

Back Calculation of Rock Mass Modulus using Finite Element Code (COMSOL)

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1. Introduction

Deformability is recognized as one of the most important parameters governing the behavior of rock masses. Quantitative values of rock mass deformation properties can be obtained through in-situ tests, empirical relations between deformation modulus and rock mass classification, and estimating from laboratory tests result [1].

For smaller size projects in-situ tests normally are ignored due to cost and time constrains. However for large projects where obtaining reliable deformation behavior of rock mass is necessary, in-situ tests are a major component of site investigation studies. There are different kinds of test available for measuring rock mass deformation modulus. Among them, Plate Load test (PLT) is frequently used in rock engineering [1].

Another method for obtaining rock mass deformation modulus is back analysis. In this method, test condition is simulated by the help of numerical software. Geometrical information along with material characteristics are applied to the model. Using measured displacement in different points inside rock mass, model can be validated. Material characteristics are changed until known displacement values are achieved.

2. Plate Load Test

PLT for rocks is simply a test in which load is applied on two opponent walls of gallery using jacks, spacers, and bearing plates. A borehole is drilled in the center of loaded area and instrumented by extensometers to measure the displacement. By recording load and related displacement, deformation modulus of rock mass can be calculated using different formulas.

PLT is classified into two main different groups: Flexible Plate Load Test and Rigid Plate Load Test.

Flexible plate load test is suggested by ISRM in 1979 while American Society for Testing and Material (ASTM) uses different standards for both methods.

In this study, results from rigid PLTs performed in Bakhtiary dam and Hydro-Power project in Iran are discussed. The bearing plate diameter is 0.915 m and ultimate pressure is 20 MPa. Figures 1 and 2 present the layout and equipment of the test.

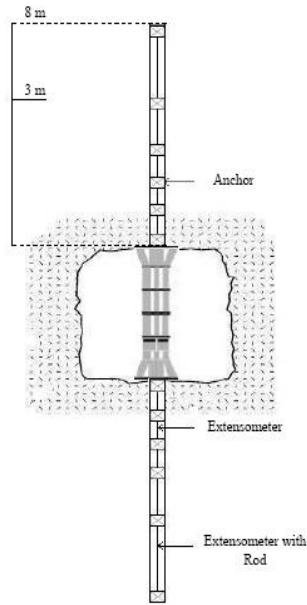


Fig. 1. Principles of plate load apparatus used at Bakhtiary site.



Fig. 2. In-situ plate load test used at Bakhtiary site.

Different modulus can be obtained from PLT tests through using various parts of the stress/stress or force/deformation curves. Choosing appropriate procedure for calculation of modulus is of great importance for designers and it usually depends on the application and the end use of the results. Peak modulus of deformation (D_{pp}) is presented by ASTM. It is the slope of the stress-displacement curve line connecting the peaks of the curves obtained from successive loading and unloading pressure cycle [2]. D_{pp} reflects the overall behavior of rock mass during the test and also is relatively conservative. Therefore, D_{pp} was used as the representative rock mass deformation modulus in Bakhtiary dam project. Figure 3 shows stress-displacement curve for one of the tests performed in Bakhtiary dam.

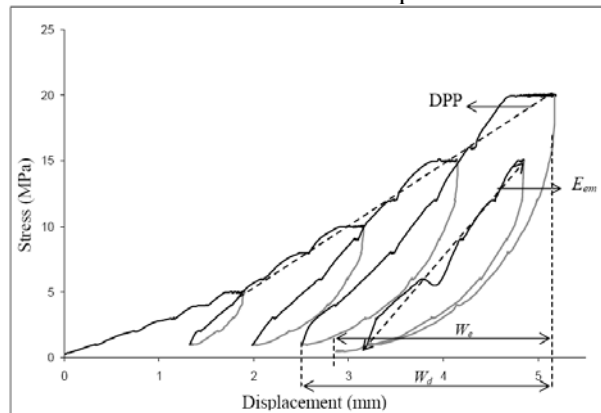


Fig. 3. Definition of Peak to Peak modulus of deformation.

