Penn State University
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Smooth Particle Hydrodynamics (SPH)

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EGEE 520 Course
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Presentation Outlines

- Introduction
- Historical Perspective
- General Principles.
- Governing Equations
- Hand-Calculation Example
- Numerical Example
- Example Applications

Smoothed Particle Hydrodynamics (SPH)

It is a computational method used for simulating the dynamics of continuum media, such as solid mechanics and fluid flows. It has been used in many fields of research, including astrophysics, ballistics, volcanology, and oceanography. It is a mesh-free Lagrangian method (where the coordinates move with the fluid)

Smoothed Particle Hydrodynamics

Smoothed → Some particle properties are determined by taking an average over neighboring particles

Particle → The fluid is represented by a particle system

Hydrodynamics → Fluid dynamics

The Smoothed Particle Hydrodynamics (SPH) method works by dividing the fluid into a set of discrete elements, referred to as particles. These particles have a spatial distance (known as the "smoothing length", typically represented in equations by h), over which their properties are "smoothed" by a kernel function

Basic Concepts of Computational Hydrodynamics

Grid Based Methods

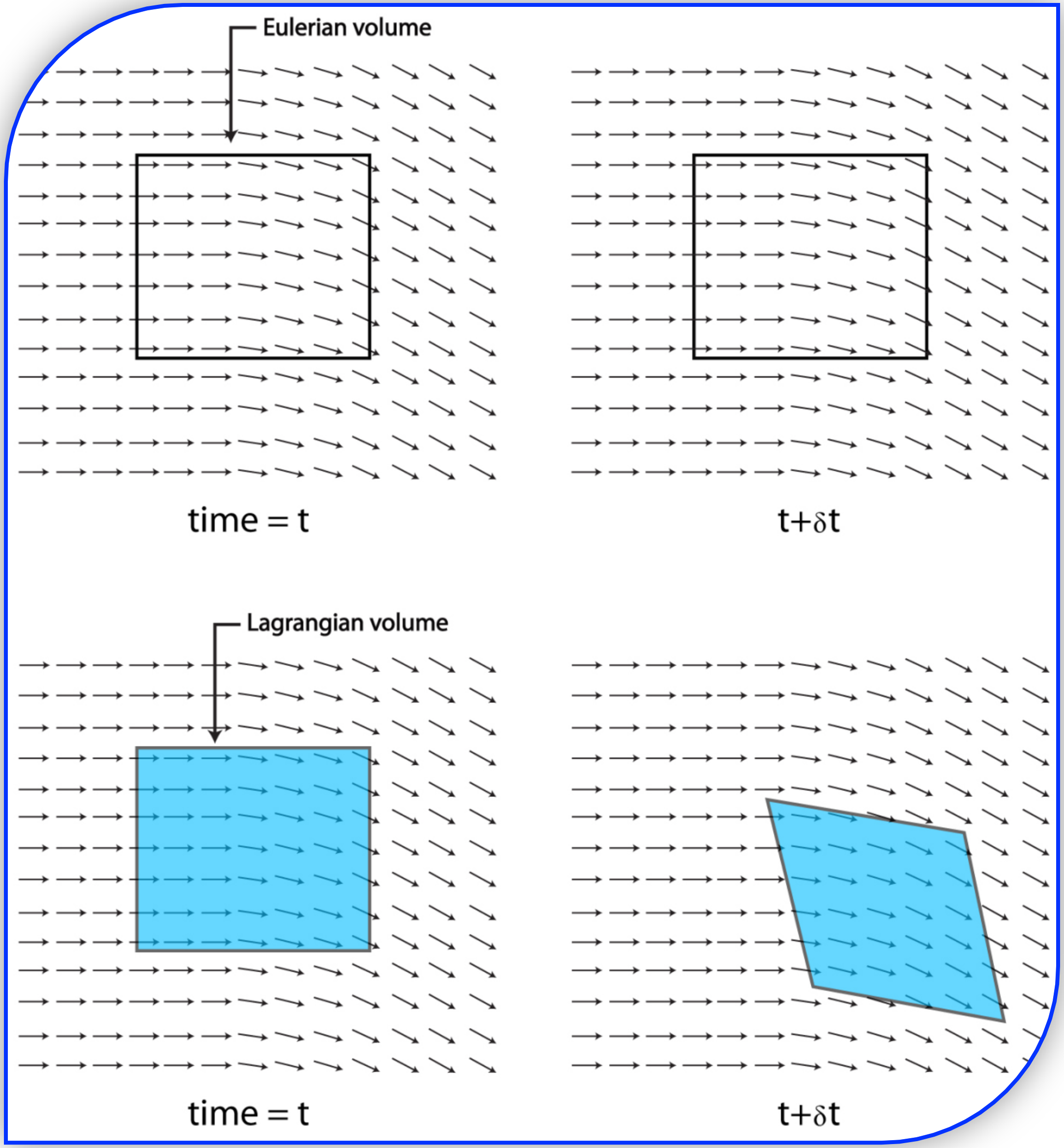
- *Eulerian approach (FDM)*
- *Lagrangian approach (FEM)*
- *Meshfree methods can use both approaches*

Particle Based Methods

- *Smoothed Particle Hydrodynamics*
- *Dissipative Particle Dynamics*
- *Brownian Dynamics*

Eulerian vs Lagrangian descriptions

Eulerian method	Lagrangian method
Concerned with fluid properties (velocity, density, pressure, temperature) at a specific space-time point (x,y,z,t).	*Concerned with a particular particle of fluid as it moves through space to reflect the behavior of the rest particles.*
Control volume approach	*Control mass approach*
Named after Swiss mathematician Leonhard Euler (1707-1783).	*Named after Italian mathematician Joseph Lagrange (1736-1813).*


Eulerian volume
time = t
t+δt
Lagrangian volume
time = t
t+δt

Eulerian vs Lagrangian descriptions

	Lagrangian Methods	Eulerian Methods
Grid	*Attaching on the moving material*	*Fixed in the space*
Track	*Movement of any point on materials*	*Mass, momentum, and energy flux across grid nodes and mesh cell boundary*
Time History	*Easy to obtain time-history data at a point attached on materials*	*Difficult to obtain time-history data at a point attached on materials*
Moving Boundary and Interference	*Easy to track*	*Difficult to track*
Irregular Geometry	*Easy to model*	*Difficult to model with good accuracy*
Large Deformation	*Difficult to handle*	*Easy to handle*

Meshfree and Meshfree Particle Methods

- *The basic idea of the meshfree methods is to discretize the continuum through a set of nodes without the connective mesh in order to follow the deformation experienced by the material and avoid the degradation of the numerical result maintaining a suitable computational effort.*

- *When the nodes assumes a physical meaning, i.e. they represent material particles carrying physical properties, the method is said to be meshfree particle and follows, in general, a Lagrangian approach.*

- *Accurate and stable numerical solutions for integral equations or PDEs with all kind of boundary conditions*

Limitations of Grid Based Methods

Not Suitable for problems involving:

- *Large displacements.*
- *Large inhomogeinities.*
- *Moving material interface.*
- *Deformable boundaries.*
- *Free surfaces.*

Historical Perspective

(Monaghan, 2005)	Computer graphics
(Springel, 2005)	Cosmological simulations
(Monaghan, 1992)	Applied to solid and fluid mechanics
(Benz, Slattery & Cameron, 1986)	Collision problems
(Miyama, Hayashi & Narita, 1984)	Fragmentation in collapsing molecular clouds
(Lucy, 1977), (Gingold & Monaghan, 1977)	Simulating astrophysics problems

Advantages of SPH Method

- *It can obtain the time history of the material particles. The advection and transport of the system can thus be calculated.*

- *The free surfaces, material interfaces, and moving boundaries can all be traced naturally in the process of simulation regardless the complicity of the movement of the particles, which have been very challenging to many Eulerian methods. Therefore, SPH is an ideal choice for modeling free surface and interfacial flow problems.*

- *The distinct meshfree feature of the SPH method allows a straightforward handling of very large deformations, since the connectivity between particles are generated as part of the computation and can change with time.*

- *SPH is suitable for problems where the object under consideration is not a continuum. This is especially true in bio- and nano- engineering at micro and nano scale, and astrophysics at astronomic scale*

- *SPH is comparatively easier in numerical implementation, and it is more natural to develop three-dimensional numerical models than grid based methods.*
- *Pure advection is treated exactly. For example, if the particles are given a colour, and the velocity is specified, the transport of colour by the particle system is exact*
- *With more than one material, each described by its own set of particles, interface problems are often trivial for SPH but difficult for finite difference schemes.*
- *Particle methods bridge the gap between the continuum and fragmentation in a natural way.*
- *The resolution can be made to depend on position and time, which makes the method very attractive for most astrophysical and many geophysical problems.*
- *SPH has the computational advantage, particularly in problems involving fragments, drops or stars that the computation is only where the matter is, with a consequent reduction in storage and calculation.*

