EGEE-520

DEM

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Introduction and Historical Perspective

What is DEM?

- Discrete Element Method
- Particle based
- Lagrangian
- Explicit time stepping
- Simulates behavior of granular materials
- Granular media are modelled with individual (discrete) particles

Origin of DEM

- Originated in geomechanics by Cundall (1971) – Progressive movements of rock masses as 2D rigid blocks
- Extended into a RBM code by Cundall (1974)
- Approximating the deformations of blocks of complex 2D geometry – code translated into FORTRAN by Cundall (1978)
- Computer codes for 3D problems developed by Cundall and Hart (1985)

Why DEM instead of Continuum?

- Continuum models do not capture:
  - Relative movements of the particles
  - Rotations of the particles
- Sophisticated constitutive models are required to capture the behavior of the granular material
- In DEM many of the mechanical response features associated with the granular materials are captured

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What is DEM?

- Discrete Element Method
- Particle based
- Lagrangian
- Explicit time stepping
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DEM Strengths

- Loads and displacements can be applied to virtual samples to simulate the physical lab tests
- Allows us to look inside the material
- Complex behavior is captured through the separately acting physical process algorithms
- Allows analysis of the mechanisms that involve large displacements

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DEM Weaknesses

- Realistic particle shapes and arrangements are difficult to create and to calibrate
- Roughness, texture, and sharp edges of particles are not modelled
- Particle breakage or chipping is usually disallowed
- Idealized contact models (Hertz-Mindlin, etc.)
- Computationally expensive

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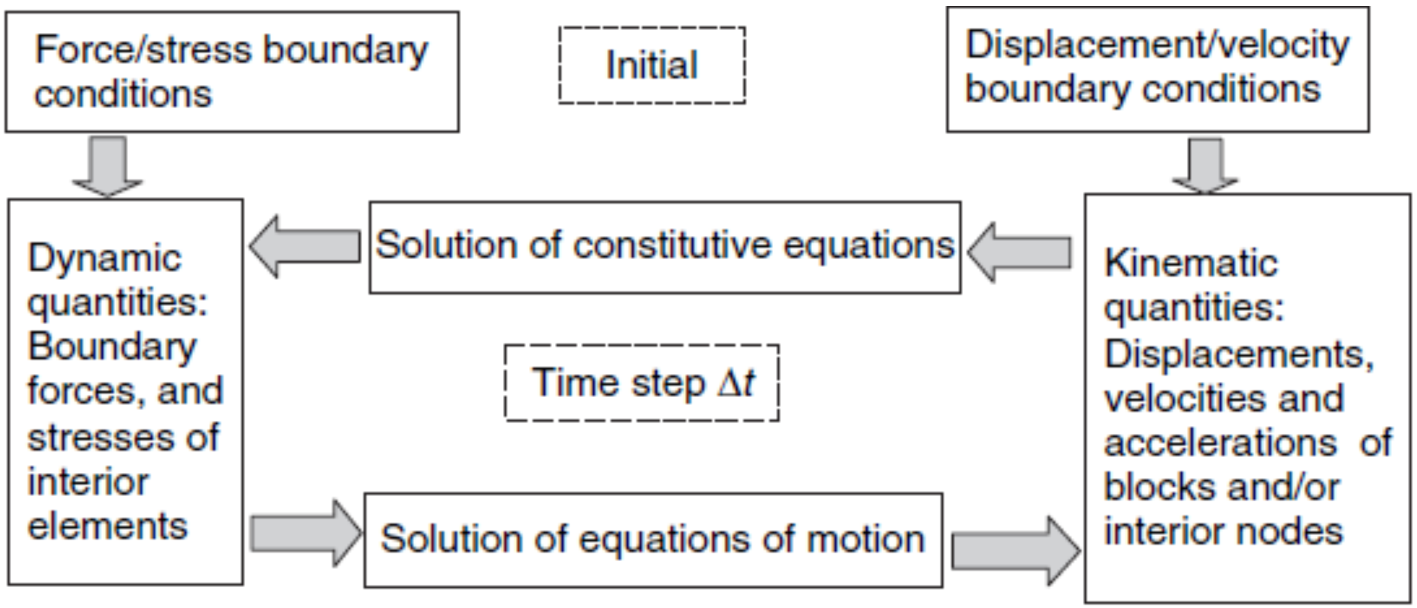
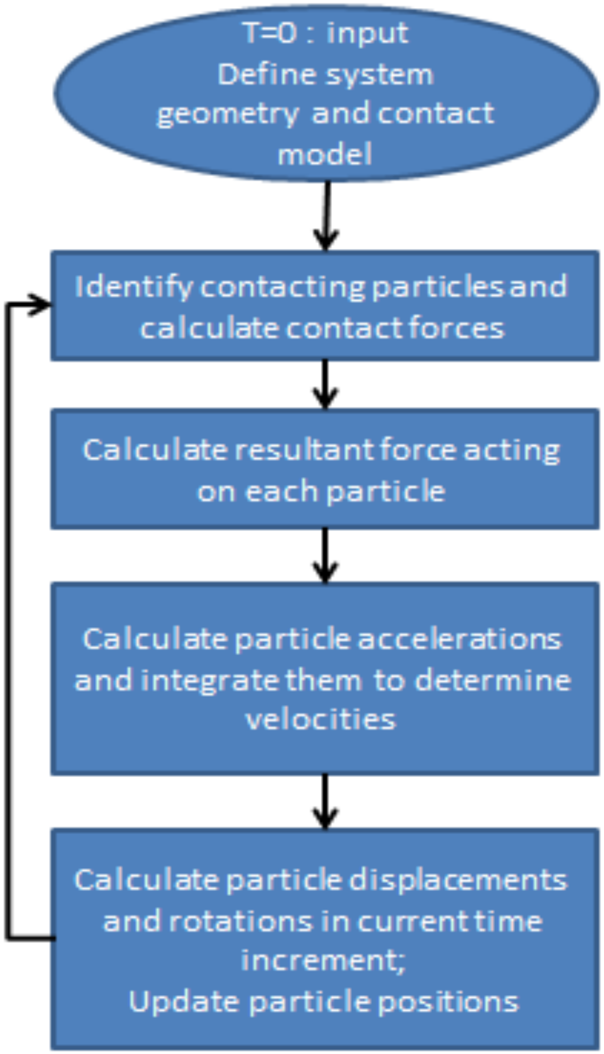
DEM Applications

- Agriculture and food handling
- Civil Engineering
- Chemical Engineering
- Oil and gas
- Mining
- Mineral processing
- Pharmaceutical
- Powder metallurgy

DEM Flow chart



DEM Flow chart



General Principles

General Principles

- Developed to study dry granular materials
  - granular mechanics uses standard contact law
  - contact law to develop creep theory
  - anisotropy of clays: contact laws + repulsive force
  - particle crushing: contact laws for cementation
  - strain localization: contact laws and granular rolling
- The one main feature:
  - complex responses controlled by **contact laws** and interparticle contacts.

Outline

- Linear normal contact model
  - introduces initial boundary conditions
- Adhesive, elasto-plastic normal contact model
  - takes into account plastic deformation
- Tangential forces
- Coupling example: sliding/stick slip model
  - rolling model
  - torsion model

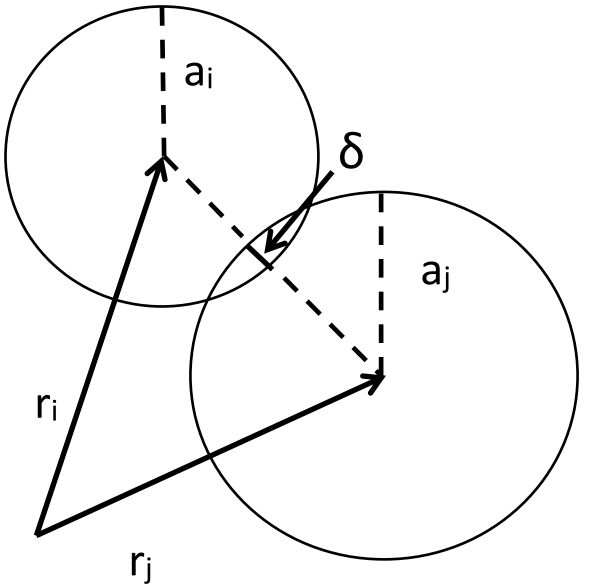
Linear normal contact model

- Interaction of 2 particles: **i** and **j**
- assumptions
  - only interact if they are in contact
  - displacement/overlap occurs (δ)

$$\delta = (a_i + a_j) - (r_i - r_j) \cdot n$$

- where $n = (r_i - r_j)/|r_i - r_j|$ vector

  pointing from **i** to **j**



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Linear contact model: Force

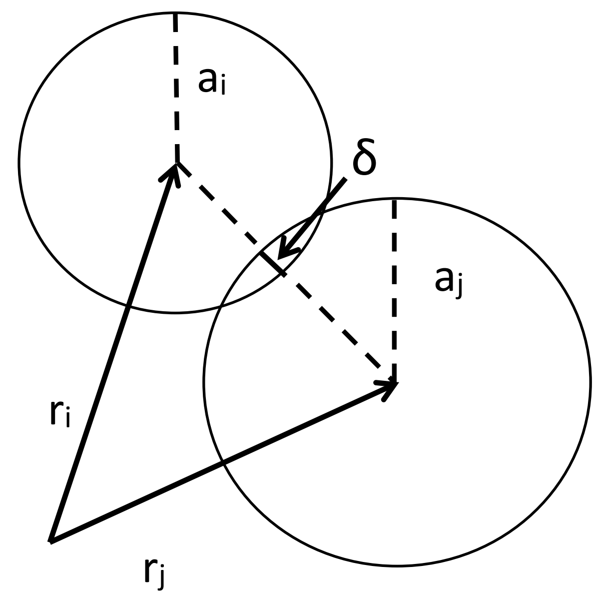
- The force from **i** and **j** at the contact (f^c) is split into:
  - normal force (f^n)
  - tangental force (f^t)

$$f^c = f^n + f^t$$

- focusing on normal force

$$f^n = k\delta + \gamma_o v_n$$

- where k is the spring stiffness
- γ_o is a viscous damping coefficient
- v_n is the velocity in the normal direction



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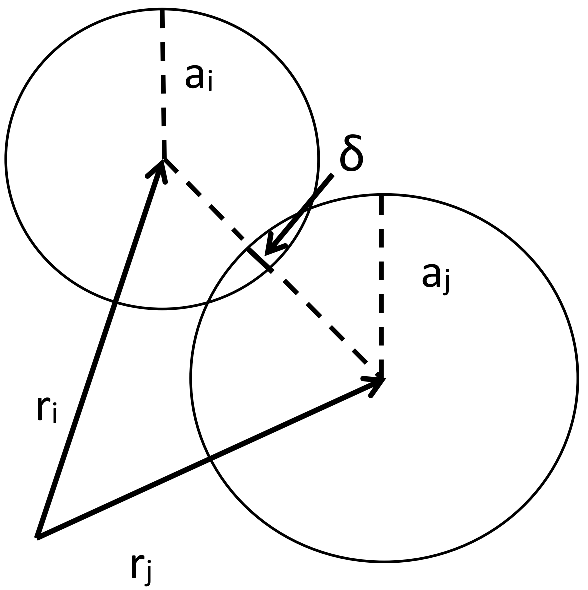
Adhesive, elasto-plastic normal contact model

- Takes into account plastic deformation

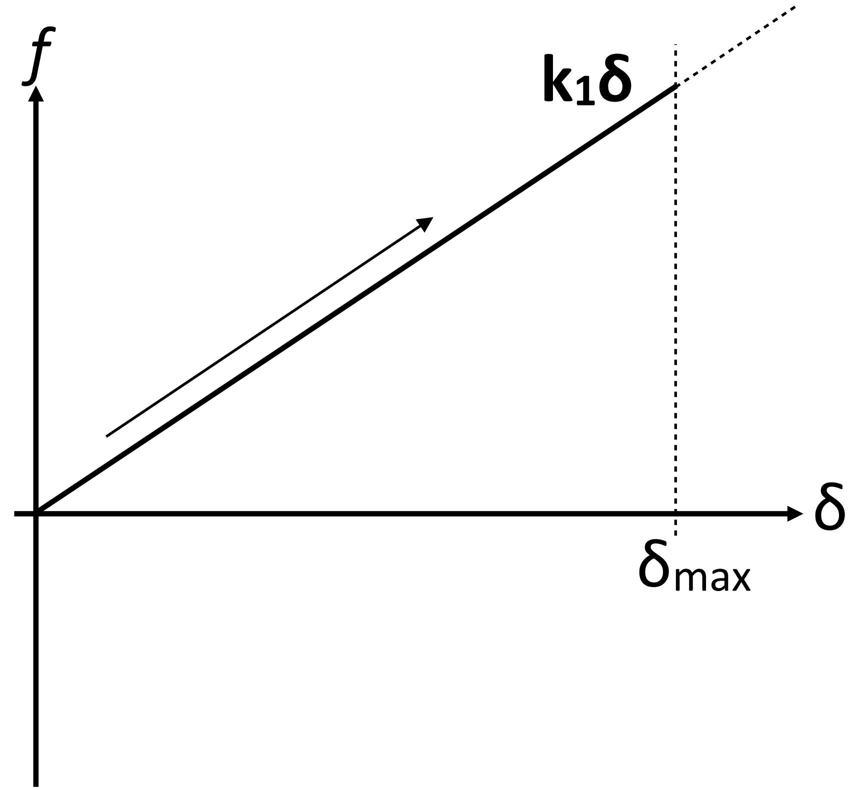
- There will be memory effects where force is

  - loading: $k_1\delta$
  - un/reloading: $k_2*(\delta-\delta_o)$
  - unload: $-k_c\delta$



