + \frac{F}{m} \cdot \frac{\partial f}{\partial c} = \Omega
\]

\[
\frac{\partial f}{\partial t} + c \cdot \nabla f = \Omega
\]

\[
f(r + c \Delta t, c + F \Delta t, t + \Delta t)drdc - f(r, c, t)drdc = \Omega(f)drdcdt
\]
The Bhatnagar, Gross, Krook and Welander (BGKW) Approximation

\[ \Omega = \omega (f^{eq} - f) \approx \frac{1}{\tau} (f^{eq} - f) \]

\[ \omega = \frac{1}{\tau} \]

\[ \frac{\partial f}{\partial t} + c \cdot \nabla f = \frac{1}{\tau} (f^{eq} - f) \]
Governing Equation

\[ \frac{\partial f_i}{\partial t} + c_i \nabla f_i = \frac{1}{\tau} (f_i^{eq} - f_i) \]

After discretizing

\[ f_i(r + c_i \Delta t, t + \Delta t) = f_i(r, t) + \frac{\Delta t}{\tau} [f_i^{eq}(r, t) - f_i(r, t)] \]
Governing Equation

Chapman-Enskog Expansion

\[
\frac{\partial T(x,t)}{\partial t} = \Gamma \frac{\partial^2 T(x,t)}{\partial x^2}
\]

\[
\frac{\partial T(x,t)}{\partial t} = \Gamma \frac{\epsilon^2 \partial^2 T(x,t)}{\partial x^2}
\]

\[
T(x,t) = \sum_{i=1}^{i=2} f_i(x,t) = f_1(x,t) + f_2(x,t)
\]

\[
f_i(x + c_i \Delta t, t + \Delta t) = f_i(x, t) + \epsilon \frac{\partial f_i}{\partial t} \Delta t + \epsilon \frac{\partial f_i}{\partial x} c_i \Delta t + \frac{1}{2 \Delta t^2} \left( \epsilon^4 \frac{\partial^4 f_i}{\partial x^4} + 2 \epsilon^3 \frac{\partial^2 f_i}{\partial x^2} c_i + \epsilon^2 \frac{\partial^2 f_i}{\partial x^2} c_i c_i \right) + O(\Delta t^3)
\]

\[
f_i^{\text{eq}} = w_i T(x,t)
\]

\[
\sum_{i=1}^{i=2} w_i = 1
\]
Governing Equation

Relative to Macroscopic View

\[ \rho(r, t) = \int mf(r, c, t) \, dc \]
\[ \rho(r, t)u(r, t) = \int mcdf(r, c, t) \, dc \]
\[ \rho(r, t)e(r, t) = \frac{1}{2} \int m\epsilon_a^2 f(r, c, t) \, dc \]
1. Our calculation example is a long pipeline of oil, whose initial pressure is zero \((t=0; \ P=0)\).

2. The pressure of the pipeline’s left boundary changes to one \((P=1)\) when time goes by \((t>0)\).

3. This example is aimed to simulate the pressure variations of the whole pipeline as time goes by.

4. This example is based on this assumption that the surroundings outside the pipeline have no influences on the pipeline’s pressure changes.
Model & Example Introduction

Our example could be processed and regarded as D1Q3 model

1. For this example, it would obey this following equation:
\[
\frac{\partial P}{\partial t} = \alpha \frac{\partial^2 P}{\partial x^2}
\]

In addition, it is feasible to set that:
\[
\alpha = \frac{Ak\bar{f}}{\mu \bar{g} c\bar{g} V\bar{b} C\bar{t}} = 1/3
\]
For this D1Q3 model, it has following character and definition:

1. In this model, each element has corresponding distribution functions, the weight factors corresponding to the distribution function $f_0$, $f_1$, $f_2$ are showed below:

   \[ w_0 = \frac{4}{6}; \quad w_1 = \frac{1}{6}; \quad w_2 = \frac{1}{6} \]