Disadvantages of SPH Method

- *The main disadvantage of SPH is its limited accuracy in multi-dimensional flows due to noise. This noise seriously messes up the accuracy that can be reached with the technique, especially for subsonic flow, and also leads to a slow convergence rate.*

- *Particularly problematic in SPH are fluid instabilities across contact discontinuities, such as Kelvin-Helmholtz instabilities. These are usually found to be suppressed in their growth.*

- *Another generic problem is that the artificial viscosity is operating at some level also outside of shocks, giving the numerical model a relatively high numerical viscosity, which limits the Reynolds numbers that can be easily reached with SPH.*

Industrial applications of SPH Method

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

Smoothed particle hydrodynamics method for fluid flows, towards industrial applications: Motivations, current state, and challenges

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The basic step of the method

(*domain discretization, field function approximation and numerical solution):*

- *The continuum:*
  A set of arbitrarily distributed particles with no connectivity (meshfree);

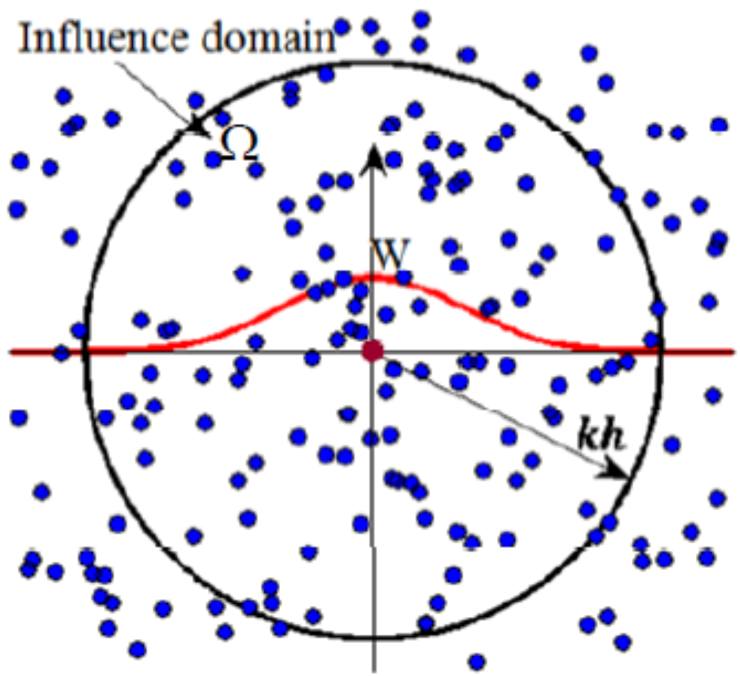
- *Field function approximation:*
  The integral representation method

- *Converting integral representation into finite summation:*
  Particle approximation

Integral representation of a function:

The continuum ⟶ **A set of arbitrarily particles**

$$f(x)=\int_{\Omega} f(x\ )W(x-x\ ,h)dx$$



Integral representation of a function:

- *The interpolation is based on the theory of integral interpolants using kernels that approximate a delta function*

- *The integral interpolant of any quantity function, A(r)*

$$A_I(r)=\int_{\Omega} A(r')W(r-r',h)\,dr'$$

- *where: r is any point in domain (Ω), W is a smoothing kernel with h as width.*

- *The width, or core radius, is a scaling factor that controls the smoothness or roughness of the kernel.*

Integral representation into finite summation

- *Numerical equivalent*

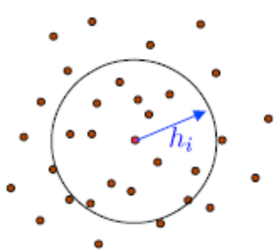
$$A_I(r)=\int_\Omega A(r')W(r-r',h)dr'$$

$$A_S(r)=\sum_j A_j V_j W(r-r_j,h)$$

- *where j is iterated over all particles, V_j is the volume attributed implicitly to particle j, r_j the position, and A_j is the value of any quantity A at r_j*

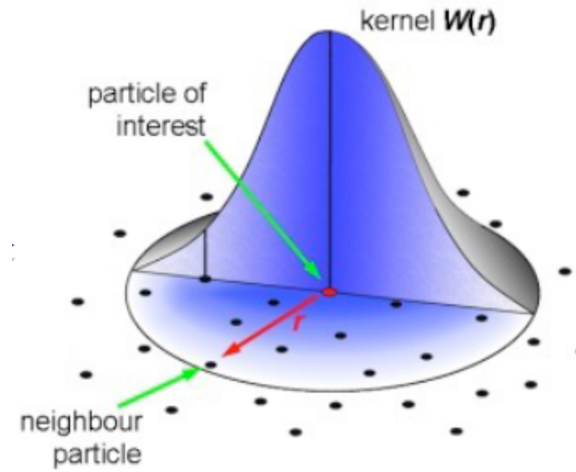
$$V=m/\rho$$

- *The basis formulation of the SPH*



$$A_S(r)=\sum_j A_j \frac{m_j}{\rho_j} W(r-r_j,h)$$

Smoothing function in support domain



- *Up to now, various kernel functions have been developed and used in the SPH method, among which the most widely used are the cubic spline kernel function and the Wendland kernel.*

$$-W(r_{ij},h)=C_h \begin{cases}(2-q)^3-4(1-q)^3 & for\ 0\le q\le 1\\ (2-q)^3 & for\ 1\le q\le 2\\ 0 & for\ q>2\end{cases}\quad C_h=15/(4\pi h^2)$$

$$W(r_{ij},h)=C_h \begin{cases}(2-q)^4(1+2q)-4(1-q)^3 & for\ 0\le q\le 2\\ 0 & for\ q>2\end{cases}\quad C_h=7/(64\pi h^2)$$

$$q=r_{ij}/h$$

Governing Equations

Each particle is specified by a state list:

mass,

velocity,

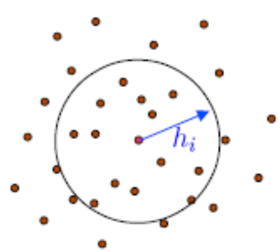
position,

force,

density,

pressure

Particle i $\longrightarrow$ $(m_i, \mathbf{v}_i, r_i, \mathbf{F}_i, \rho_i, p_i)$



Governing Equations

The acceleration of a particle is

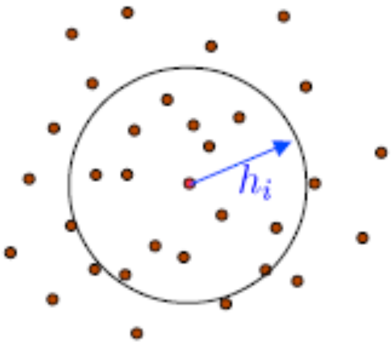
$$\boxed{\frac{dv_i}{dt} = a_i^{pressure} + a_i^{viscosity} + a_i^{interactive} + a_i^{gravity}}$$

Particle mass

Assume we have the same mass for all particles, $m_i = m$

The mass m is calculated by

$$m = \frac{(Density\ of\ fluid)\cdot(Total\ volume)}{Total\ number\ of\ particles}$$

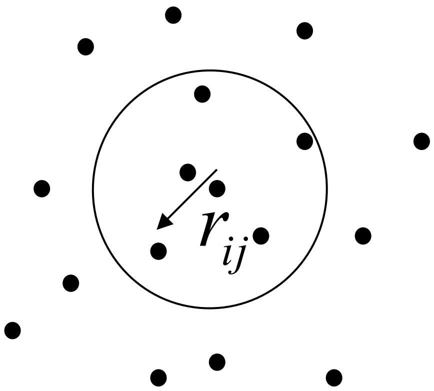


Let us now go back to the weighted averages…

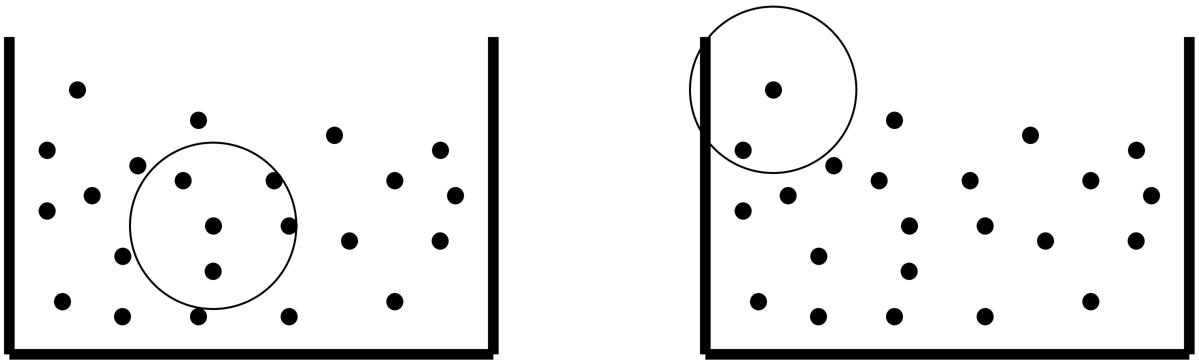
How to determine the density of a particle?