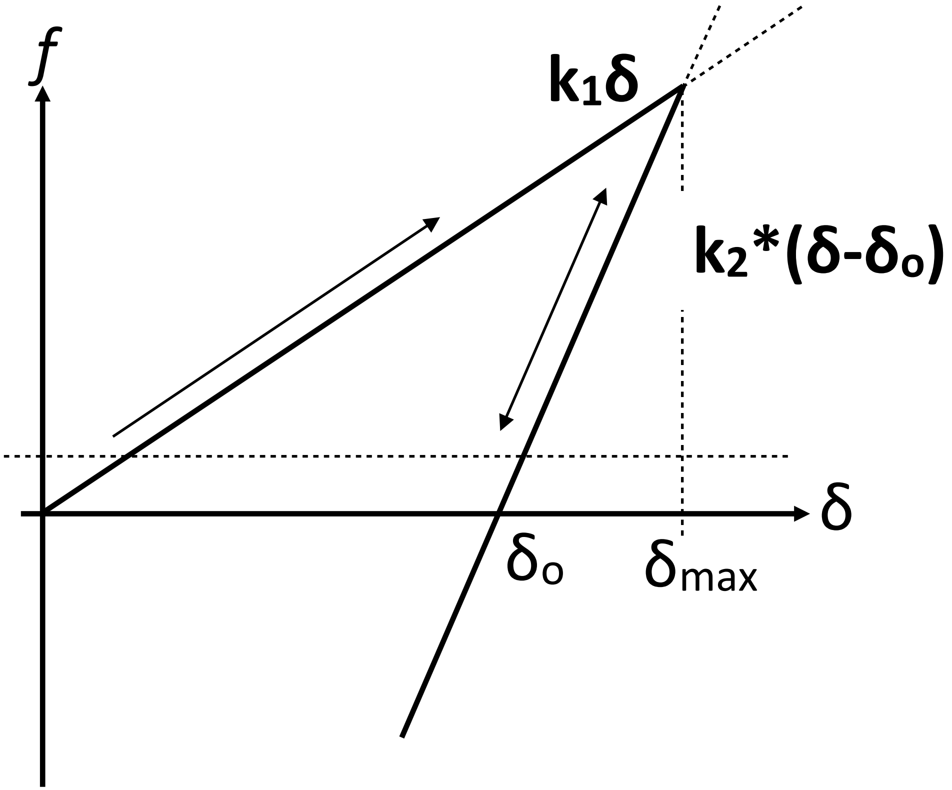
Memory effects

- loading: f increases linearly with δ until δ_{max} is reached
  - δ_{max} = memory parameter
- the line with slope k_1 defines the maximum f for a given δ



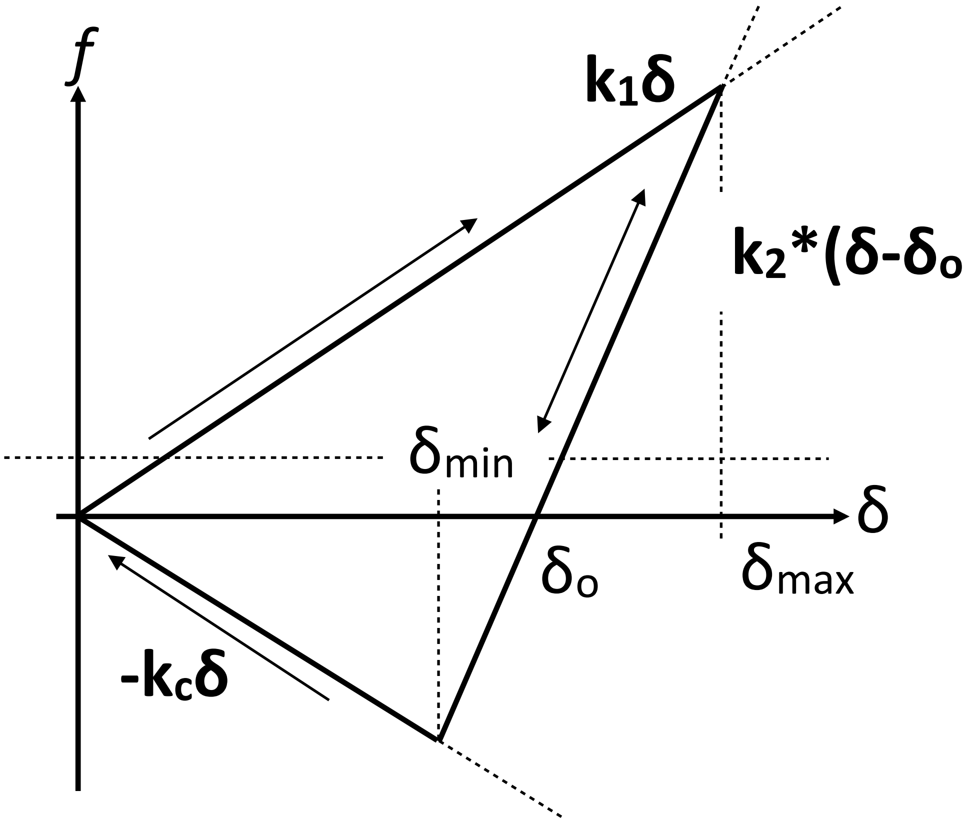
Memory effects continued

- unloading: f decreases down to 0 at δ_o along the line k_2
- reloading at any instant leads to an increase of f along the same line
- once f_{max} is exceeded, the force follows the slope k_1 and δ_{max} is adjusted



Memory effects continued

- below δ_o = negative attractive forces until the $f = -k_c\delta_{min}$
- k_1 and $-k_c$ define the range of possible force values
  - deviation from that takes place in un/reloading phases and follows k_2



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Tangential forces

- For the tangential degrees of freedom, there are three different force and torque laws

- Friction
  - dynamic and static sliding behavior

- Rolling resistance
  - distance two particles roll over each other without sliding
  - activates torque (2 particles rotating anti-parallel)

- Torsion resistance
  - when two particles are rotating anti-parallel with spins parallel to the normal direction

Sliding/stick-slip model

- One example of how they can be coupled is through a sliding/ stick slip model
- The f^t is coupled with the f^n through Coulomb's law:
  - $f^t \leq f_C = u^s f^n$
- Where, for the sliding case, the dynamic friction is:
  - $f^t = f^t{}_C = u^d f^n$
    - With $u^d \leq u^s$
- For an adhesive contact (seen earlier), the Coulomb law is modified so that
  - $f^n = f^n + k_c \delta$

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Sliding/stick-slip model

- If you have an active contact, need to project a tangential spring into the tangential plane
- $\xi = \xi' - n(n\xi')$
  - where ξ' is the spring in the previous step
- to compute the changes in the spring, a tangential test force is computed:
  - $f^t_o = -k_t \xi + \gamma_t v_t$
- If $f_o^t \leq f_C^s$ with $f_C^s = u^s(f^n + k_c\delta)$ you have static friction
- if $f_o^t > f_C^s$ then sliding friction becomes active

Summary

- The four parameters in the friction law are k_t, u_s, $\phi = u_s/u_d$, and γ_t
  - accounting for tangential stiffness

I. project tangential spring into the tangential plane

II. compute the changes in the spring via tangential test force

III. continuously iterate at time steps and plug the equations into each other for each step and track the state of friction

Summary

- The rolling resistance model
  - the three new parameters (k_r, u_r, γ_r with $\phi_r = \phi_d$) will be used like in the friction law
  - these parameters account for a rolling stiffness, static rolling friction coefficient, and a rolling viscosity

- The torsion resistance model
  - the three new parameters (k_O, u_O, γ_O with $\phi_r = \phi_O$) will be used like in the friction law
  - these parameters account for a torsion stiffness, static torsion friction coefficient, and a torsion viscosity

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Other applications

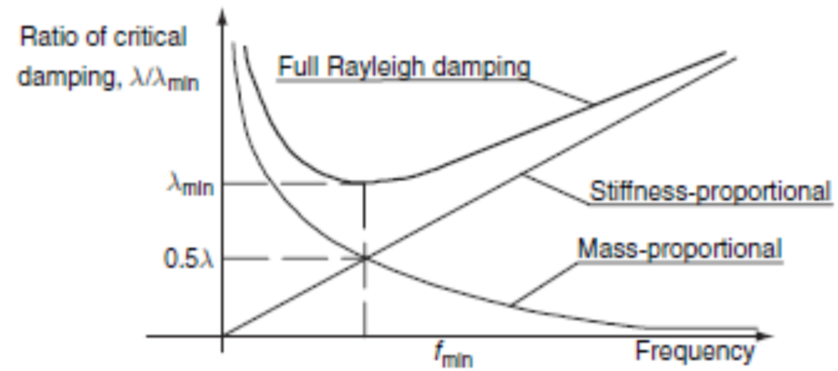
- This was only looking at 2 grains…
- real applications look at thousands of grains to analyze:
  - propagation and mechanics of fractures
  - Mine structure and rock reinforcement
  - underground civil structure
  - glacial loading/unloading
  - crustal deformation
- All of these examples need to look at the stress state, strength, and stiffness of the material

Governing Equations

Damping:

Mass proportional damping:

$$d_i^m = -\alpha \partial u_i / \partial t \, m$$



Stiffness proportional damping:

$$d_i^s = \beta K_{ij} \partial u_j / \partial t$$

Critical damping ratio using Bathe and Wilson equation:

$$\lambda = 1/2 \ (\alpha/\omega + \beta\omega)$$

$$\lambda_{\min} = \sqrt{\alpha/\beta} = \omega_{min}$$

$$f_{min} = \omega_{min} / 2\pi$$

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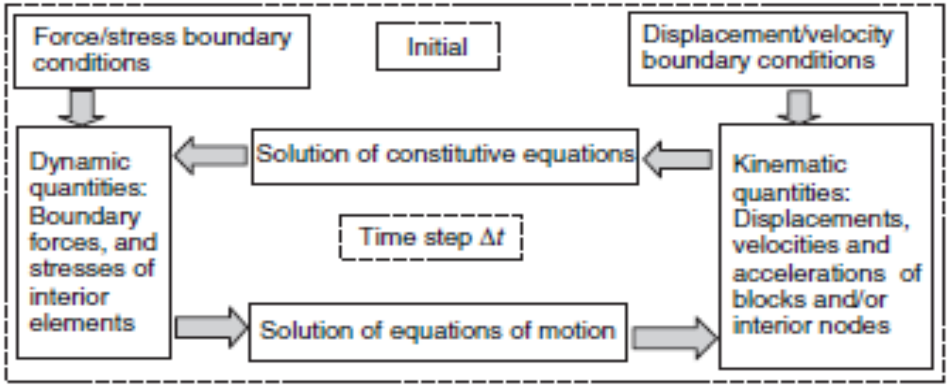
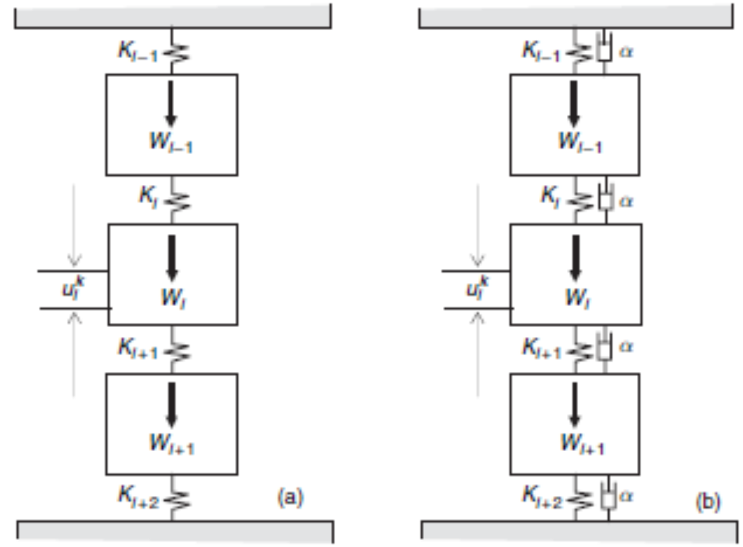
Dynamic Relaxation:





From Newton's Second Law:

$$ma_x + \alpha m v_x = F_x \qquad ma_y + \alpha m v_y = F_y \qquad I\frac{d^2\theta}{dt^2} + \alpha I \frac{d\theta}{dt} = T$$

From Hooke's Law:

$$F_i = K_i\left[(u_i^{k+1} - u_{i-1}^{k+1}) + (u_i^{k} - u_{i-1}^{k}) \ldots\ldots\ldots + (u_i^{1} - u_{i-1}^{1})\right] = K_i\left[\sum_{j=1}^{K+1} (u_i^j - u_{i-1}^j)\right]$$

$$F_{i+1} = K_{i+1}\left[\sum_{j=1}^{K+1} (u_i^j - U_{i+1}^j)\right]$$

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Dynamic Relaxation (cont'd):

<u>Doing a force balance:</u>

$$m_i \frac{d^2 u_i^k}{dt^2} + \alpha m_i * \left(\frac{du_i^k}{dt}\right) = W_i - K_i \sum_{j=1}^{k-1} \left(u_i^j - u_{i-1}^j\right) - K_{i+1} \sum_{j=1}^{k-1} \left(u_{i+1}^j - u_i^j\right) = f_i^{k-1}$$

$$\frac{d^2 u_i^k}{dt^2} = \frac{u_i^{k+1} - 2u_i^k + u_i^{k-1}}{(\Delta t)^2} \quad ; \quad \frac{du_i^k}{dt} = \frac{u_i^{k+1} - u_i^{k-1}}{2\Delta t}$$

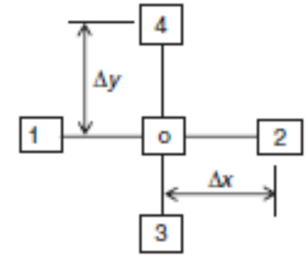
$$u_x^{k+1} = \left(1+\frac{\Delta t}{2}\alpha\right)^{-1} \left\{ \frac{f_x^{k-1} (\Delta t)^2}{m} + 2u_x^k - \left(1-\frac{\Delta t}{2}\alpha\right) u_x^{k-1} \right\}$$

$$u_y^{k+1} = \left(1+\frac{\Delta t}{2}\alpha\right)^{-1} \left\{ \frac{f_y^{k-1} (\Delta t)^2}{m} + 2u_y^k - \left(1-\frac{\Delta t}{2}\alpha\right) u_y^{k-1} \right\}$$

$$\theta^{k+1} = \left(1+\frac{\Delta t}{2}\alpha\right)^{-1} \left\{ \frac{T^{k-1} (\Delta t)^2}{I} + 2\theta^k - \left(1-\frac{\Delta t}{2}\alpha\right) \theta^{k-1} \right\}$$

$$a_i = \frac{u_i^{k+1} - 2u_i^k + u_i^{k-1}}{(\Delta t)^2} \quad ; \quad v_i = \frac{u_i^{k+1} - u_i^{k-1}}{2\Delta t} \quad \text{such that}; \quad i: x, y \text{ or } \theta$$