2. The velocity vectors are defined as follow:

   \[ c_0 = 0; \quad c_1 = 1; \quad c_2 = -1 \]

3. The displacement and time interval are defined as follow:

   \[ \Delta t = 1; \quad \Delta x = 1 \]
Hand-Calculation Example

The flow chart of hand-calculation
Initialization

1. Initialize macroscopic properties:

As assumed before, when time goes by, the pressure of the left boundary is one ($P_{\text{Boundary}} = 1$)

2. Start the iteration calculation with suitable initialization of distribution function:

For this case, distribution function $f_{\downarrow i}$ is set as $w_{\downarrow i}$ in first element, and in the second and third elements distribution function $f_{\downarrow i}$ set as $C_{\downarrow i}$ initially.
Initialization Calculation------Equilibrium Distribution Function Calculation

As initialization:
\[ f_{↓0}(1,0) = \frac{4}{6}; \quad f_{↓1}(1,0) = \frac{1}{6}; \quad f_{↓2}(1,0) = \frac{1}{6}; \]

Because,
\[ P(x,t) = \sum_{i=1}^{3} f_{↓i}(x,t) \]
So,
\[ P(1,0) = f_{↓0}(1,0) + f_{↓1}(1,0) + f_{↓2}(1,0) = 1; \]

In addition, the same as the process above:
\[ f_{↓0}(2,0) = 0; \quad f_{↓1}(2,0) = 1; \quad f_{↓2}(2,0) = -1; \]
\[ f_{↓0}(3,0) = 0; \quad f_{↓1}(3,0) = 1; \quad f_{↓2}(3,0) = -1; \]

Therefore:
\[ P(2,0) = 0; \quad P(3,0) = 0 \]
Initialization Calculation------Equilibrium Distribution Function Calculation

Because,  
\[
f↓i↑eq(x,t) = w↓i P(x,t)
\]
So,  
\[
f↓0↑eq(1,0) = w↓1 \times P(1,0) = \frac{4}{6} \times 1 = \frac{4}{6};
\]

In the same way, the following the result values of equilibrium distribution function could be obtained:

\[
f↓1↑eq(1,0) = \frac{1}{6}; \quad f↓2↑eq(1,0) = \frac{1}{6};
\]

\[
f↓0↑eq(2,0) = f↓1↑eq(2,0) = f↓2↑eq(2,0) = 0;
\]

\[
f↓0↑eq(3,0) = f↓1↑eq(3,0) = f↓2↑eq(3,0) = 0;
\]
Collisions Calculations

1. When calculate collisions, the following equation is obeyed:

\[ f_{\downarrow i \uparrow} (x,t) = (1 - \omega) f_{\downarrow i} (x,t) + \omega f_{\downarrow i \text{eq}} (x,t) \]

This model uses BGK Approximation for the collision calculation.

For this example,

\[ \alpha = \tau - \Delta t / 2 = 1 / 3 \; ; \; \Delta t = 1 \; ; \; \omega = \Delta t / \tau \; ; \]

\[ \therefore \omega = 6 / 5 \]

Therefore,

\[ f_{\downarrow 0 \uparrow} (1,0) = (1 - 6 / 5) \times \frac{4}{6} + 6 / 5 \times \frac{4}{6} = 2 / 3 \; ; \]

\[ f_{\downarrow 1 \uparrow} (1,0) = (1 - 6 / 5) \times \frac{1}{6} + 6 / 5 \times \frac{1}{6} = 1 / 6 \; ; \]

\[ f_{\downarrow 2 \uparrow} (1,0) = (1 - 6 / 5) \times \frac{1}{6} + 6 / 5 \times \frac{1}{6} = 1 / 6 \; ; \]
Collisions Calculations