$$\rho_i = \sum_j m_j W(r_{ij})$$

Weight function or Kernel function



Surface tracking



We can find the surface by monitoring the density
If the density at a particle deviates too much compared to expected density
we tag it as a surface particle

Pressure

We get the pressure from the relation:

$$p_i = c_s^{\,2}\left(\rho_i - \rho_0\right)$$

where Cs is the speed of sound and ρ0 is the fluid reference density

$$(\boxed{m_i}, \mathbf{v}_i, r_i, \mathbf{F}_i, \boxed{\rho_i}, \boxed{p_i})$$

- *The next property we focus on is the force*
- *Velocities and positions are calculated from the forces in a way similar to an ordinary particle system*
- *But before we go into that we need to learn more about taking averages…*

In SPH, the average is formally defined as follows:

$$\langle A(r) \rangle = \int_V A(r')W(r-r')dr'$$

For numerical solutions, the summation is used in stead of integration.

$$\langle A \rangle_i \approx \sum_j \frac{m_j}{\rho_j} A_j W(r_{ij})$$

$$\langle \nabla A \rangle_i \approx \sum_j \frac{m_j}{\rho_j} A_j \nabla W(r_{ij})$$

$$\langle \nabla^2 A \rangle_i \approx \sum_j \frac{m_j}{\rho_j} A_j \nabla^2 W(r_{ij})$$

$$\langle \rho \rangle_i \approx \sum_j \frac{m_j}{\rho_j} \rho_j W(r_{ij}) \approx \sum_j m_j W(r_{ij})$$

Meshless method!!

Velocities and Forces

Motion equation in elasticity:

$$\frac{dv}{dt} = \frac{1}{\rho}\nabla\cdot\sigma + \frac{1}{\rho}F_{ext}$$

where:

$$\sigma = C\varepsilon$$

$$\sigma = -p\mathrm{I} + \mu\dot{\varepsilon}$$

All this together produces the following fluid equation called

Navier-Stokes equation

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla\cdot\nabla\mathbf{v} + \frac{1}{\rho}F_{ext} + \mathbf{g}$$

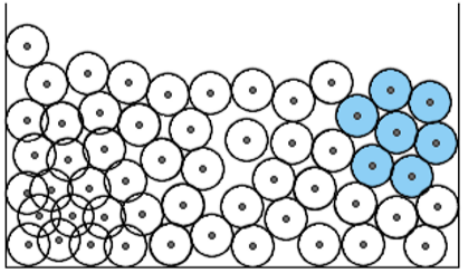
Convert each term on the RHS in Navier-Stokes to SPH-averages

- *First term (pressure) becomes:*

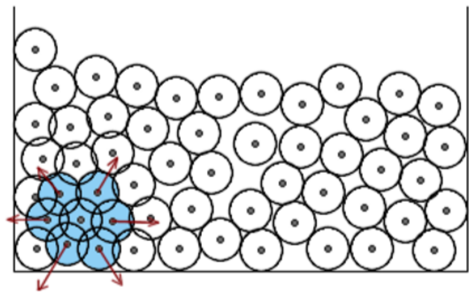
$$\left\langle -\frac{1}{\rho}\nabla p \right\rangle_i \approx \sum_j P_{ij} \nabla W(r_{ij})$$

where

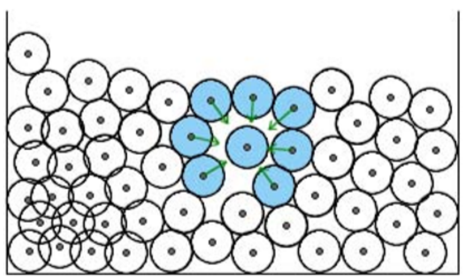
$$P_{ij} = -\frac{m_j}{\rho_j}\left(\frac{p_i}{\rho_i^{\,2}} + \frac{p_j}{\rho_j^{\,2}}\right)$$



a) Balanced mass-density in the marked region, hence no produced pressure forces.



b) High mass-density in the marked region will produce repulsive pressure forces.



c) Low mass-density in the marked region will produce attractive pressure forces.

$$\left\langle -\frac{1}{\rho}\nabla p \right\rangle_i \approx \sum_j P_{ij} \nabla W(r_{ij})$$

The second term (viscosity):

$$\left\langle -\frac{\mu}{\rho}\nabla\cdot\nabla\mathbf{v}\right\rangle_i \approx \sum_j \mathbf{V}_{ij}\boxed{\nabla^2 W(r_{ij})}$$

where

$$\mathbf{V}_{ij} = -\mu\frac{m_j}{\rho_j}\left(\frac{\mathbf{v}_i}{\rho_i^{\,2}}+\frac{\mathbf{v}_j}{\rho_j^{\,2}}\right)$$

Summary

- *The acceleration of a particle can now be written:*

$$\boxed{\frac{dv_i}{dt} = a_i^{pressure} + a_i^{viscosity} + a_i^{external} + a_i^{gravity}}$$

$$a_i^{pressure} \approx \sum_j P_{ij} \nabla W(r_{ij})$$

$$a_i^{viscosity} \approx \sum_j V_{ij} \nabla W(r_{ij})$$

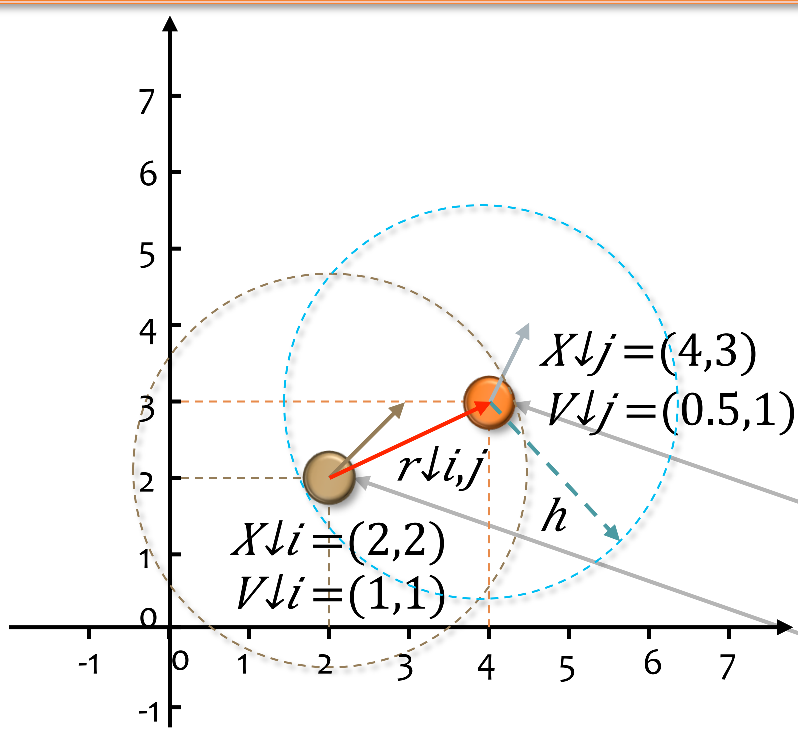
$$a_i^{external} \approx \frac{1}{\rho_i} F_i^{external}$$

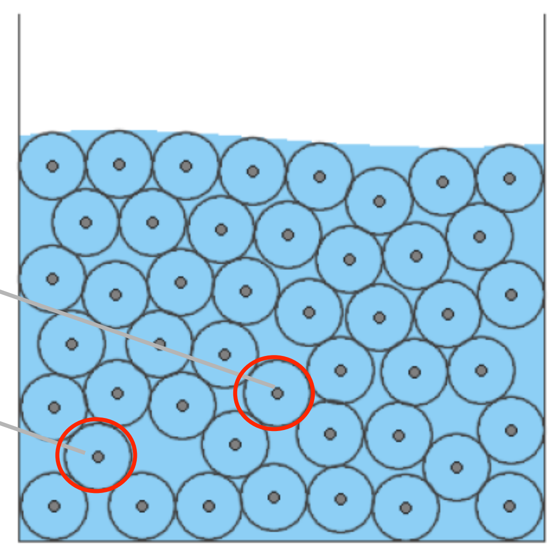
$$a_i^{gravity} \approx (0,0,-g)$$

Hand-Calculation Example

Problem Description

	Mass /m	Density /ρ	Pressure /P	Viscosity /μ	Velocity /v	Location /(x,y)	h	Δt
Particle i	1	1	1	1	(1,1)	(2,1)	2.5	1
Particle j	1	1	1	1	(0.5,1)	(1,2)	2.5	1