Dynamic relaxation for fluid flow in porous media:



$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)+s=c\frac{\partial h}{\partial t}+\rho\frac{\partial^2 h}{\partial t^2}$$

$$q=\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)+s$$

$$v=\frac{\partial h}{\partial t}$$

$$q=cv+\frac{\partial v}{\partial t}$$

$$q_{o,t}=\frac{K_x}{(\Delta x)^2}\left[h_1-2h_0+h_2\right]_t+\frac{K_y}{(\Delta y)^2}\left[h_3-2h_0+h_4\right]_t+\frac{K_z}{(\Delta z)^2}\left[h_5-2h_0+h_6\right]_t+s_{o,t}$$

$$RHS=\frac{c}{2}\left[v_{o,\,t+\Delta t/2}+v_{o,\,t-\Delta t/2}\right]+\frac{\rho}{\Delta t}\left[v_{o,\,t+\Delta t/2}-v_{o,\,t-\Delta t/2}\right]$$

$$h_{o,\,t+\Delta t}=h_{o,t}+(\Delta t)\,v_{o,\,t+\Delta t/2}$$

Dynamic relaxation for fluid flow in porous media: (cont'd)

$$\Delta t \leq \Delta t^{c} = \frac{1}{2}\, c / [\max(K_x, K_y, K_z)]\, [\min(\Delta x, \Delta y, \Delta z)]^{2}$$

$$\omega = 2\pi / N\Delta t \quad (rad/s)$$

$$c = 2\rho\omega$$

Dynamic relaxation for Solid Mechanics:

1-D problem (elastic problems) :

$$zz = E\partial w/\partial z$$

$$\partial zz/\partial z = \rho(\partial w/\partial t + K/\Delta t\, w)$$

$$zz_{k}^{r} = E/\Delta z\ (w_{k+1}^{r} - w_{k}^{r})$$

$$zz_{k}^{r} - zz_{k-1}^{r}/\Delta z = \rho/\Delta t\ (w_{k}^{r+1} - w_{k}^{r}) + k\rho/\Delta t\,.1/2\ (w_{k}^{r+1} + w_{k}^{r})$$

$$w_{k}^{r+1} = 1/1+K/2\ [\,(1-K/2)w_{k}^{r} + \Delta t/\rho\Delta z\ (zz_{k}^{r} - zz_{k-1}^{r})]$$

Dynamic relaxation for Solid Mechanics: (cont'd):

Stress/Strain relations:

$xx = (\lambda + 2\mu)\partial u/\partial x + \lambda \partial v/\partial y$

$yy = \lambda \partial u/\partial x + (\lambda + 2\mu)\partial v/\partial y$

$xy = \mu(\partial u/\partial y + \partial v/\partial x)$

Dynamic damped equilibrium equations:

$\rho(\partial u/\partial t + K/\Delta t \,.\, u) = \partial xx/\partial x + \partial xy/\partial y$

$\rho(\partial v/\partial t + K/\Delta t \,.\, v) = \partial xy/\partial x + \partial yy/\partial y$

Static Relaxation:

Simply, implicit solution for the same equations of motion and constitutive laws based on contacts.

Two major profound aspects:

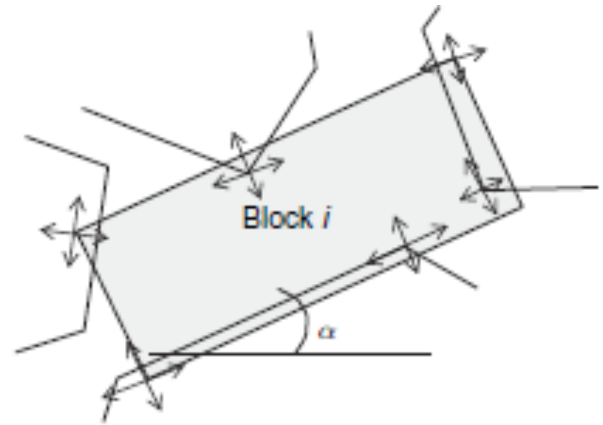
(1-) No consideration of viscous damping forces.

(2-) Balance of forces for an object is considered

Static Relaxation (cont'd):

$$\Delta u_n^i = -\Delta u_x^i \sin\alpha_i + \Delta u_y^i \cos\alpha_i$$

$$\Delta u_t^i = \Delta u_x^i \cos\alpha_i + \Delta u_y^i \sin\alpha_i$$



$$\Delta F_x^i = -K_n (\Delta u_x^i \sin\alpha_i - \Delta u_y^i \cos\alpha_i) \sin\alpha_i + K_t (\Delta u_x^i \cos\alpha_i + \Delta u_y^i \sin\alpha_i) \cos\alpha_i$$

$$\Delta F_y^i = -K_n (\Delta u_x^i \sin\alpha_i - \Delta u_y^i \cos\alpha_i) \cos\alpha_i + K_t (\Delta u_x^i \cos\alpha_i + \Delta u_y^i \sin\alpha_i) \sin\alpha_i$$

Static Relaxation (cont'd):

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} \Delta u_x^c \\ \Delta u_y^c \\ \ldots \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta u_x^c \\ \Delta u_y^c \\ \Delta \theta^c \end{Bmatrix}_k = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & \ldots \end{bmatrix}$$

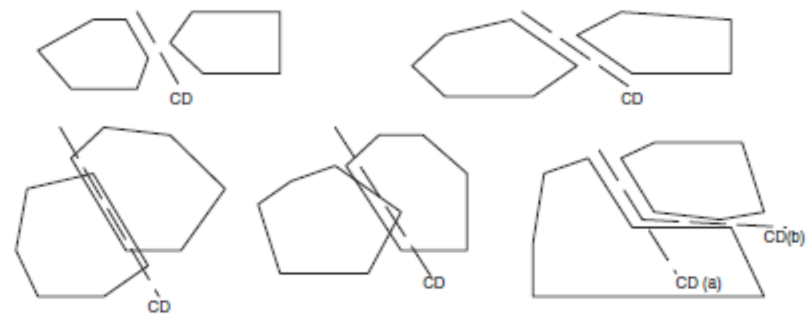
$$F_x = \sum_{i=1}^{M} F_x^i + F_x^e$$

$$F_y = \sum_{i=1}^{M} F_y^i + F_y^e \quad ; \quad T = \sum_{i=1}^{M} F_y^i \left[x^i - x^c\right] - \sum_{i=1}^{M} F_x^i \left[y^i - \ldots\right.$$

Contact Types and Detection in DEM

Table 8.2 Types of contacts for 2D polygons and 3D polyhedral blocks

Block shape	Contact type
General 2D polygons (convex or concave, singly or multiply connected)	Vertex-to-vertex Vertex-to-edge Edge-to-edge
Convex 3D polyhedra	Vertex-to-vertex Vertex-to-edge Vertex-to-face Edge-to-edge Edge-to-face Face-to-face



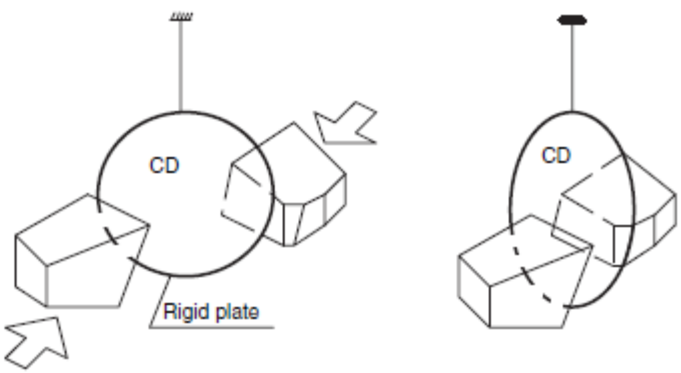


Table 8.3 Contact types associated with the common plane

Number of vertices in contact, Block *A*	Number of vertices in contact, Block *B*	Contact types
0	0	null
1	1	vertex-to-vertex
1	2	vertex-to-edge
1	>2	vertex-to-face
2	1	edge-to-vertex
2	2	edge-to-edge
2	>2	edge-to-face
>2	1	face-to-vertex
>2	2	face-to-edge
>2	>2	face-to-face

Hand Calculation Example

4. Hand-Calculation Examples

The discrete element method (DEM) is a finite difference scheme.

1-D examples:

1. Pendulum Motion
2. Heat Transfer

Strategy:

- Nonlinear becomes linear
- Continuous becomes discrete

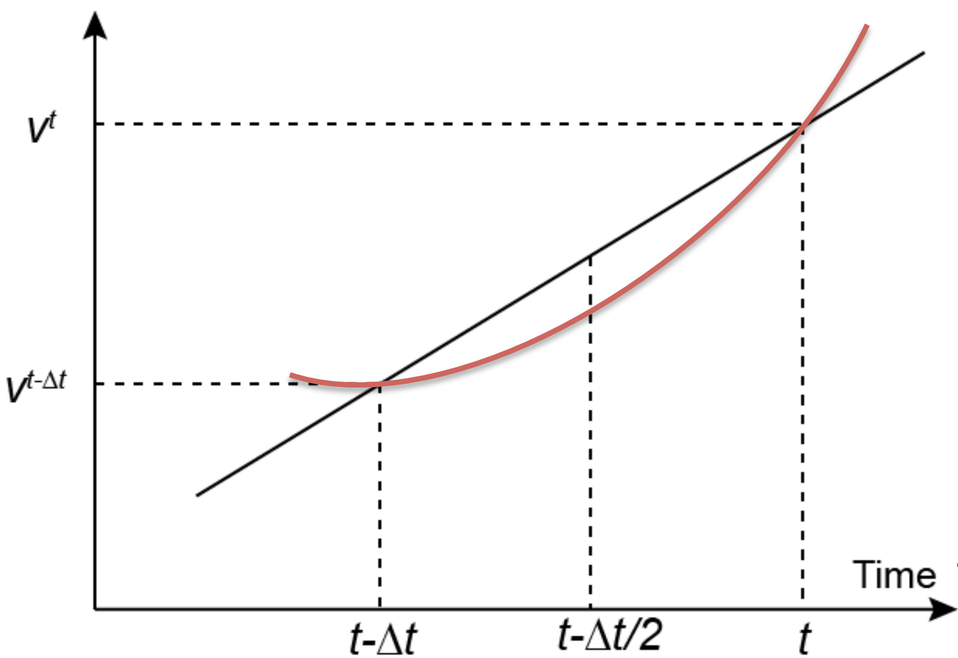
Finite Difference Scheme

- A method to approximate differential equation.

Forward difference $\frac{u(x+h)-u(x)}{h}$

Backward difference $\frac{u(x)-u(x-h)}{h}$

Centered difference $\frac{u(x+h)-u(x-h)}{2h}$



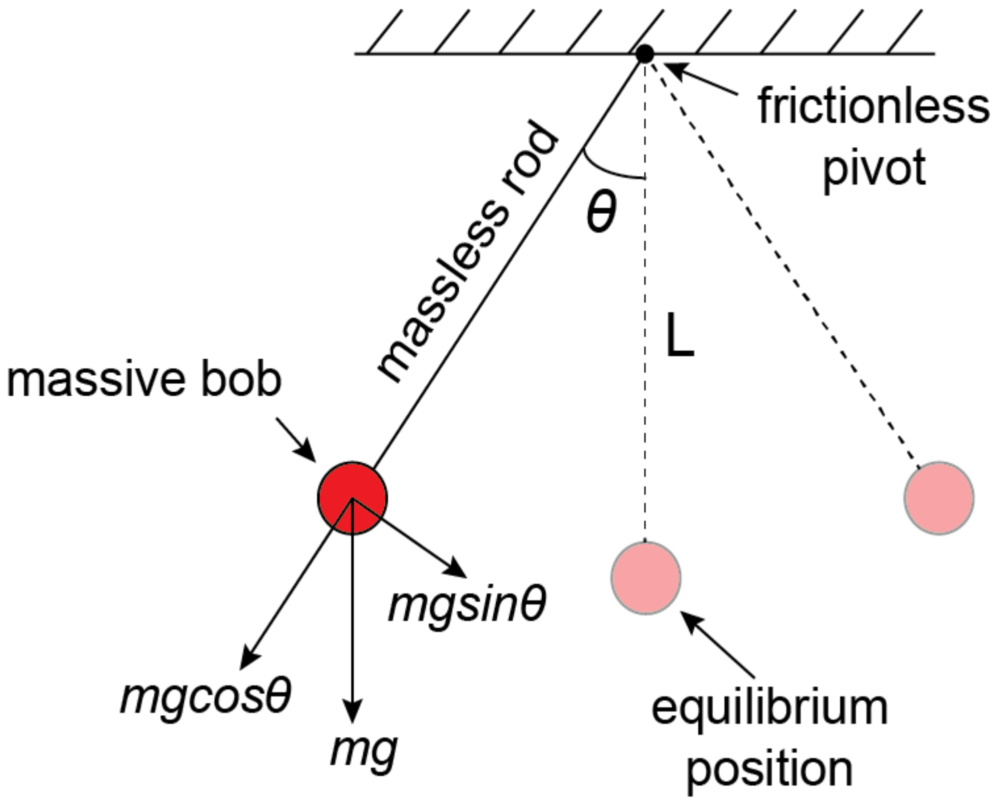
- Transform nonlinear into linear
- Make complex question simple

$$(1)\ v^t = v^{t-\Delta t} + a^{t-\frac{1}{2}\Delta t} \cdot \Delta t$$

$$(2)\ u^t = u^{t-\Delta t} + v^{t-\frac{1}{2}\Delta t} \cdot \Delta t$$

$$\vdots$$

1-D Example (1): Simple Pendulum Motion



Simple Pendulum Motion

Assumption:

No damping is considered

Calculate:

1. angular displacement ϑ
2. angular velocity ω
3. angular acceleration α

Fundamental Relations

$$\text{angular velocity: } \omega = \dot{\theta} = \frac{\partial \theta}{\partial t}$$

$$\text{angular acceleration: } \alpha = \ddot{\theta} = \frac{\partial^2 \theta}{\partial t^2} \approx \frac{\Delta \omega}{\Delta t}$$

$$\text{linear displacement: } u = L\theta$$

$$\text{linear velocity: } v = L\omega$$

$$\text{linear acceleration: } a = L\alpha$$

Newton's 2nd Law: F=ma

The only force for driving the motion:

(3) $F=-mg \cdot \sin\theta$

According to F=ma, we have:

(4) $Lm\alpha^{t} = -mg \cdot \sin\theta^{t}$

$$L \cdot \frac{\partial^2 \theta}{\partial t^2} + g \sin\theta = 0$$

Discrete time-step based equations:

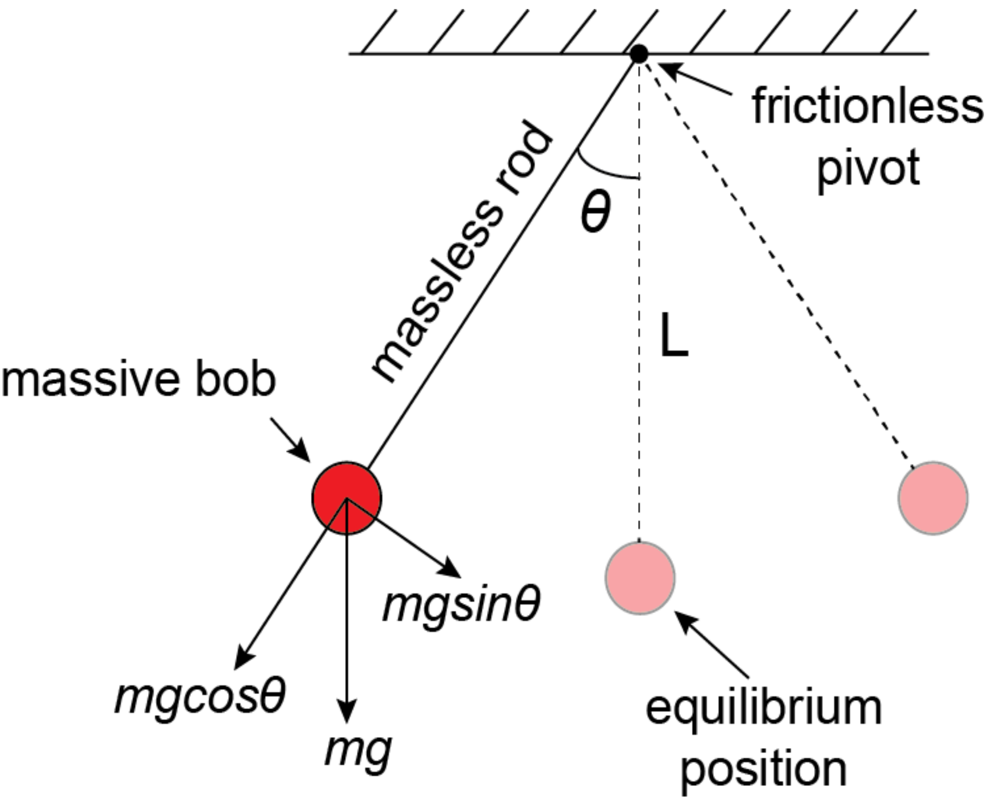
By Forward Difference we get:

$$(5)\ \omega^{t} = \omega^{t-\Delta t} + \alpha^{t-\Delta t} \cdot \Delta t$$

$$(6)\ \theta^{t} = \theta^{t-\Delta t} + \omega^{t-\Delta t} \cdot \Delta t$$

$$(7)\ \alpha^{t} = -\frac{mg\sin\theta^{t}}{L}$$

Initial Conditions



Bob is held at $t=0^-$ but released at $t=0^+$

$m=1kg$

$g=10m/s^2$

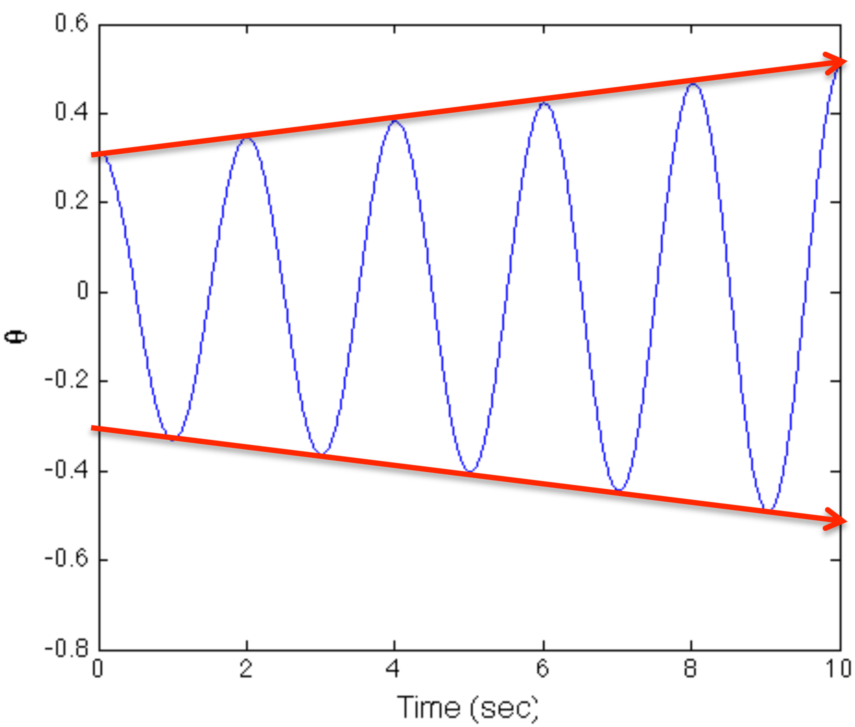
$L=1m$

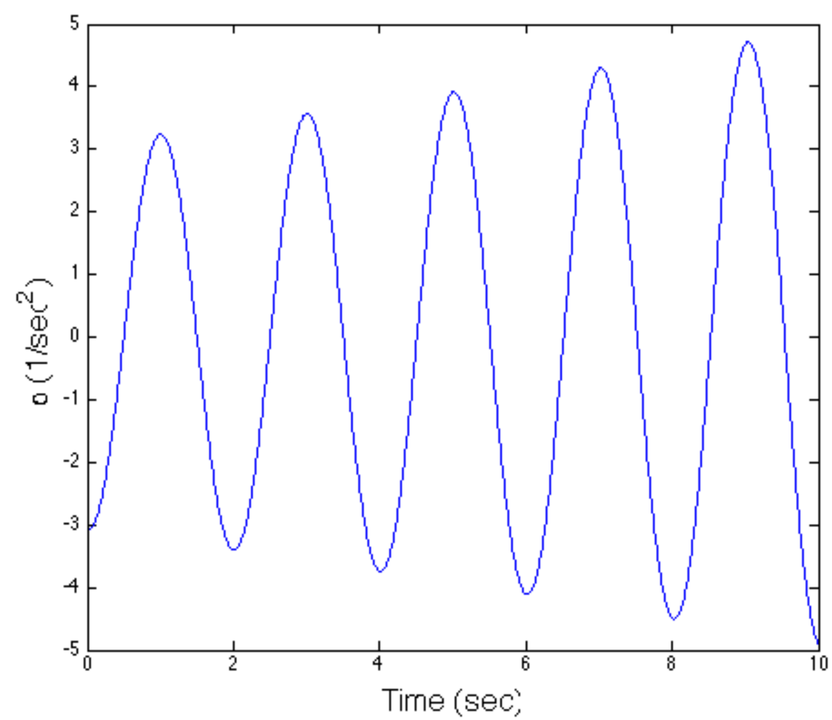
$\vartheta^{(0)}=\pi/10$

Calculation Results

Time step: Δt=0.01s

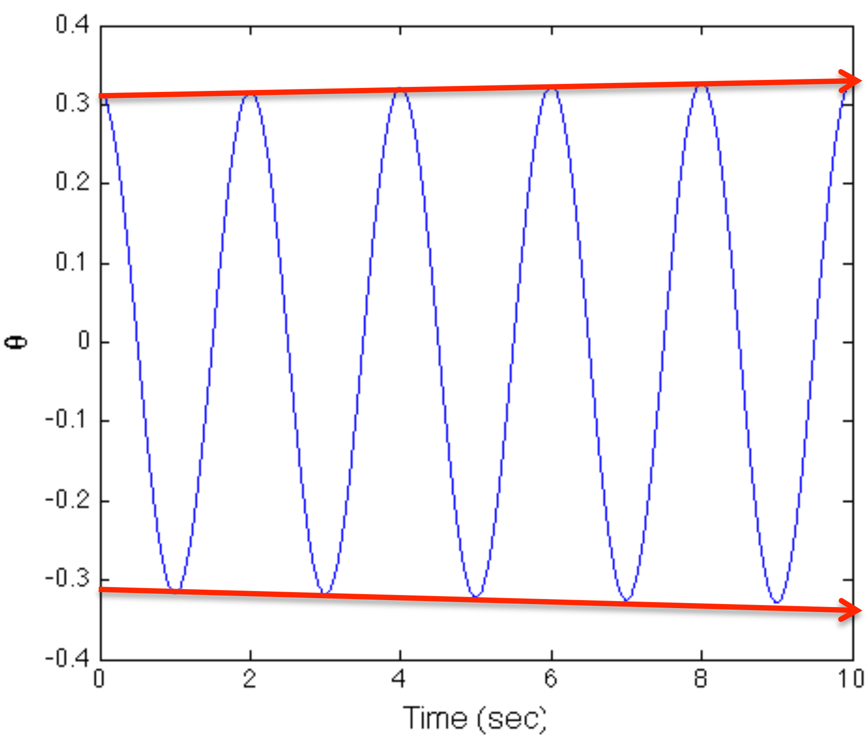
t	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	…
ϑ	0.3142	0.3142	0.3139	0.3132	0.3123	0.3111	0.3095	0.3077	0.3055	0.3031	…
ω	0	-0.039	-0.062	-0.093	-0.124	-0.154	-0.185	-0.215	-0.276	-0.306	…
α	-3.090	-3.090	-3.087	-3.081	-3.073	-3.061	-3.046	-3.029	-3.001	-2.985	…

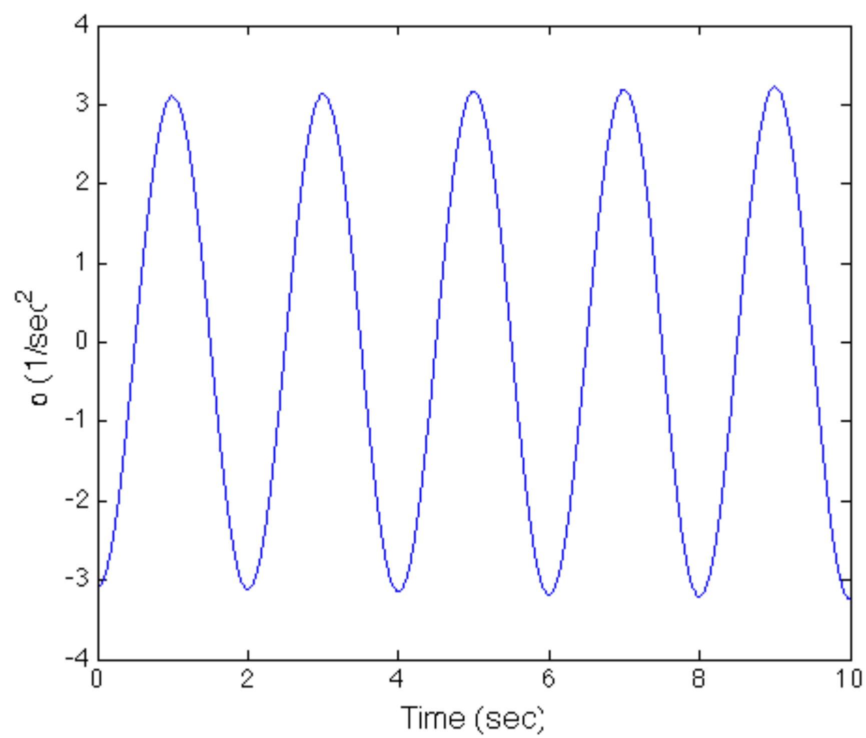




Time step: Δt=0.001s

t	**0**	**0.001**	**0.002**	**0.003**	**0.004**	**0.005**	**0.006**	**0.007**	**0.008**	**0.009**	**…**
ϑ	0.3142	0.3142	0.3142	0.3141	0.3141	0.3141	0.3141	0.3141	0.3140	0.3140	…
ω	0	-0.0031	0.0062	-0.0093	-0.0124	-0.0155	-0.0185	-0.0216	-0.0247	-0.0278	…
α	-3.0902	-3.0896	-3.0901	-3.0901	-3.0900	-3.0899	-3.0897	-3.0896	-3.0893	-3.0891	…