In the same way, the following values could be calculated:

\[
f_{\downarrow 0} \uparrow \ast (2,0) = (1 - \frac{6}{5}) \times 0 + \frac{6}{5} \times 0 = 0;
\]

\[
f_{\downarrow 1} \uparrow \ast (2,0) = (1 - \frac{6}{5}) \times 1 + \frac{6}{5} \times 0 = -\frac{1}{5};
\]

\[
f_{\downarrow 2} \uparrow \ast (2,0) = (1 - \frac{6}{5}) \times (-1) + \frac{6}{5} \times 0 = \frac{1}{5};
\]

And,

\[
f_{\downarrow 0} \uparrow \ast (3,0) = (1 - \frac{6}{5}) \times 0 + \frac{6}{5} \times 0 = 0;
\]

\[
f_{\downarrow 1} \uparrow \ast (3,0) = (1 - \frac{6}{5}) \times 1 + \frac{6}{5} \times 0 = -\frac{1}{5};
\]

\[
f_{\downarrow 2} \uparrow \ast (3,0) = (1 - \frac{6}{5}) \times (-1) + \frac{6}{5} \times 0 = \frac{1}{5};
\]
When calculate streaming, the following equation is obeyed:
\[ f_{\downarrow i} (x + c_{\downarrow i} \Delta t, t + \Delta t) = f_{\downarrow i}^\uparrow (x, t) \]

\[ C_{\downarrow 0} = 0; \quad C_{\downarrow 1} = 1; \quad C_{\downarrow 2} = -1 \]

Therefore,
\[ f_{\downarrow 0} (1,1) = f_{\downarrow 0}^\uparrow (1,0) = \frac{2}{3}; \]
\[ f_{\downarrow 0} (2,1) = f_{\downarrow 0}^\uparrow (2,0) = 0; \]
\[ f_{\downarrow 0} (3,1) = f_{\downarrow 0}^\uparrow (3,0) = 0; \]

Similarly,
\[ f_{\downarrow 1} (3,1) = f_{\downarrow 1}^\uparrow (2,0) = -\frac{1}{5}; \]
\[ f_{\downarrow 1} (2,1) = f_{\downarrow 1}^\uparrow (1,0) = \frac{1}{6}; \]
\[ f_{\downarrow 1} (1,1) = f_{\downarrow 1}^\uparrow (1,0) = \frac{1}{6}; \quad \text{(Boundary Condition)} \]
Similarly,
\[ f_{\downarrow 2} (3,1) = f_{\downarrow 2} \uparrow (3,0) = 1/5; \]
\[ f_{\downarrow 2} (2,1) = f_{\downarrow 2} \uparrow (3,0) = 1/5; \]
\[ f_{\downarrow 2} (1,1) = f_{\downarrow 2} \uparrow (1,0) = 1/6; \text{ (Boundary Condition)} \]

Because they are under boundary condition:
\[ f_{\downarrow 1} (1,1) = f_{\downarrow 1} \uparrow (1,0) = 1/6; \text{ (Boundary Condition)} \]
\[ f_{\downarrow 2} (1,1) = f_{\downarrow 2} \uparrow (1,0) = 1/6; \text{ (Boundary Condition)} \]
Hand-Calculation Example

Re-Calculation of Macroscopic Properties

As presented above:
\[ P(x,t) = \sum_{i=1}^{3} f_{i} (x,t) \]
Submit the latest value of \( f_{i} (x,t) \) into this equation,
Therefore,
\[ P(1,1) = f_{0} (1,1) + f_{1} (1,1) + f_{2} (1,1) = \frac{2}{3} + \frac{1}{6} + \frac{1}{6} = 1; \]
\[ P(2,1) = f_{0} (2,1) + f_{1} (2,1) + f_{2} (2,1) = 0 + 0 + \frac{1}{5} = \frac{1}{5}; \]
\[ P(3,1) = f_{0} (3,1) + f_{1} (3,1) + f_{2} (3,1) = 0 - \frac{1}{5} + \frac{1}{5} = 0; \]
Re-Calculation------Equilibrium Distribution Function Calculation