Calculation of ∇W_{ij}

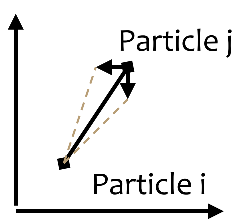
Using the numerical method to solve $\nabla W_{i,j}$

$$\frac{\partial W_{ij}}{\partial x} = \left(\frac{-45*((2-0.001)^2+1^2)^{3/2}}{4\pi*h^5} + \frac{45*((2-0.001)^2+1^2)}{2\pi*h^4} - \frac{15}{\pi*h^2}\right) - \left(\frac{-45*5^{3/2}}{4\pi*h^5} + \frac{45*5}{2\pi*h^4} - \frac{15}{\pi*h^2}\right) / 0.001 = -2.43*10^{-1}$$

$$\frac{\partial W_{ij}}{\partial y} = \left(\frac{-45*(2^2+(1-0.001)^2)^{3/2}}{4\pi*h^5} + \frac{45*(2^2+(1-0.001)^2)}{2\pi*h^4} - \frac{15}{\pi*h^2}\right) - \left(\frac{-45*5^{3/2}}{4\pi*h^5} + \frac{45*5}{2\pi*h^4} - \frac{15}{\pi*h^2}\right) / 0.001 = -1.21*10^{-1}$$

$$-W(r,h) = C_h \begin{cases} (2-q)^3 - 4(1-q)^3 & for\ 0 \le q \le 1 \\ (2-q)^3 \end{cases}$$

$$q = \frac{r_{j,i}}{h} = \frac{\sqrt{5}}{2.5} <$$



$$W_{ij} = \frac{45 r_{ij}^3}{4\pi h^5} - \frac{45 r_{ij}^2}{2\pi h^4} + \frac{15}{\pi h^2}$$

$$\nabla W_{i,j} = \frac{\partial W_{ij}}{\partial x}, \frac{\partial W_{ij}}{\partial y} = (-2.43*10^{-1}, -1.21*10^{-1});$$

Calculation of ∇W_{ji}

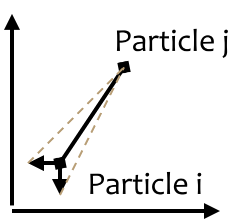
Using the numerical method to solve ∇W_{ji}

$$\frac{\partial W_{ji}}{\partial x} = \left(\frac{-45*((-2-0.001)^2+1^2)^{3/2}}{4\pi*h^5}+\frac{45*((-2-0.001)^2+1^2)}{2\pi*h^4}-\frac{15}{\pi*h^2}\right)-\left(\frac{-45*5^{3/2}}{4\pi*h^5}+\frac{45*5}{2\pi*h^4}-\frac{15}{\pi*h^2}\right)/0.001 = -2.43*10^{-1}$$

$$\frac{\partial W_{ji}}{\partial y} = \left(\frac{-45*(2^2+(-1-0.001)^2)^{3/2}}{4\pi*h^5}+\frac{45*(2^2+(-1-0.001)^2)}{2\pi*h^4}-\frac{15}{\pi*h^2}\right)-\left(\frac{-45*5^{3/2}}{4\pi*h^5}+\frac{45*5}{2\pi*h^4}-\frac{15}{\pi*h^2}\right)/0.001 = -1.21*10^{-1}$$

$$-W(r,h)=C_h \begin{cases}(2-q)^3-4(1-q)^3 & for\ 0\le q\le 1 \\ (2-q)^3 \end{cases}$$

$$q=\frac{r_{j,i}}{h}=\frac{\sqrt{5}}{2.5} <$$



$$W_{ji}=\frac{45 r_{ji}^3}{4\pi h^5}-\frac{45 r_{ji}^2}{2\pi h^4}+\frac{15}{\pi h^2}$$

$$\nabla W_{ji}=\left(\frac{\partial W_{ji}}{\partial x},\frac{\partial W_{ji}}{\partial y}\right)=(-2.43*10^{-1},-1.21*10^{-1});$$

Calculation of $\nabla^2 W_{i,j}$

Using the numerical method to solve $\nabla^2 W_{i,j}$

$$\nabla^2 W_{ij}|_x = \frac{W_{i+0.001,j} - 2W_{ij} + W_{i-0.001,j}}{0.001^2} = \frac{-0.257075 - 2*(-0.257317) - 0.257558}{0.001^2} = 5.1*10^{-1}$$

$$\nabla^2 W_{ij}|_y = \frac{W_{i,j+0.001} - 2W_{ij} + W_{i,j-0.001}}{0.001^2} = \frac{-0.257196 - 2*(-0.257317) - 0.257438}{0.01^2} \quad -2.36*10^{-1}$$

$$-W(r,h) = C_h \begin{cases} (2-q)^3 - 4(1-q)^3 & for\ 0 \le q \le 1 \\ (2-q)^3 \end{cases}$$

$$q = r_{j,i}/h = \sqrt{5}/2.5 < 1$$

$$W_{ji} = \frac{45 r_{ji}^3}{4\pi h^5} - \frac{45 r_{ji}^2}{2\pi h^4} + \frac{15}{\pi h^2}$$

$$\nabla^2 W_{i,j} = \nabla^2 W_{ij}|_x, \nabla^2 W_{ij}|_y = (-5.1*10^{-1}, -2.36*10^{-1});$$

Calculation of $\nabla^2 W_{j,i}$

Using the numerical method to solve $\nabla^2 W_{j,i}$

$$\nabla^2 W_{ji}|_x = \frac{W_{j+0.001,i} - 2W_{ji} + W_{j-0.001,i}}{0.001^2} = \frac{-0.257558-2*(-0.257317)-0.257075}{0.001^2} = -5.1*10^{-1}$$

$$\nabla^2 W_{ji}|_y = \frac{W_{j,i+0.001} - 2W_{ji} + W_{j,i-0.001}}{0.001^2} = \frac{-0.257438-2*(-0.257317)-0.257196}{0.001^2} = -2.36*10^{-1}$$

$$-W(r,h)=C_h \begin{cases} (2-q)^3 - 4(1-q)^3 & for\ 0\le q\le 1 \\ (2-q)^3 & \end{cases}$$

$$q = \frac{r_{j,i}}{h} = \frac{\sqrt{5}}{2.5} < 1$$

$$W_{ji} = \frac{45 r_{ji}^3}{4\pi h^5} - \frac{45 r_{ji}^2}{2\pi h^4} + \frac{15}{\pi h^2}$$

$$\nabla 2W_{ji} = \nabla^2 W_{ji}|_x , \nabla^2 W_{ji}|_y = (-5.1*10^{-1}, -2.36*10^{-1});$$

Calculation of Acceleration, Velocity and Location

$$a_i = a_i^{pressure} + a_i^{viscosity} + a_i^{external} + a_i^{gravity} = \sum_j P_{ij} \nabla W(r_{ij}) + \sum_j V_{ij} \nabla^2 W(r_{ij}) + (1/\rho_i) F_i^{external} + (0,-g) = (-2.43*10^{-1}, -1.21*10^{-1}) - 3.5(-5.1*10^{-1}, -2.36*10^{-1}) + (0,0) + (0,0) = (1.542, 0.705)$$

$$\boldsymbol{V_{i\,new} = V_i + a_i * \Delta t = (1,1) + (1.542, 0.705) = (2.542, 1.705)}; \quad \boldsymbol{X_{i\,new} = X_i + V_{i\,new} * \Delta t = (2,2) + (2.542, 1.705) = (4.542, 3.705)};$$