Numerical Solution vs. Analytical Solution

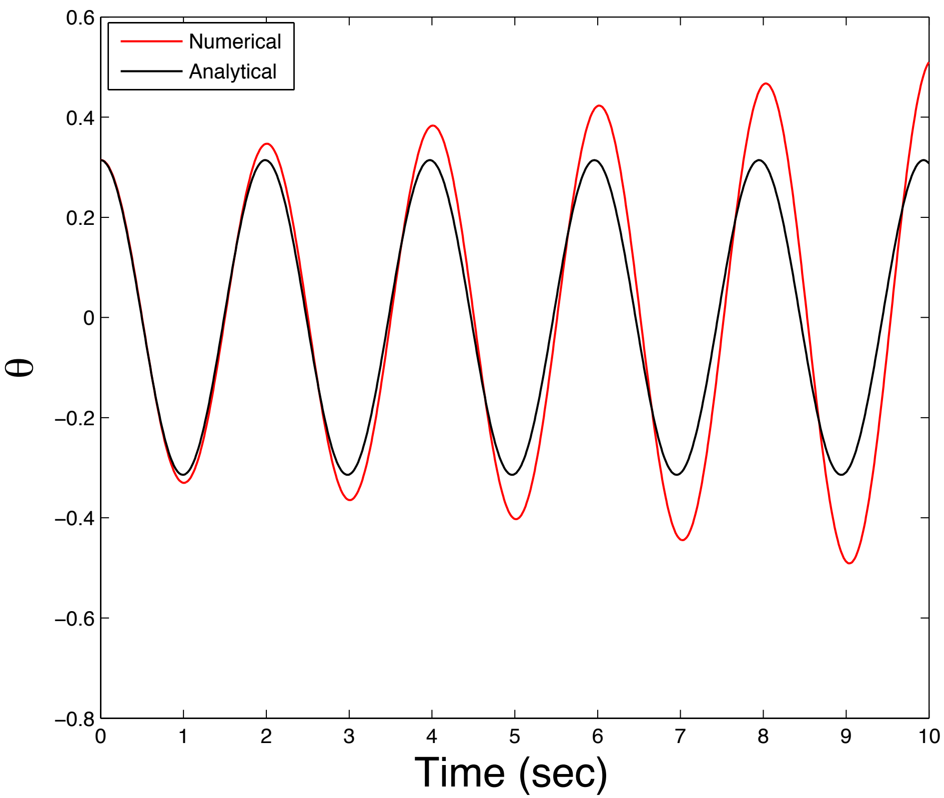
$$L \cdot \frac{\partial^2 \theta}{\partial t^2} + g \sin\theta = 0$$

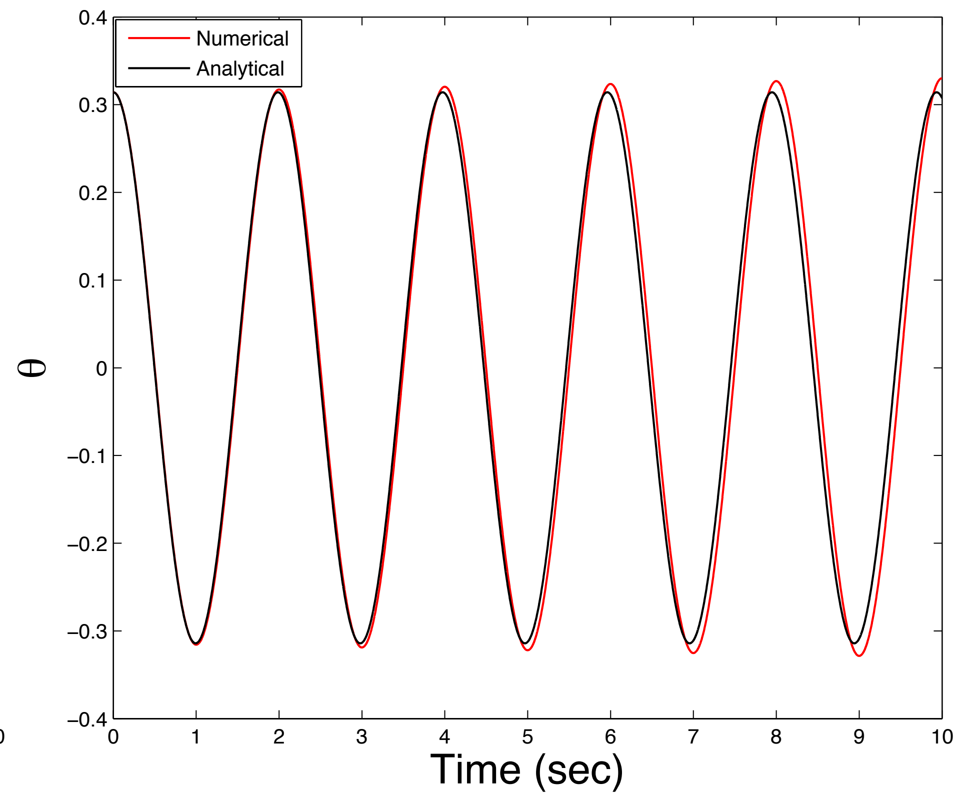
→

$$\theta = A \cdot \cos(\omega_0 t + \delta)$$

$$A = \theta_0$$

$$\omega_0 = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$





It is important to have the <u>error</u> within an acceptable level!

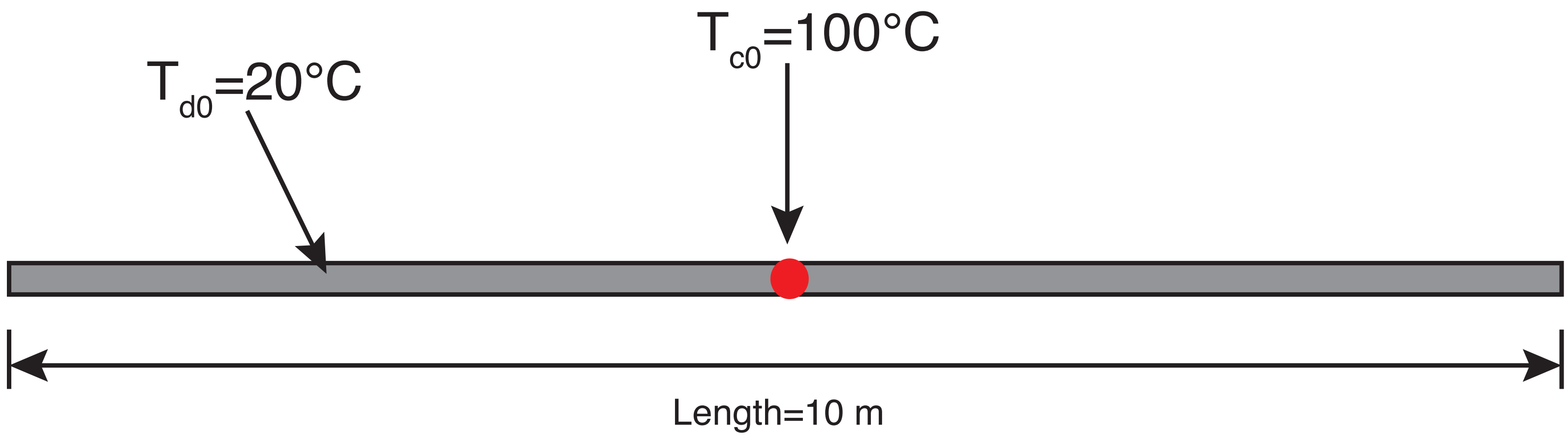
Forward	$u(x+h) = u(x) + hu'(x) + \frac{1}{2}h^2u''(x) + \frac{1}{6}h^3u'''(x) + \cdots$
Backward	$u(x-h) = u(x) - hu'(x) + \frac{1}{2}h^2u''(x) - \frac{1}{6}h^3u'''(x) + \cdots$

Truncation Error

The forward and backward difference are both first order accurate. Because {item} as a leading error has first power of h, which is the time interval (step) in this example.

1-D Example (2): Heat Transfer

Heat Transfer Within a Pyrolytic Graphite



Assumptions:

The surface of the bar is perfectly thermally insulated.

Consider the 1-D, transient heat conduction equation without heat generating sources:

$$(1)\ \rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \cdot \frac{\partial T}{\partial x}\right)$$

ρ: density

c_p: heat capacity

k: thermal conductivity

T: temperature

x: distance

t: time

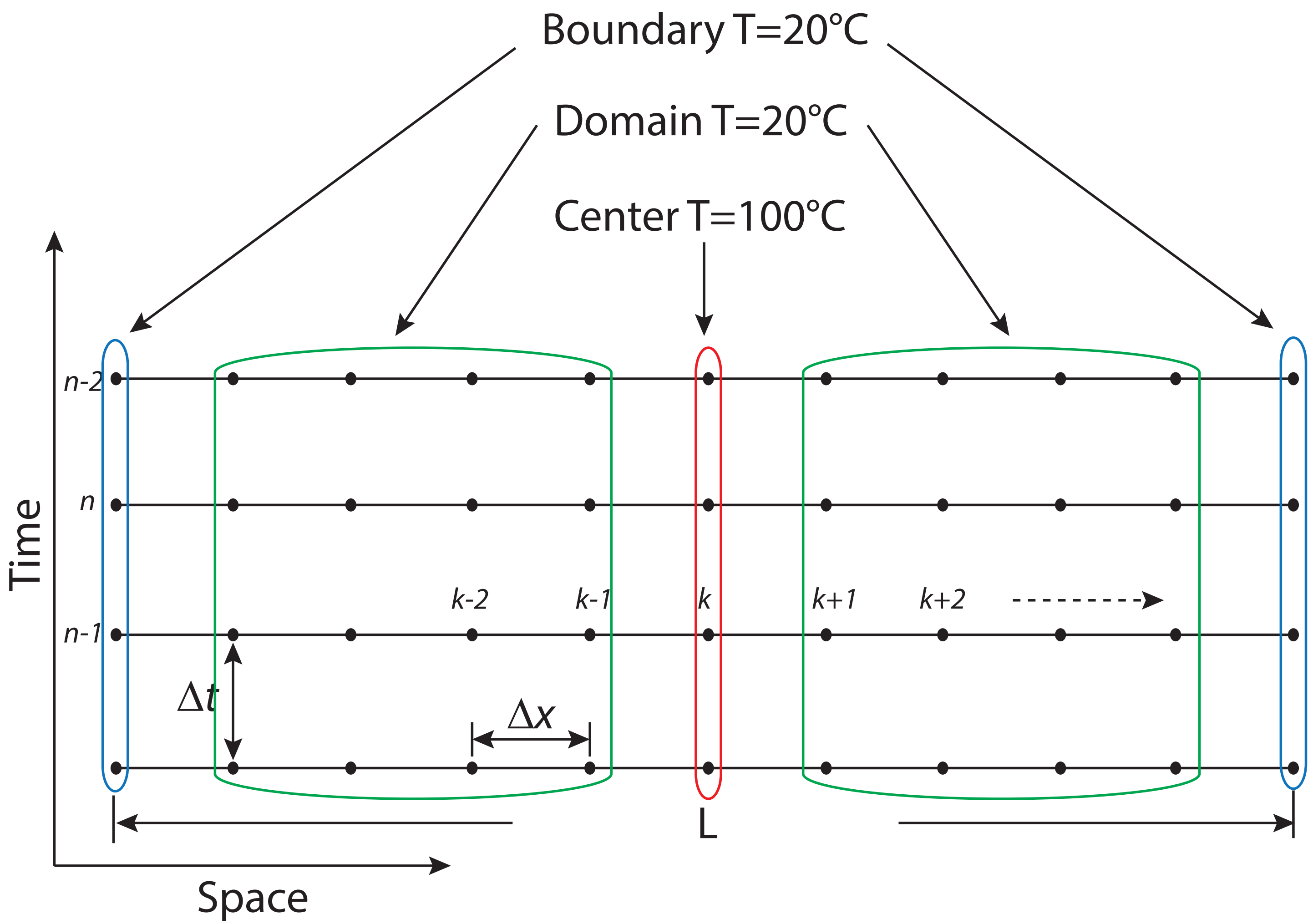
If we have constant density, heat capacity, thermal conductivity over the model domain, we can simplify the Eq. (1).

$$(2)\quad \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

where: $\kappa = \dfrac{k}{\rho c_p}$ is the thermal diffusivity

The temperature here becomes a function of space and time, which satisfies Eq. (2).

The first step use DEM is to construct a grid with points (called discretization).



Time and Space Discretization

Forward difference approximation of Eq. (2):

$$(3)\quad \frac{\partial T}{\partial t} \approx \frac{T_k^{new} - T_k^{current}}{t^{new} - t^{current}} = \frac{T_k^{n+1} - T_k^{n}}{t^{n+1} - t^{n}} = \frac{T_k^{n+1} - T_k^{n}}{\Delta t}$$

$$(4)\quad \frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \approx \frac{\dfrac{T_{k+1}^{n} - T_k^{n}}{\Delta x} - \dfrac{T_k^{n} - T_{k-1}^{n}}{\Delta x}}{\Delta x}$$

Combine Eq. (3) and Eq. (4), we will get Eq. (5):

$$T_k^{n+1} = T_k^n + \kappa \cdot \Delta t \cdot \frac{T_{k+1}^n - 2T_k^n + T_{k-1}^n}{(\Delta x)^2}$$

$$\text{let } \alpha = \frac{\kappa \cdot \Delta t}{(\Delta x)^2}, \text{ then}$$

$$T_k^{n+1} = \alpha \cdot (T_{k+1}^n + T_{k-1}^n) + (1 - 2\alpha) T_k^n$$

Stability Condition:

$$(1-2\alpha)>0 \text{ and } \alpha=\frac{\kappa\cdot\Delta t}{(\Delta x)^2}>0$$

Physical Parameters:

Parameters	Values	Physical Meaning
L [m]	10	Length
kappa [m^2/sec]	1e-3	Thermal Diffusivity
T_c [°C]	100	Temperature of Central Point
T_d [°C]	20	Temperature of Domain
T_b [°C]	20	Temperature of Boundaries
t_{final} [sec]	1500	Total Time of Heat Conduction