As presented above:

\[ f_{\text{down}} \uparrow_{\text{up}} (x,t) = w_{\text{down}} P(x,t) \]

\[ w_{\text{down}} = \frac{4}{6}; \quad w_{\text{up}}^{1} = \frac{1}{6}; \quad w_{\text{up}}^{2} = \frac{1}{6} \]

Submit the latest value of \( P(x,t) \) into this equation, therefore:

\[ f_{\text{down}} \uparrow_{\text{up}} (1,1) = w_{\text{down}} P(1,1) = \frac{4}{6} \times 1 = \frac{4}{6} \]

\[ f_{\text{down}} \uparrow_{\text{up}} (2,1) = w_{\text{down}} P(2,1) = \frac{4}{6} \times \frac{1}{5} = \frac{2}{15} \]

\[ f_{\text{down}} \uparrow_{\text{up}} (1,1) = w_{\text{up}}^{1} P(1,1) = \frac{1}{6} \times 1 = \frac{1}{6} \]

\[ f_{\text{down}} \uparrow_{\text{up}} (2,1) = w_{\text{up}}^{1} P(2,1) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30} \]

\[ f_{\text{down}} \uparrow_{\text{up}} (1,1) = w_{\text{up}}^{2} P(1,1) = \frac{1}{6} \times 1 = \frac{1}{6} \]

\[ f_{\text{down}} \uparrow_{\text{up}} (2,1) = w_{\text{up}}^{2} P(2,1) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30} \]
Hand-Calculation Example

Re-Calculation------Equilibrium Distribution Function Calculation

And similarly:
\[ f_{\downarrow i \uparrow \text{eq}}(x,t) = w_{\downarrow i} P(x,t) \]

\[ f_{\downarrow 0 \uparrow \text{eq}}(3,1) = w_{\downarrow 0} P(3,1) = \frac{4}{6} \times 0 = 0 \]
\[ f_{\downarrow 1 \uparrow \text{eq}}(3,1) = w_{\downarrow 1} P(3,1) = \frac{1}{6} \times 0 = 0 \]
\[ f_{\downarrow 2 \uparrow \text{eq}}(3,1) = w_{\downarrow 2} P(3,1) = \frac{1}{6} \times 0 = 0 \]

That is a whole process of one iteration, aiming to obtain the final result, more iterations are needed, here is just an example. When the latest macroscopic properties are pretty closed to the previous macroscopic result, get out of the iterations and output the final result.
Matlab Implementation

Define Velocity Vector

%define es

\begin{align*}
  e_x(0) &= 0; \quad e_y(0) = 0 \\
  e_x(1) &= 1; \quad e_y(1) = 0 \\
  e_x(2) &= 0; \quad e_y(2) = 1 \\
  e_x(3) &= -1; \quad e_y(3) = 0 \\
  e_x(4) &= 0; \quad e_y(4) = -1 \\
  e_x(5) &= 1; \quad e_y(5) = 1 \\
  e_x(6) &= -1; \quad e_y(6) = 1 \\
  e_x(7) &= -1; \quad e_y(7) = -1 \\
  e_x(8) &= 1; \quad e_y(8) = -1
\end{align*}
Matlab Implementation

% D2Q9 LATTICE CONSTANTS
\[ t = \left[ \frac{4}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36} \right]; \]
\[ Cx = [0, 1, 0, -1, 0, 1, -1, -1, 1]; \]
\[ Cy = [0, 0, 1, 0, -1, 1, 1, -1, -1]; \]
\[ Opp = [1, 4, 5, 2, 3, 8, 9, 6, 7]; \]
\[ col = [2:(ly-1)]; \]
\[ [y,x] = meshgrid(1:ly,1:lx); \]
\[ obst = (x-obst_x)^2 + (y-obst_y)^2 \leq obst_r^2; \]
\[ obst(:,[1,ly]) = 1; \]
\[ bbRegion = find(obst); \]