Calculation of Acceleration, Velocity and Location

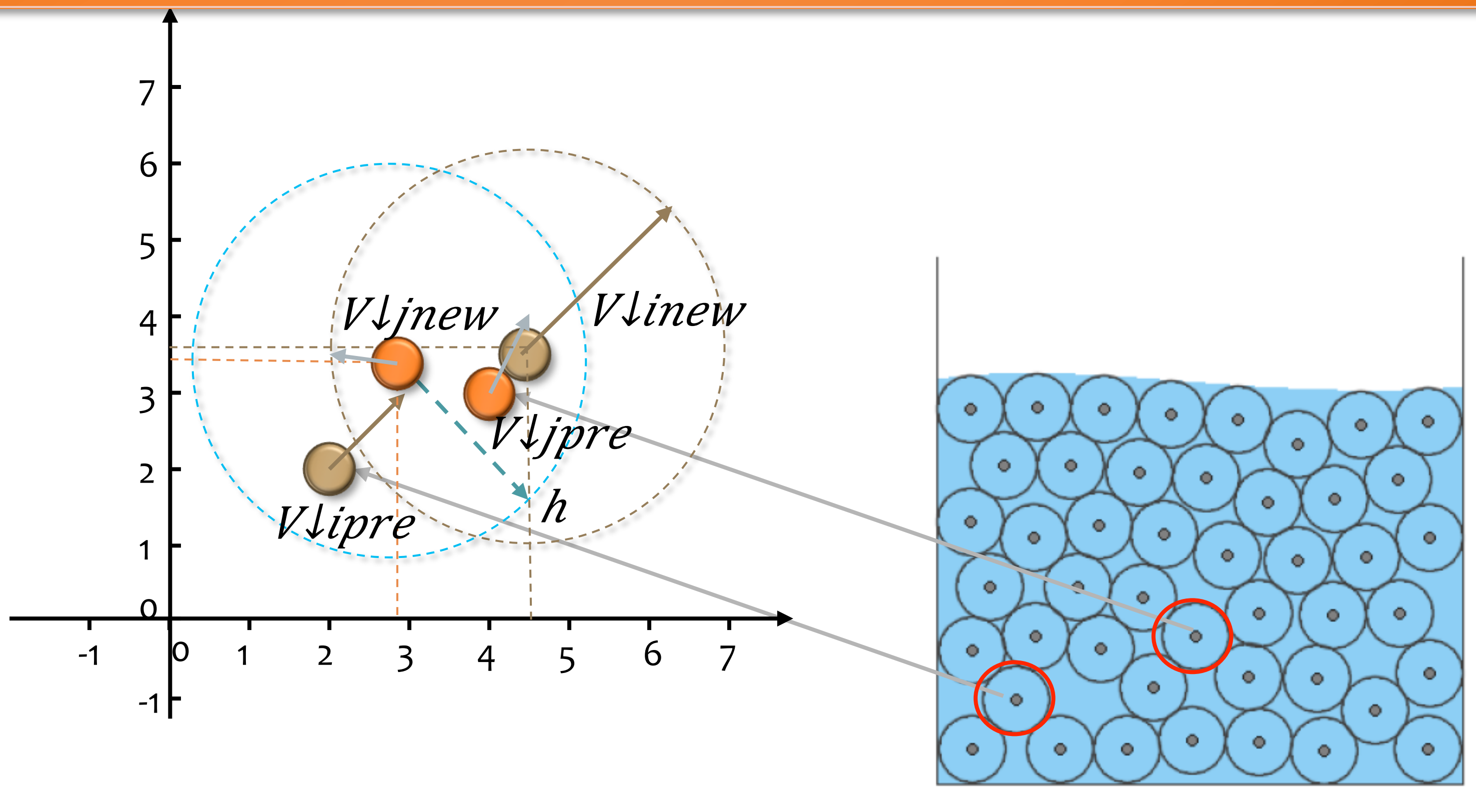
$$a_j = a_j^{pressure} + a_j^{viscosity} + a_j^{external} + a_j^{gravity} = \sum_i P_{ji} \nabla W(r_{ji}) + \sum_i V_{ji} \nabla^2 W(r_{ji}) + (1/\rho_j) F_j^{external} + (0,-g) = -(-2.43*10^{-1}, -1.21*10^{-1}) + 3.5(-5.1*10^{-1}, -2.36*10^{-1}) + (0,0) + (0,0) = (-1.542, -0.705)$$

$$\boldsymbol{V_{j\,new} = V_j + a_j * \Delta t = (0.5,1) + (-1.542,-0.705) = (-1.042,0.295)};$$

$$\boldsymbol{X_{j\,new} = X_j + V_{j\,new} * \Delta t = (4,3) + (-1.042,0.295) = (2.958,3.295)};$$

Calculation Results

	Mass /m	Density /ρ	Pressure /P	Viscosity /μ	Velocity /v	Location /(x,y)	h	Δt
Particle i	1	1	1	1	(2.5,1.7)	(2,1)	2.5	1
Particle j	1	1	1	1	(-1.0,0.3)	(1,2)	2.5	1



For each time step:

- *Discrete from integration to summation*
- *Numerical approximation for Kernel Function*
- *Calculate density for each particle*
- *Calculate pressure for each particle*
- *Calculate all type of accelerations for each particle, and sum it up*
- *Find new velocities and positions by using the same summation method as before…*

Numerical Example

Simulation Example

- Software Introduction
- Software preparation
- Numerical Computation
- Parameter Setup
- Coding
- Visualization of result

Software Introduction

- Code source: Open source.
- Dimension: 2-D & 3-D.
- Numerically computation.
- Visualization routines with ParaView.

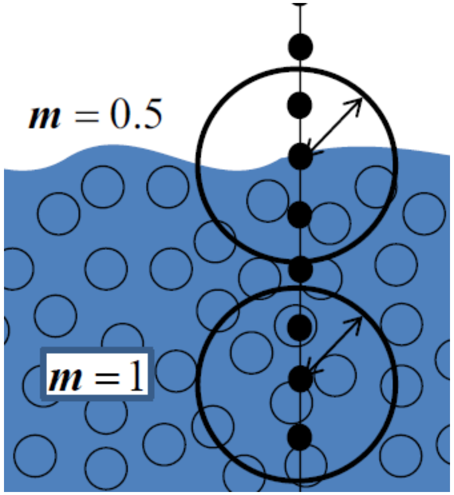
Software preparation

- Windows: Intel Visual Fortran
  Silverfrost FTN95
  GNU gfortran compiler on Cygwin

- Mac: GNU gfortran

- Linux: GNU gfortran compiler on Cygwin
  ParaView 5.3.0 (Visualization)

Numerical Computation

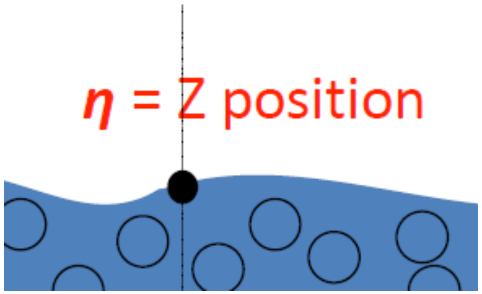


Numerically compute free-surface elevation:

1) For a given (X,Y) position.

2) Compute numerical MASS at different Z positions using MASS values of neighbouring fluid particles:

$$m_a = \sum_b m_b W_{ab}$$



3) We will choose as wave elevation:
the Z value for which m=0.5*m(reference)

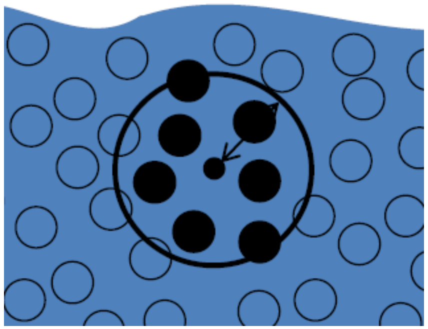
Numerical Computation

Numerically compute velocity:

1). For a given location.

2). Compute numeical velocity using velocity values of neighbouring fluid particles:

$$V_a = \frac{\sum_b V_b W_{ab}}{\sum_b W_{ab}}$$



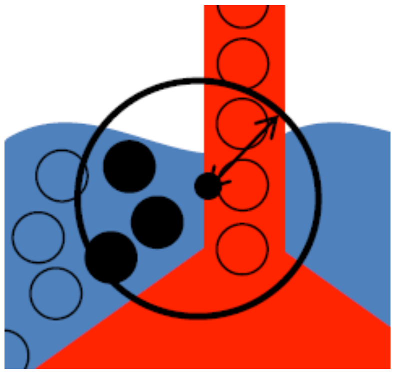
Numerical Computation

Numerically compute pressure:

1). For a given location.