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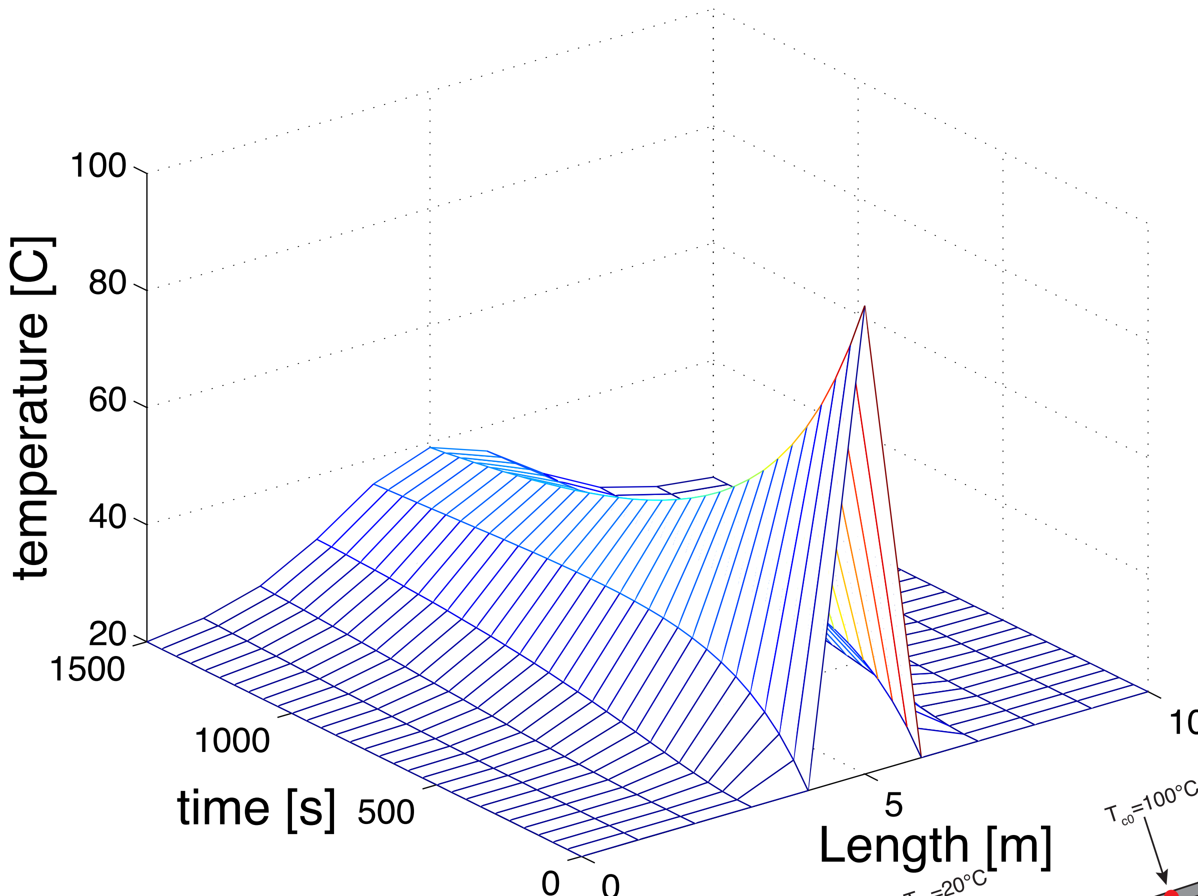
Physical Parameters:

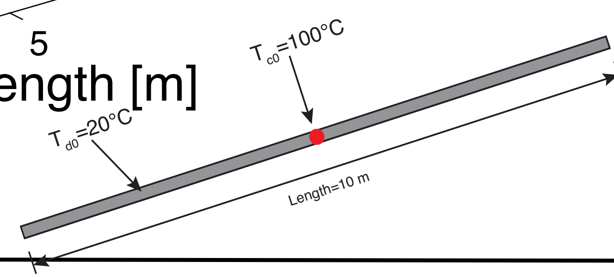
Parameters	Values	Numerical Meaning
kx	10	Number of Space Steps
dx	1	Space Step
nx	11	Number of Gridpoints
nt	30	Number of Time Steps
dt	50	Time Step

Calculation:

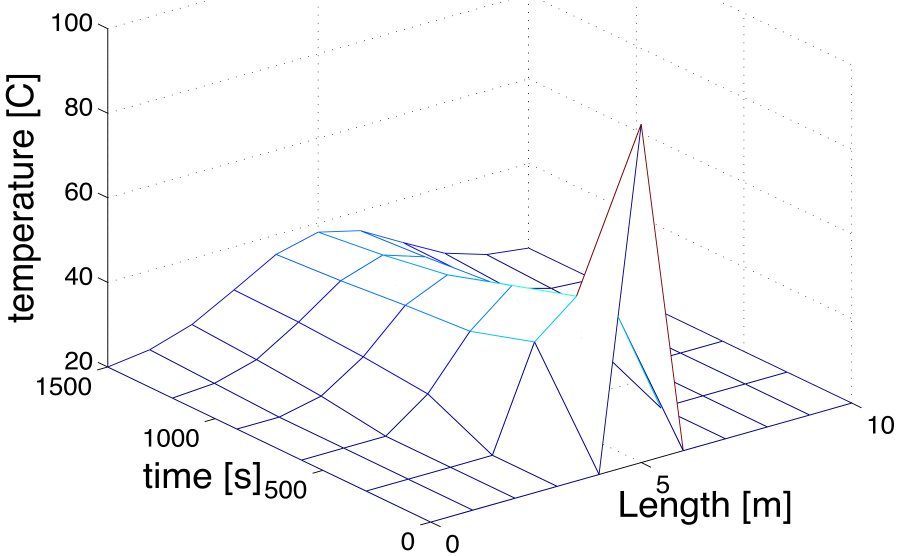
T(°C)	n=1	n=2	n=3	n=4	n=5	n=6	n=7
k=1	20	20	20	20	20	20	20
k=2	20	20	20	20	20.0005	20.0023	20.0061
k=3	20	20	20	20.01	20.0360	20.0811	20.1465
k=4	20	20	20.2	20.54	20.9740	21.4670	21.9926
k=5	20	24	27.2	29.75	31.7720	33.3652	34.6105
k=6	100	92	85.2	79.40	74.4350	70.1687	66.4884

Temperature Evolution

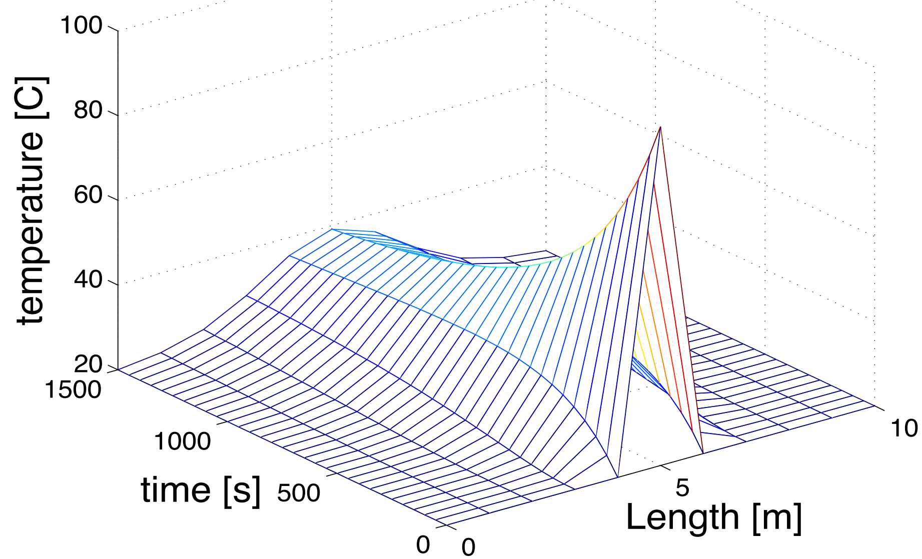




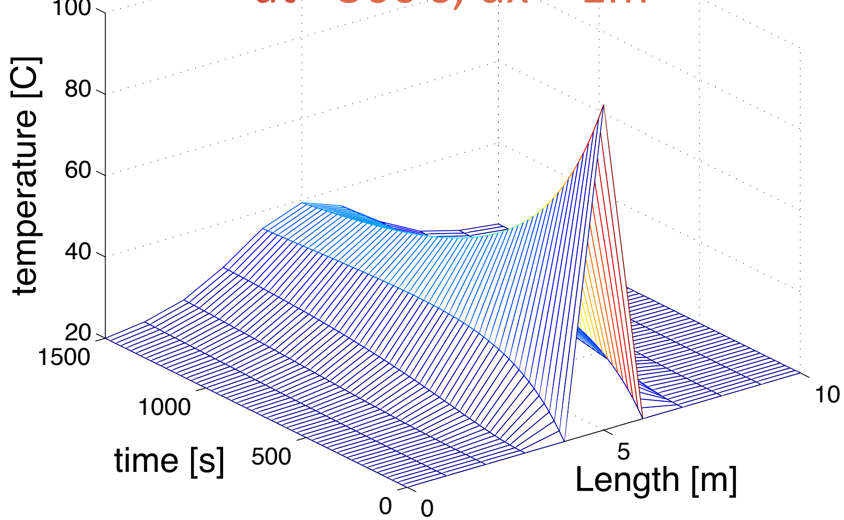
dt =25 s; dx =1 m

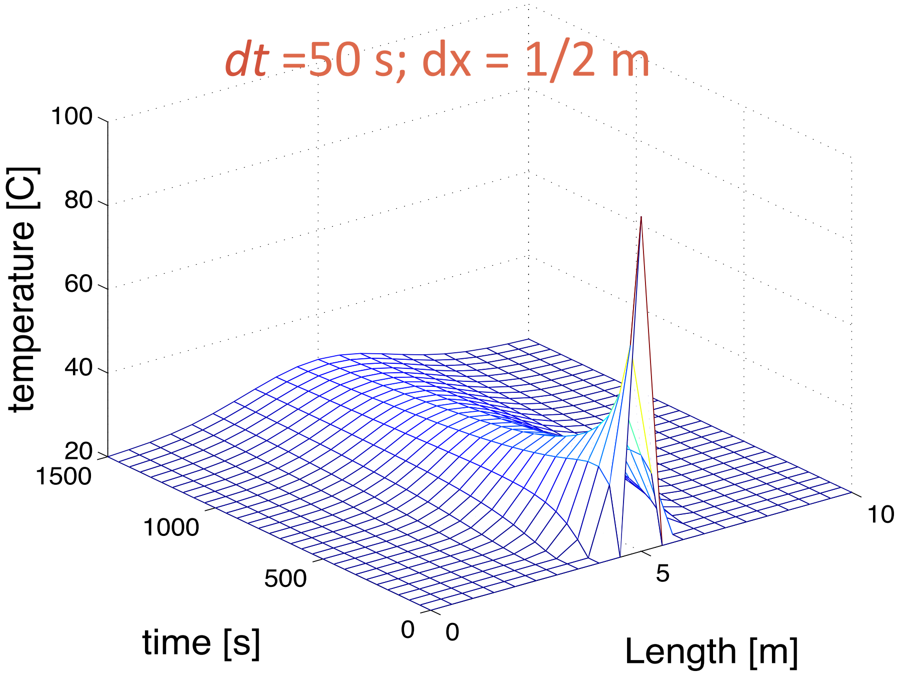


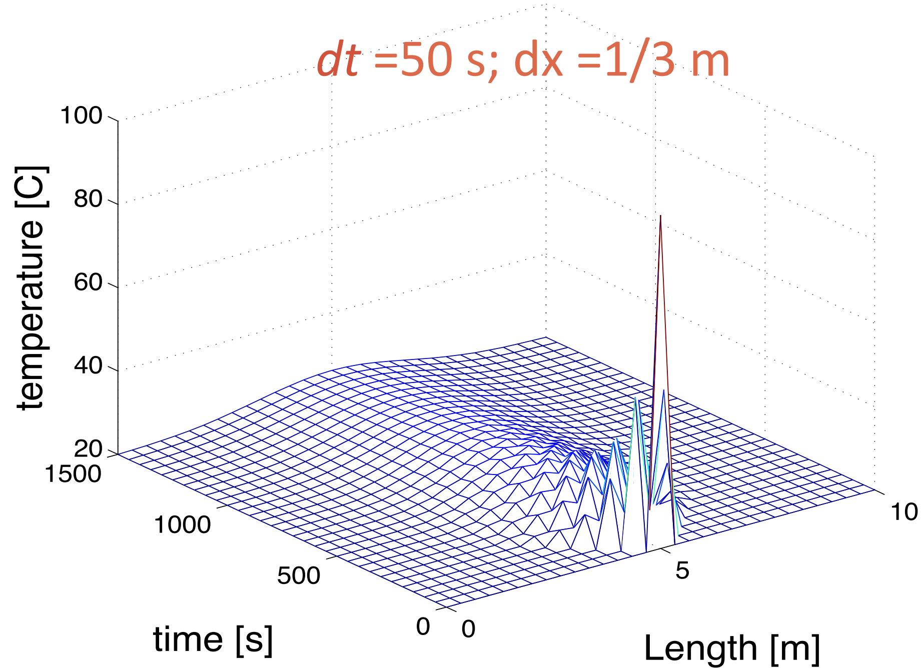
dt =50 s; dx =1 m

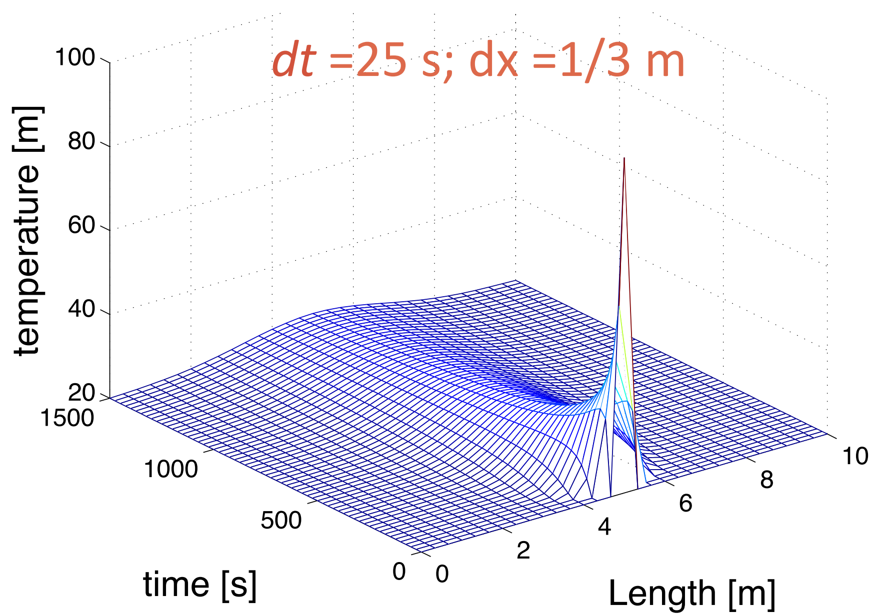


dt =500 s; dx = 1m




dt =50 s; dx = 1/2 m
temperature [C]
100
80
60
40
20
1500
1000
500
0
0
5
10
time [s]
Length [m]


dt =50 s; dx =1/3 m
temperature [C]
100
80
60
40
20
1500
1000
500
0
0
5
10
time [s]
Length [m]


dt =25 s; dx =1/3 m
temperature [m]
100
80
60
40
20
1500
1000
500
0
0
2
4
6
8
10
time [s]
Length [m]

Summary from these examples

1. Solving the right equations
   - Use right/best equations that best describe the physical laws.

2. Solving the equations right
   - Appropriate initial/boundary conditions
   - Control the numerical error and see how the output depends on the input.
   - Accuracy and stability.