% INITIAL CONDITION: (\rho=0, u=0) ==> fIn(i) = t(i)
\[ fIn = \text{reshape}(t' \ast \text{ones}(1,lx*ly), 9, lx, ly); \]
% MAIN LOOP (TIME CYCLES)
for cycle = 1:maxT

% MACROSCOPIC VARIABLES
rho = sum(fIn); % Density
Ux = reshape(...
(cx * reshape(fIn,9,lx*ly)), 1,lx,ly) ./rho;
Uy = reshape(...
(cy * reshape(fIn,9,lx*ly)), 1,lx,ly) ./rho
Matlab Implementation

% MACROSCOPIC (DIRICHLET) BOUNDARY CONDITIONS
% Inlet: Poiseuille profile
L = ly-2; y = col-1.5;
ux(:,1,col) = 4 * uMax / (L*L) * (y.*L-y.*y);
uy(:,1,col) = 0;
rho(:,1,col) = 1 ./ (1-ux(:,1,col)) .* ( ... sum(fIn([1,3,5],1,col)) + 2*sum(fIn([4,7,8],1,col)) );

% Outlet: Zero gradient on rho/ux
rho(:,lx,col) = rho(:,lx-1,col);
uy(:,lx,col) = 0;
ux(:,lx,col) = ux(:,lx-1,col);
Matlab Implementation

% COLLISION STEP
for i=1:9
    cu = 3*(cx(i)*ux+cy(i)*uy);
    fEq(i,:,:)= rho.* t(i) .* ...
        ( 1 + cu + 1/2*(cu.*cu) ...
        -3/2*(ux.^2+uy.^2) );
    fOut(i,:,:)= fIn(i,:,:)- ...
        omega .* (fIn(i,:,:)-fEq(i,:,:));
end
Matlab Implementation

% MICROSCOPIC BOUNDARY CONDITIONS
for i=1:9

% Left boundary
fOut(i,1,col) = fEq(i,1,col) + ...  
18*t(i)*cx(i)*cy(i)* ( fIn(8,1,col) - ...  
fIn(7,1,col)-fEq(8,1,col)+fEq(7,1,col) );

% Right boundary
fOut(i,lx,col) = fEq(i,lx,col) + ...  
18*t(i)*cx(i)*cy(i)* ( fIn(6,lx,col) - ...  
fIn(9,lx,col)-fEq(6,lx,col)+fEq(9,lx,col) );

% Bounce back region
fOut(i,bbRegion) = fIn(opp(i),bbRegion);
end
% STREAMING STEP
for i=1:9
    fIn(i,:,:) = ... 
circshift(fOut(i,:,:), [0,cx(i),cy(i)]);
end
% VISUALIZATION
if (mod(cycle,tPlot)==0)
    u = reshape(sqrt(ux.^2+uy.^2),lx,ly);
    u(bbRegion) = nan;
    imagesc(u');
    axis equal off; drawnow
end
end
% end main loop
Results
Other simulation
Flow around moving boundary
Field Application

- Stimulate Material crystal condensation and diffusion
- Application of urban development planning
Simulate rules of the surface growth process, the probability cellular automata rules of the model forest fire, and the sand pile rules, even simulating the basic accumulation and collapse of particles like sand grains.

This method has been widely used in studying the recrystallization of metallic materials and dendritic growth of metal solidification process.
Field Application

The lattice Boltzmann method is based on the same idea of **cellular automata**.

**Application** of Lattice-Boltzmann method:

1. Land-cover variations
2. Human-land relationships
3. Urban development planning
Field Application

Fig. Numerical simulation of the different forms of the rock cranny

Fig. The arching phenomenon in crowd pedestrian flow

Fig. Simulation result on things of one kind come together
Thank You!
Further information and resources

- Master’s and PhD theses around the lattice Boltzmann method: wiki.palabos.org/literature:theses
- Forum for questions and discussions around lattice Boltzmann: palabos.org/forum/

Additional simulation examples

- Calculation of mixing quality in static fluid mixers: www.flowkit.com/showcases/static-mixers