2). Compute numerical PRESSURE
using PRESSURE values of neighbouring fluid particles:

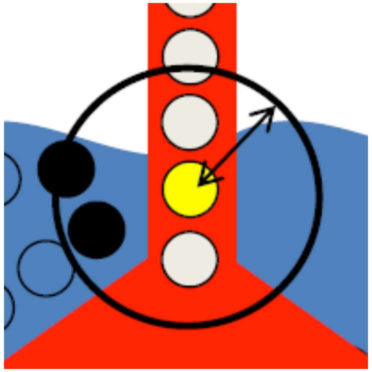
$$\boldsymbol{P}_a = \frac{\sum_b \boldsymbol{P}_b W_{ab}}{\sum_b W_{ab}}$$



Numerical Computation

Numerically compute forces:

1) For a range of boundary particles



2) We compute numerical ACCELERATION of those boundary particles solving the particle interactions with fluid neighbouring particles :

$$\frac{d\mathbf{v}_a}{dt} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi_{ab} \right) \nabla_a W_{ab} + \mathbf{g}$$

3) Summation of ACCELERATION values of those boundary particles:

$$\mathbf{F} = m \sum \frac{d\mathbf{v}_a}{dt}$$

Parameter Setup

Kernel Function:

- Gaussian
- Quadratic
- Cubic
- Wendland

Parameter Setup

Time-stepping algorithm:

- Predictor-Corrector
- Verlet
- Symplectic
- Beeman

Parameter Setup

Viscosity treatment:

- Artificial viscosity
- Laminar
- Laminar viscosity+ Sub-Particle Scale

Parameter Setup

Density Filter:

- Zeroth Order – Shepard Filter
- First Order – Moving Least Squares (MLS)

Other choices

- Kernel correction (None)
- Equation of state (Tait’s equation)
- Boundary conditions (Dalrymple)
- Geometry of the zone (Box)
- Initial fluid particle structure (BCC)

Coding

In Cygwin, with virtual Linux platform:

- Compiles SPHysicsgen_2D by SPHysicsgen.make.
- Run SPHysicsgen_2D with input file(Case1.txt), then obtain a output file (Case1.out) .
- Compile and generate SPHysics_2D by SPHysics.make, places the SPHysics_2D executable.
- Execute SPHysics_2D.
- Visualize by ParaView.

Coding-bug and solution

Bug:

'Xilink' Command in SPHYSICS.mak file cannot be recognize by new version of Cygwin.

Solution:

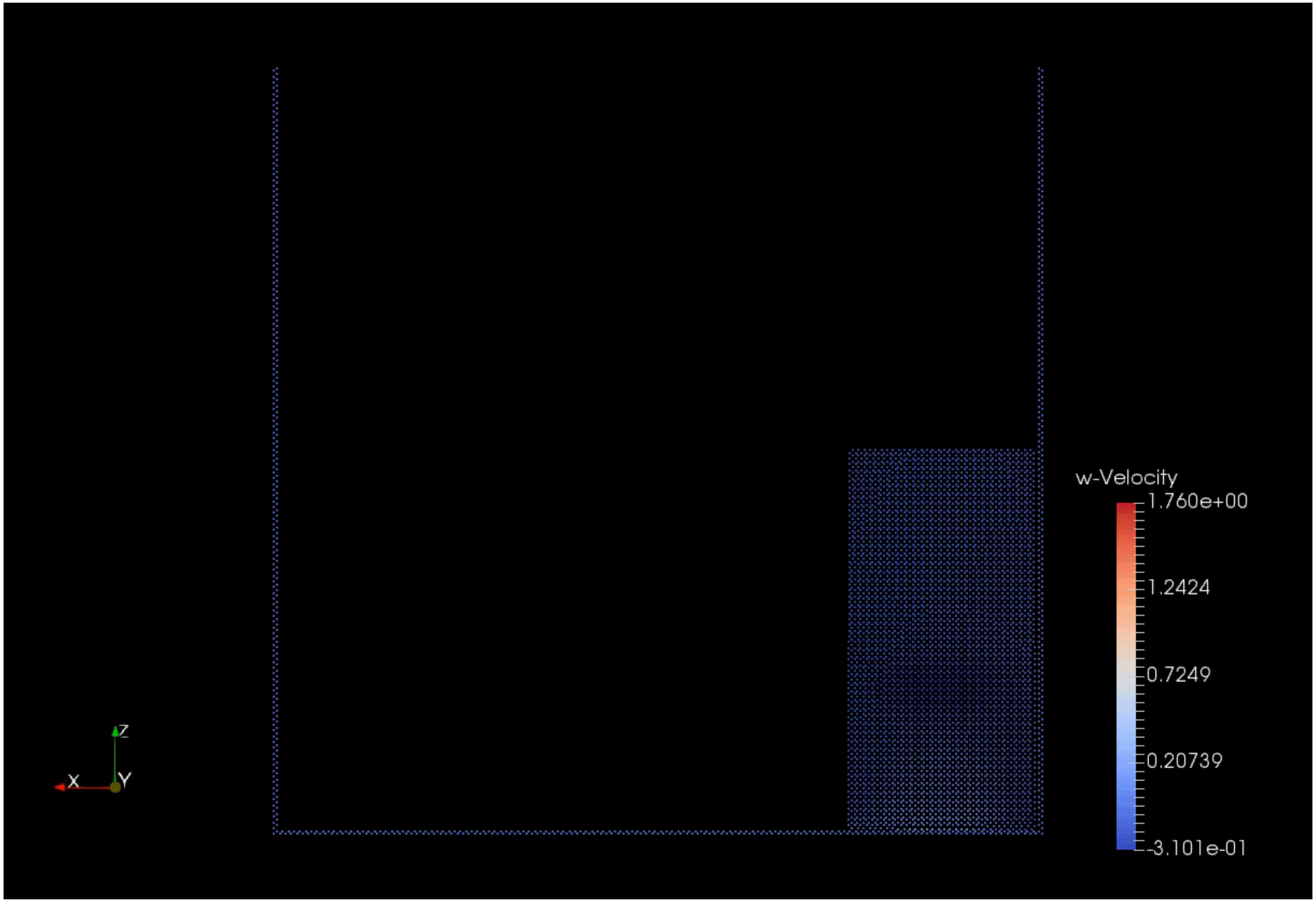
Remodify the PHYSICS.mak file as follows:

```
.f.obj:
	ifort $(OPTIONS) $(COPTIONS) /O3 /c $<

SPHYSICS_2D.exe: $(OBJFILES)
	xilink /OUT:$@ $(OPTIONS) $(OBJFILES)

clean:
	del *.mod *.obj
```

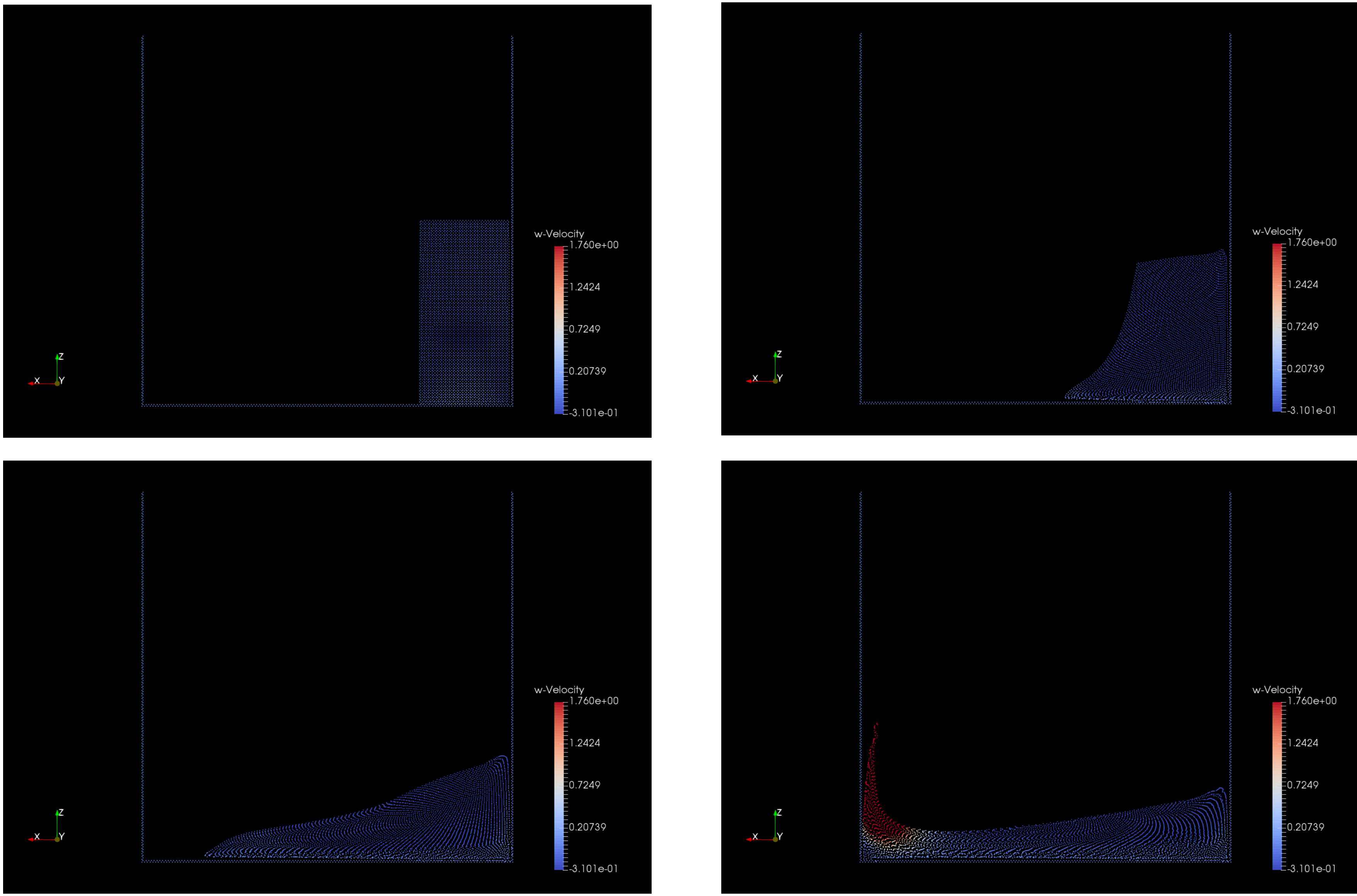
→

```
.f.obj:
	ifort $(OPTIONS) $(COPTIONS) /O3 /c $<

SPHYSICS_2D.exe:
	gfortran -o SPHYSICS_2D.exe $(OBJFILES)

clean:
	rm *.mod *.o
```