Numerical Example

5. Numerical Example

A DEM commercial software was used to develop the example models:

What it can do…

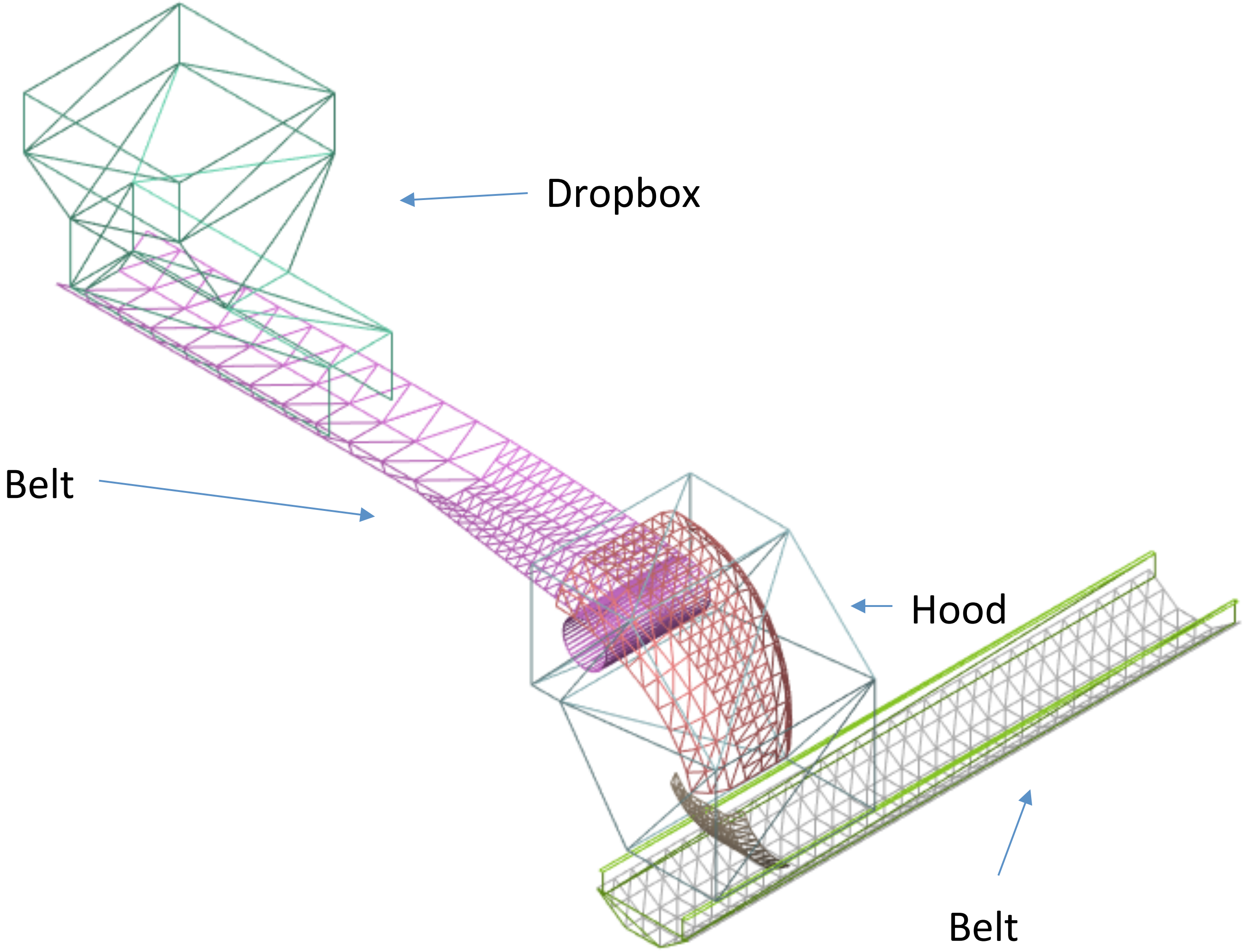
- Model three-dimensional behavior of complex material flows
- Analyze various material properties and fundamental parameters

Model Proposed

Two groups of particle flow simulations:

- Dry and wet particle flow on declines
- Belt transport problem with dry and wet particles

Geometry



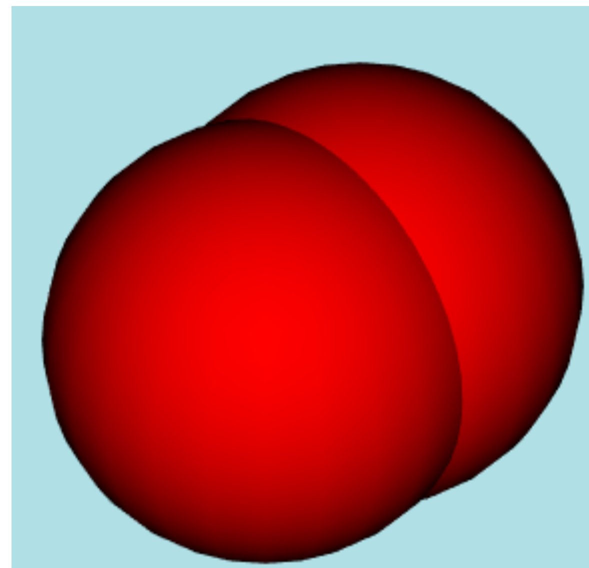
Particle Properties

- Particle-Particle Friction Coefficient
- Coefficient of Restitution
- Rotational Damping
- Particle-Particle Cohesion Factor

Ratchet Effect

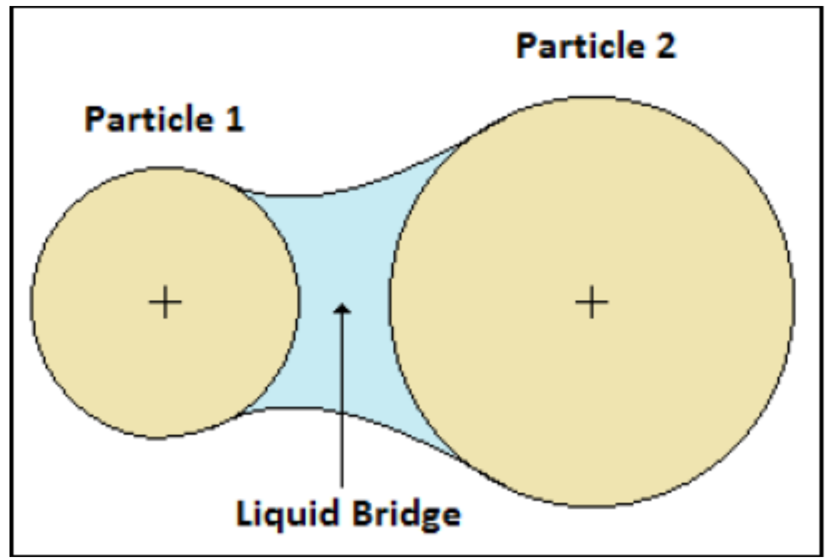
- Normally, virtual spring is created when two particles overlap

- Ratchet effect can create a second spring to pull the two particles back



Liquid Bridge Effect

- Moisture between two particles
- Small force applied on this bridge
- Bridge collapse when one particle moves apart



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Input Parameters

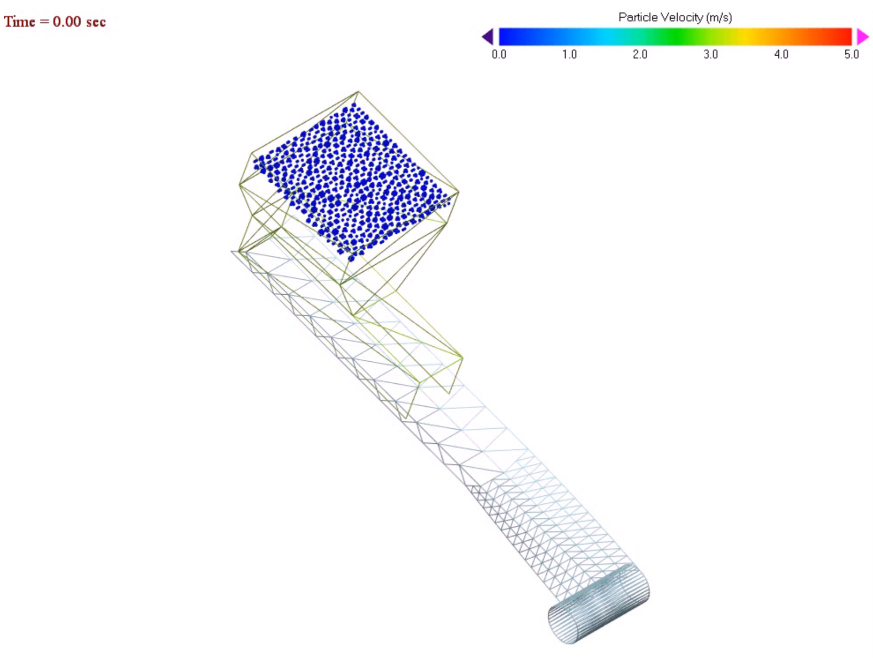
Wet Particles

General Properties	A
Particle-Particle Friction Coefficient (0-1)	0.85
Particle-Boundary Friction Coefficient (0-1)	0.75
Coefficient of Restitution (0.075-1)	0.100
Rotational Damping (0-10)	1.00
Ratchet Effect	
Use Ratchet Effect	YES
Particle-Particle Cohesion Factor (0-0.25)	0.150
Particle-Boundary Cohesion Factor (0-0.25)	0.150
Liquid Bridge	
Use Liquid Bridge	YES
Surface Tensions (0.05 - 0.50 J/m²)	0.50
Water Content (%)	15.0
Boundary Surface Tension Multiplier (0-10)	2.00
Equivalent Sphere Size Ratio	15.00

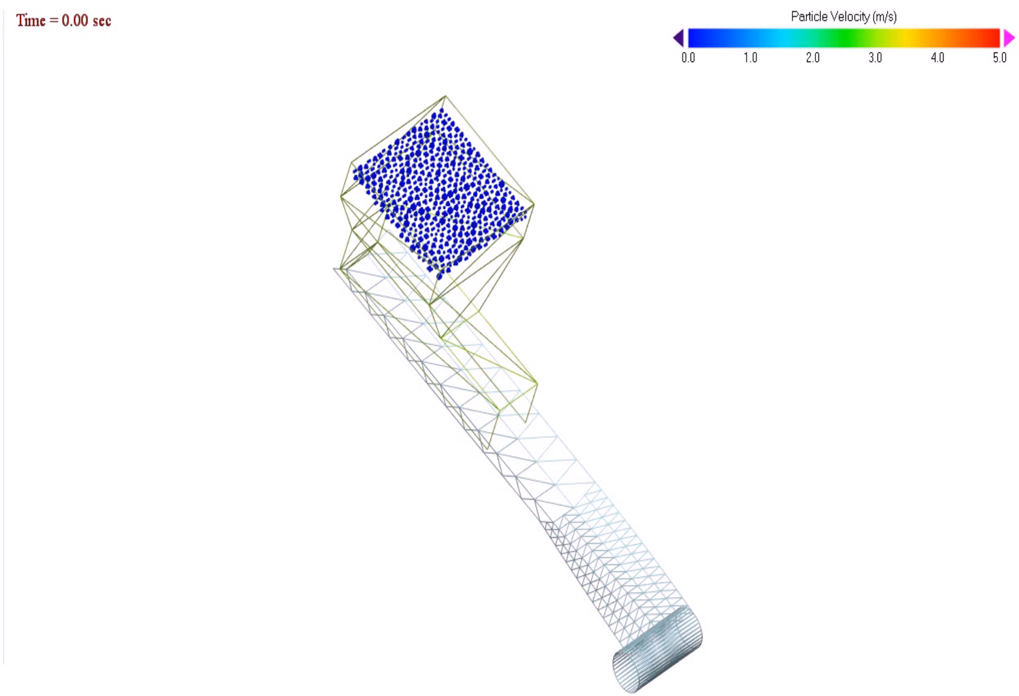
Dry Particles

General Properties	A
Particle-Particle Friction Coefficient (0-1)	0.30
Particle-Boundary Friction Coefficient (0-1)	0.20
Coefficient of Restitution (0.075-1)	0.150
Rotational Damping (0-10)	2.00
Ratchet Effect	
Use Ratchet Effect	NO
Particle-Particle Cohesion Factor (0-0.25)	
Particle-Boundary Cohesion Factor (0-0.25)	
Liquid Bridge	
Use Liquid Bridge	NO
Surface Tensions (0.05 - 0.50 J/m²)	
Water Content (%)	
Boundary Surface Tension Multiplier (0-10)	
Equivalent Sphere Size Ratio	