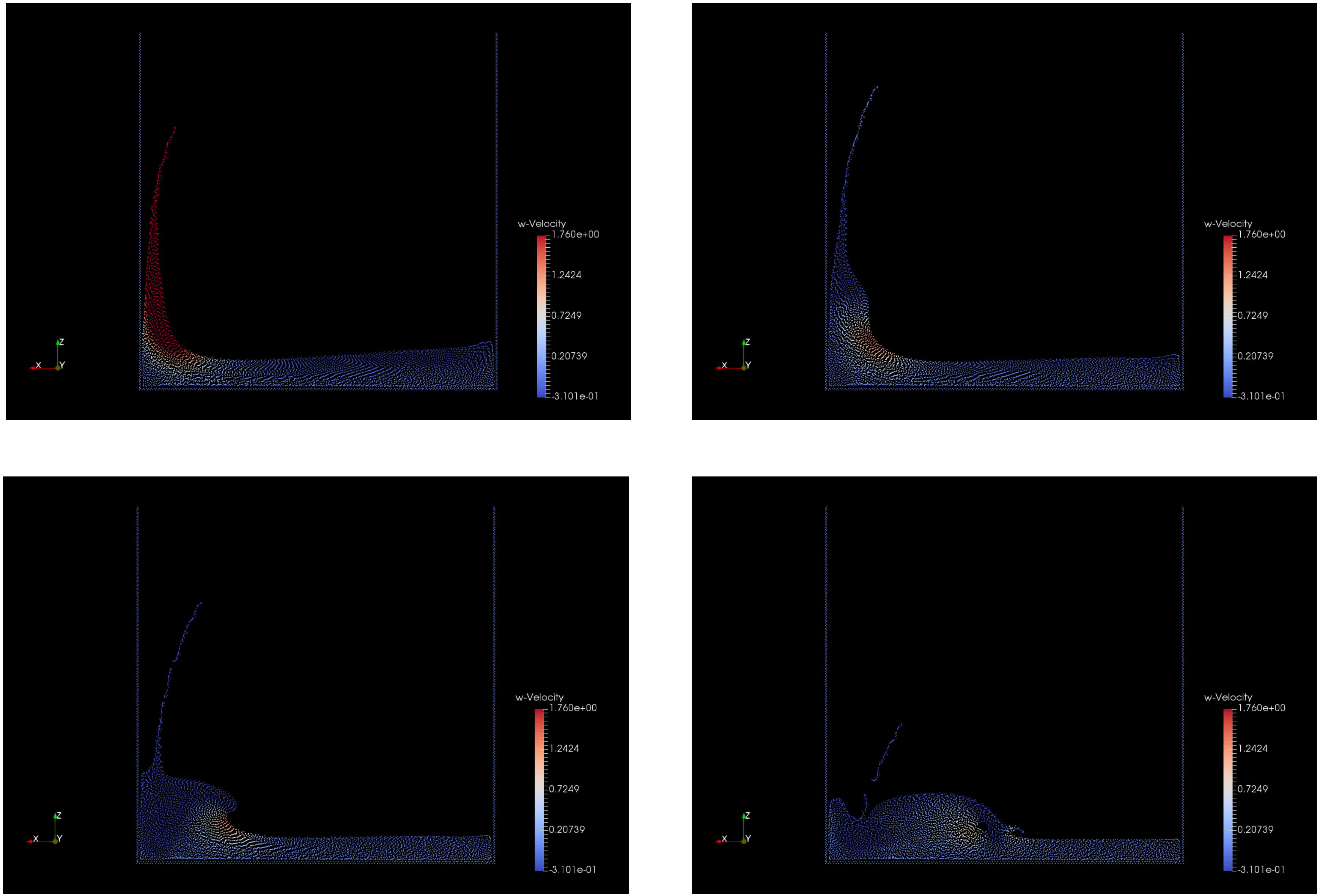
Visualization of result



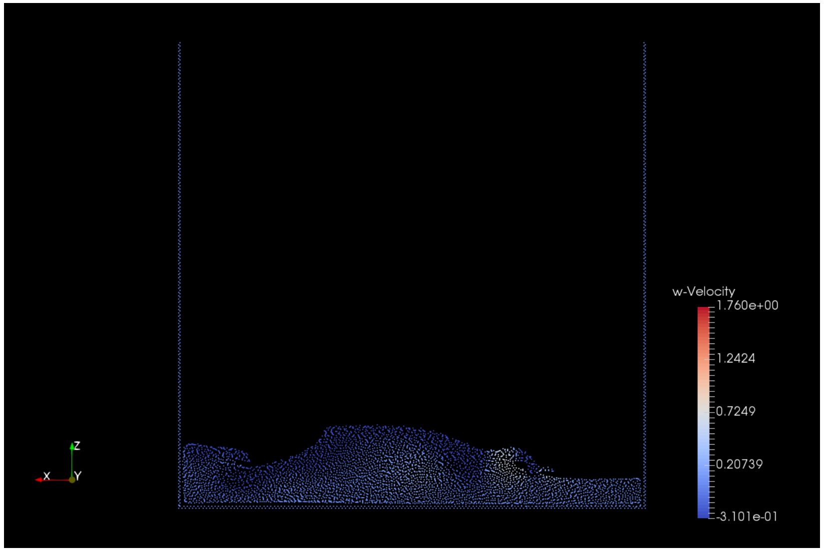
Visualization of result (t=0, 15,30,45)

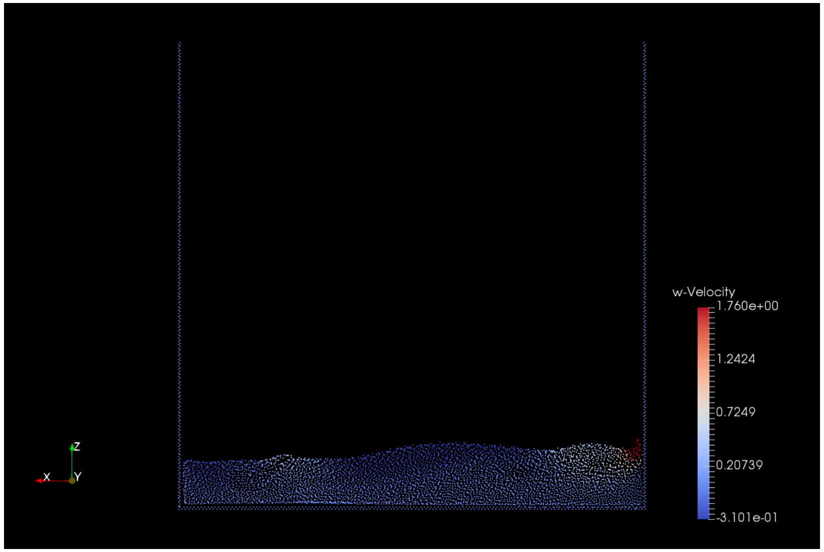


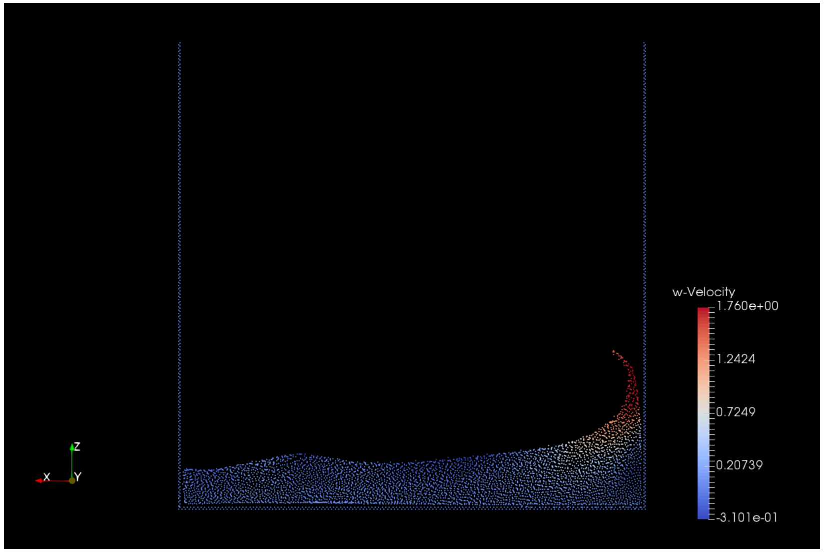
Visualization of result (t=60, 75,90,105)