Animation Results

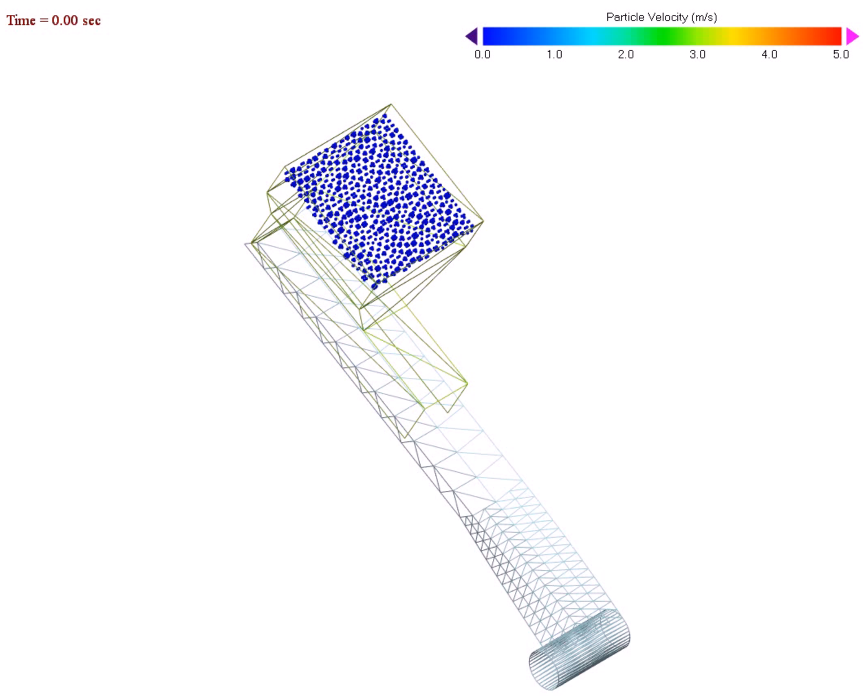


Dry particles on 20↑o decline

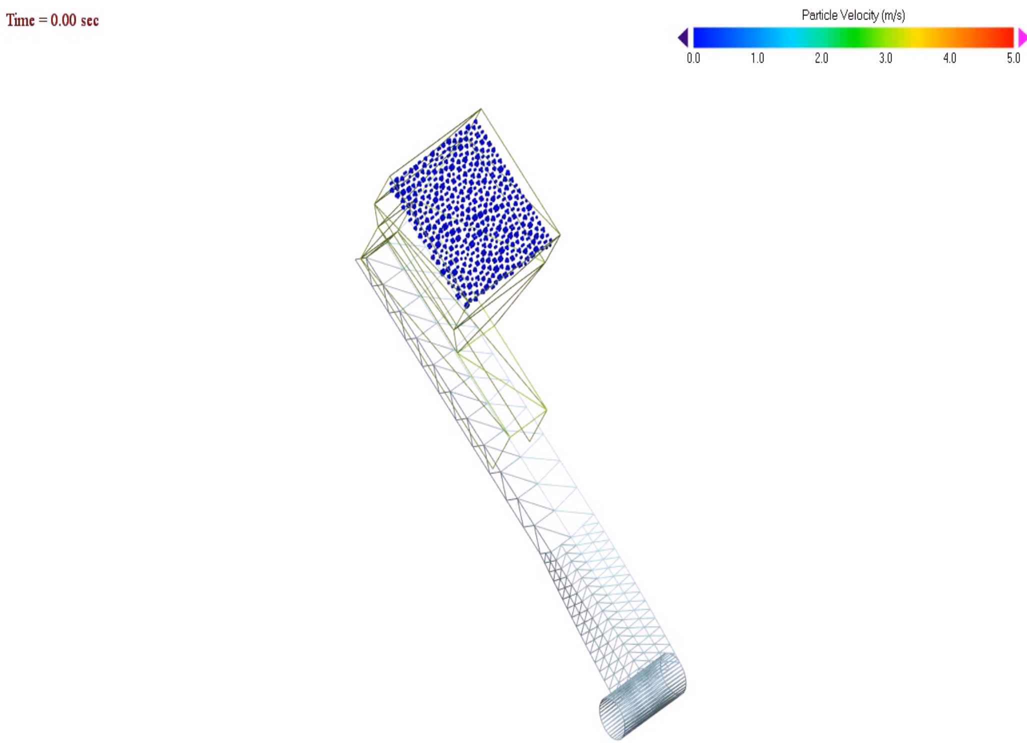


Wet particles on 20↑o decline

Animation Results

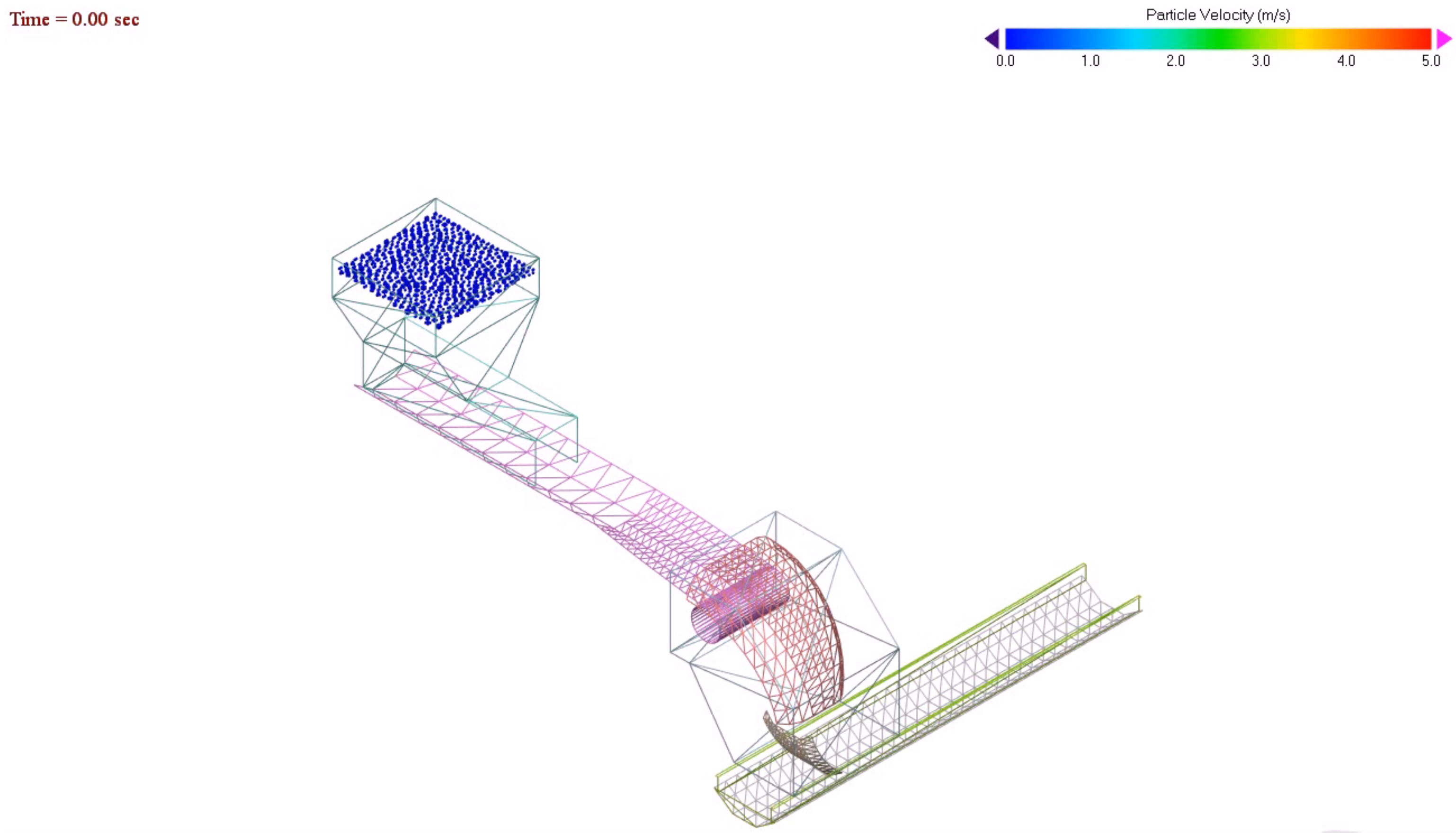


Dry particles on 30↑o decline



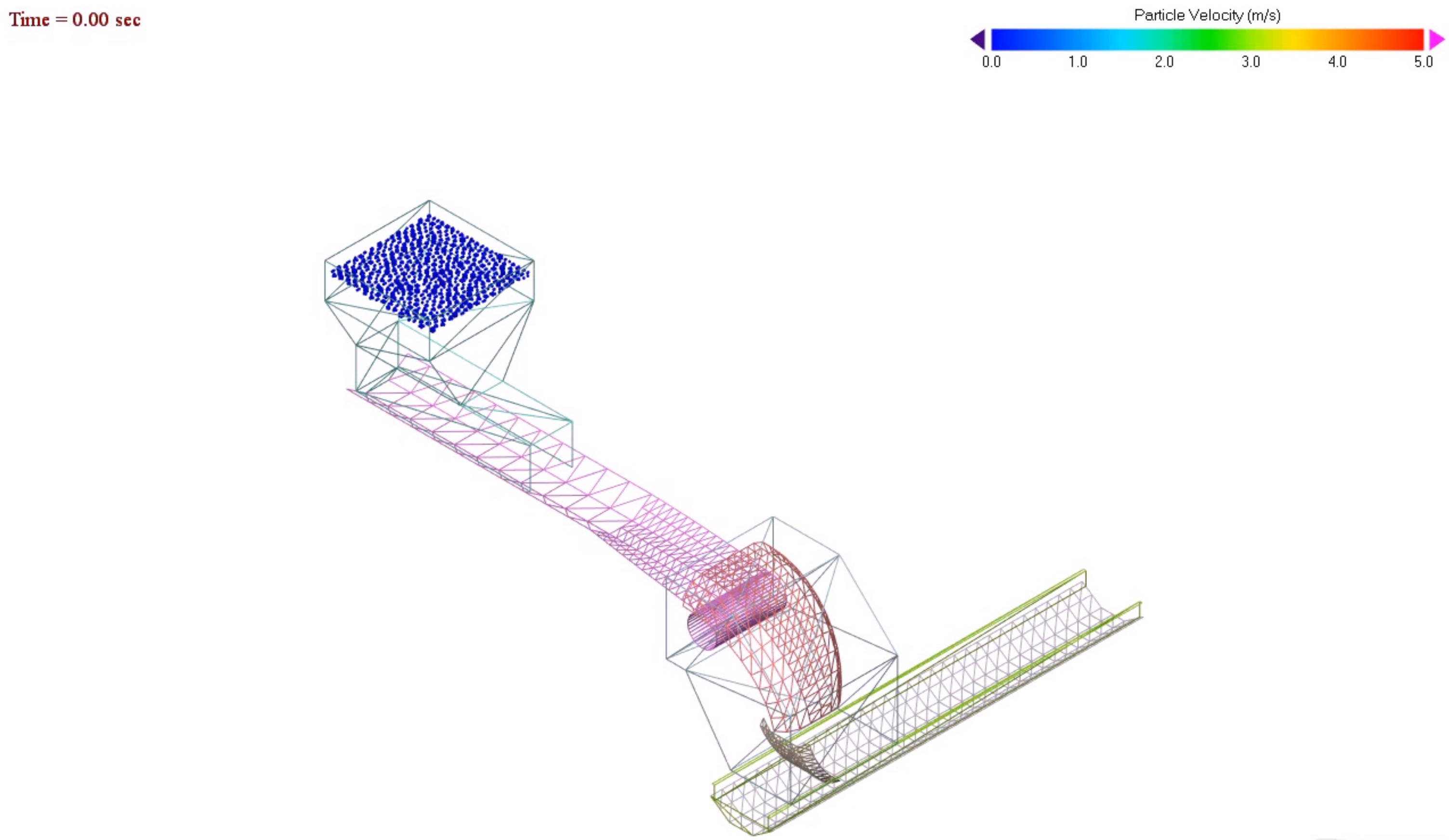
Wet particles on 30↑o decline

Animation Results



Dry particles transport on belt with a velocity of 2m/s

Animation Results



Wet particles transport on belt with a velocity of 2m/s

6. Example Applications

DEM has been used in many applications:

- Geophysics/Seismology
- Rock fracture
- Soil mechanics
- Ice blocks floating into bridge supports
- Industrial/commercial applications

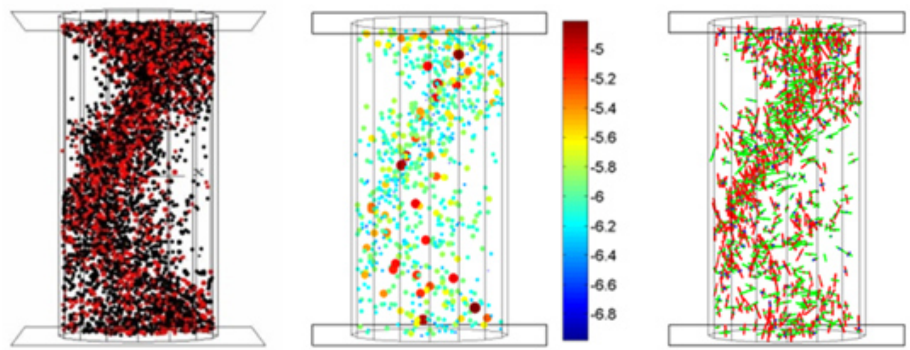
Example 1

- Various cases of DEM simulation

https://www.youtube.com/watch?v=y2Otlge_YaY&list=LL5LLRt-U8nlyfRhpm5lBEHQ&index=2

DEM results are much more than cartoons.

- The animations and videos are the visual display of a large amount of data
- Particle data
  - Position
  - Stress
  - Velocity
- System boundary data
- All data is available to the user for further analysis



Micro-cracks (black: tensile failure red : shear failure)

Acoustic emissions recorded

Moment tensors (red : maximum pressure direction green : maximum tension direction)

What kind of problems can DEM predict?

- Plugging
- Material loss
- Material stagnation
- Dust production
- Wear on machine/structure
- Mixing
- Other inadequacies

Example 2

- Thermal flow simulation & melting simulation in twin screw extruder

https://www.youtube.com/watch?v=v4aMrxJOdU0

Example 3

- Rotating Bullet simulation

https://www.youtube.com/watch?v=qFHW0Q6wRy8

Thank you !