Visualization of result (t=120, 135,149)



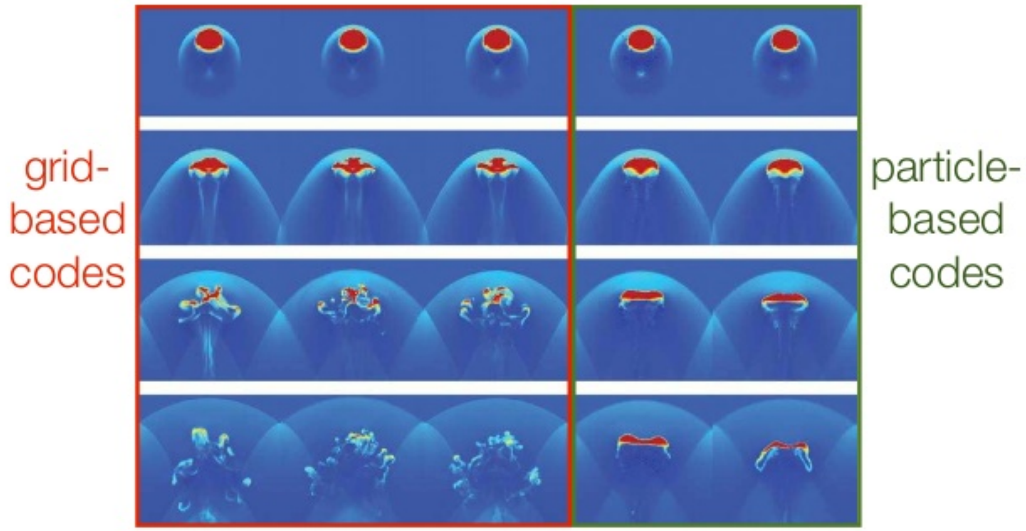




Example Applications

Uses in astrophysics

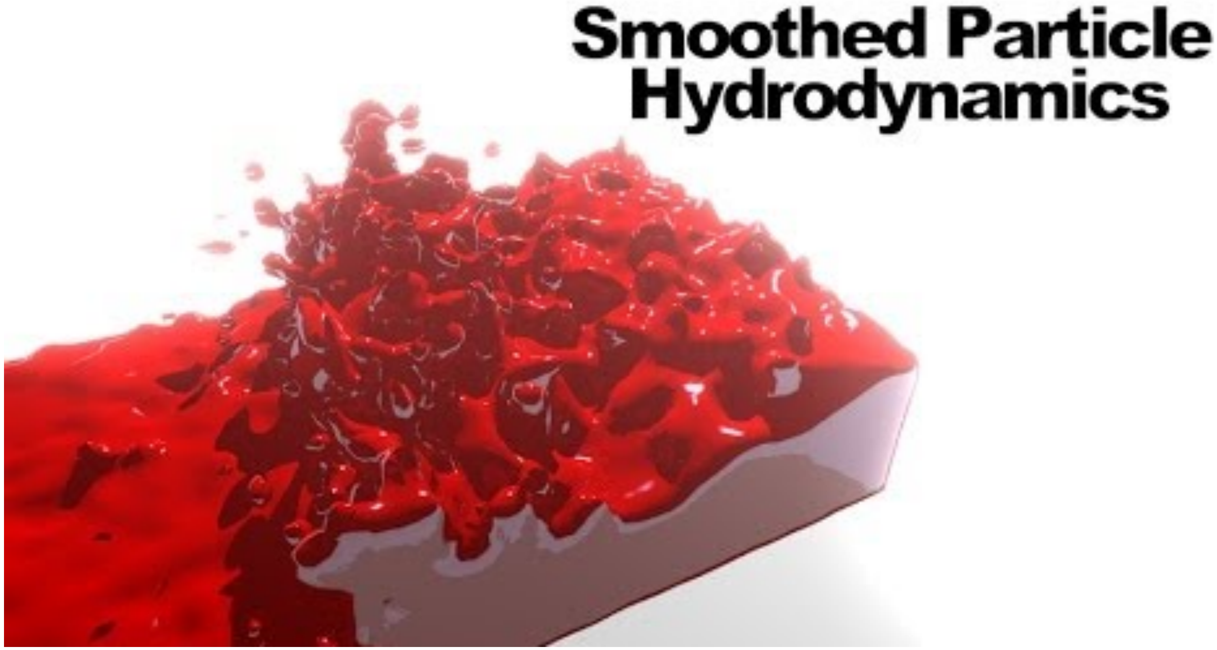
Smoothed-particle hydrodynamics's adaptive resolution, numerical conservation of physically conserved quantities, and ability to simulate phenomena covering many orders of magnitude make it ideal for computations in theoretical astrophysics.



Agertz et al 2007, "Fundamental Differences Between SPH and Grid Methods", *Monthly Notices of the Royal Astronomical Society*

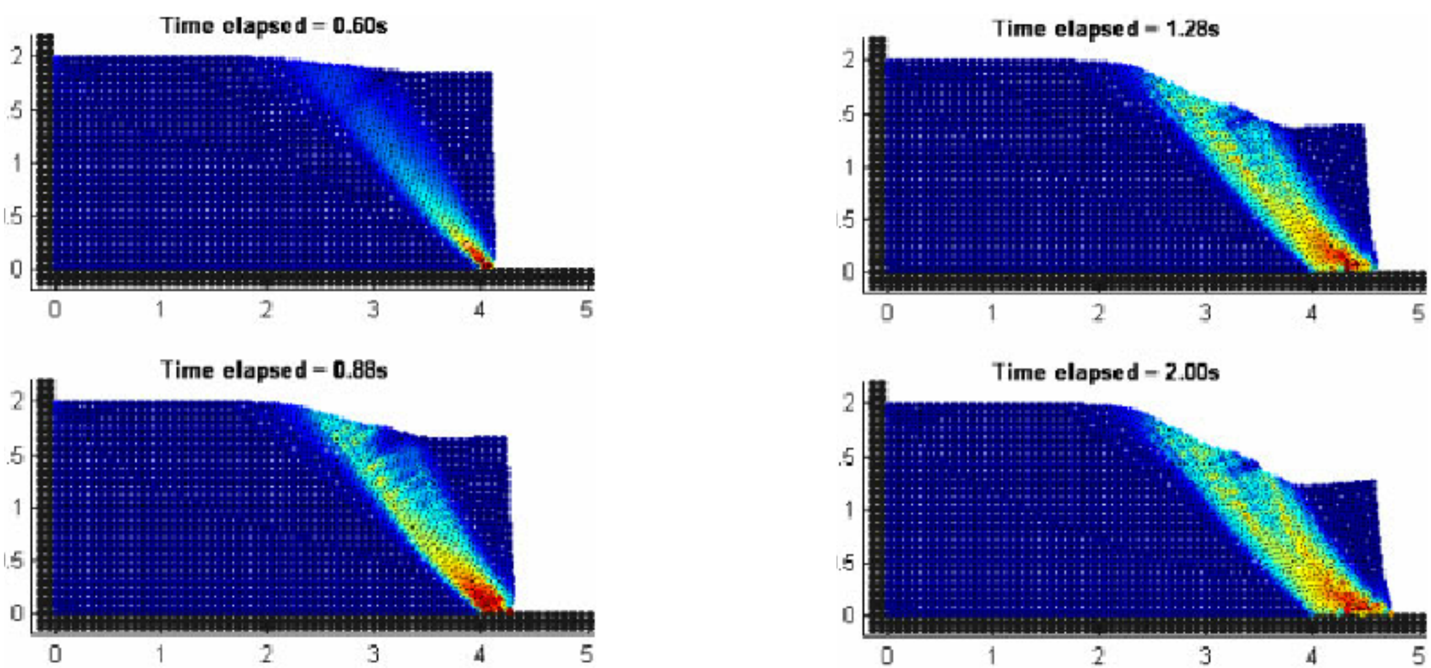
Uses in fluid simulation

First, SPH guarantees conservation of mass without extra computation. Second, SPH computes pressure from weighted contributions of neighboring particles. Finally, SPH creates a free surface for two-phase interacting fluids directly.For these reasons it is possible to simulate fluid motion using SPH in real time.



Uses in solid mechanics

This feature has been exploited in many applications in Solid Mechanics: metal forming, impact, crack growth, fracture, fragmentation, etc.



[Questions]