There are different formulas available to calculate rock mass modulus from the PLT test. Most important formulas are listed as follow:

- ASTM Standard [2]
- UNAL Method [1]

➤ *ASTM Standard D-4394-04*

The formula which were used in calculation of rock mass deformation modulus from PLT tests were the ones suggested by ASTM D4394–04. The suggested formula is as follows [2]:

$$E = \frac{(1 - \nu^2) \cdot P}{2W_a \cdot R} \quad (1)$$

where:

ν = Poisson's ratio of the rock,

P = total load on the rigid plate, kN (lbf),

W_a = average deflection of the rigid plate, mm (in.), and

R = radius of the rigid plate, mm. (in.).

As can be seen, Eq. (1) only uses the total displacement of the rock mass and does not incorporate displacement changes in depth. This formula actually ignores benefits of using multiple point extensometers.

➤ *Unal Method*

This method suggests three different modulus definitions [1]:

- Instant Modulus which gives the modulus at any given point inside the rock mass.
- Interval Modulus which gives the modulus between any two given points inside the rock mass.
- Overall Modulus which gives the deformability of the total rock mass.

Overall modulus calculation incorporates the measurements made at depth and also uses mathematical approach to have the best possible output. Therefore, it seems that it provides the most reliable value for rock mass modulus. A brief overview of the procedure is as follows.

Displacement at any given point in the rock mass can be predicted using Eq. (3) [1].

$$y_{pi} = W_z = \frac{q_{av} \cdot a}{2 \cdot E_m} \cdot [2(1 - \nu^2) \cdot \arccot z_i + (1 + \nu) \cdot \frac{z_i}{z_i^2 + 1}] \quad (3)$$

where:

W_z = deflection at depth z (mm),

q_{av} = Average stress on the loaded surface (MPa),

a = radius of the loaded circular area (m),

E_m = modulus (GPa),

ν = Poisson's ratio,

z_i = depth from the surface (m).

Using least square method, function ϕ can be defined as follow [1]:

$$\phi = \sum (y_{pi} - y_{mi})^2 \quad (4)$$

Where, y_{mi} is measured displacement at any depth (including surface) at a certain stress level. By differentiating equation with respect to μ and E_m two equations will be derived. These equations should be set to zero in order to minimize the difference between theoretical and measured displacement values. By solving these equations, μ and E_m can be calculated [1]

3. Review of In-Situ Tests Results at Bakhtiary Dam Project

Using ASTM Standard and Unal (Overall modulus) method, Peak to Peak Rock Mass Modulus calculated for 35 PLT tests at Bakhtiary dam project.

Poisson's ratio was estimated by using Petite Seismic testing. It was recommended $\nu=0.3$ as rock mass Poisson's ratio. Therefore this value used for estimation of rock mass modulus in formulas (even in Unal method). It should be noted that where displacement in opposing walls in the same test were found to be different, rock mass modulus was calculated individually for each wall. Therefore, 69 data point were analyzed (one of the walls does not have the data due to practical issues during the test).

Formulas used have their own advantages, disadvantages, and limitations. They also work with different assumptions.

Using back analysis method, Young's modulus were calculated by COMSOL Multiphysics 3.5 software.

4. Governing Equations and Formulation

COMSOL software is a finite element code that uses different governing equations for various problems. In this study solid mechanics governing equations are used.

According to definition, an elastic body is in equilibrium, if, for any set of virtual displacement, \bar{u} , the virtual work of the external forces is equal to the virtual strain energy of the system (i.e. internal work) [3].

$$VW_E = VW_I \quad (1)$$

VW_E = Boundary forces (f) × Boundary Displacements (virtual) (u)

$$VW_E = \sum_{i=1}^n \bar{u}_i f_i = \bar{u}^T \underline{f} \quad (2)$$

VW_I = Body strain resulting from virtual displacement (\square) × Stress change due to boundary conditions (σ)

$$VW_I = \int_v \bar{\underline{\varepsilon}}^T \underline{\sigma} dv \quad (3)$$

Also, we have:

$$\underline{\sigma} = \underline{D} \cdot \underline{\varepsilon} \quad (4)$$

And,

$$\underline{\varepsilon} = \underline{a} \cdot \underline{u} \quad (5)$$

Therefore,

$$VW_I = \bar{u}^T \int_v \underline{a}^T \underline{D} \underline{a} dv \underline{u} \quad (6)$$

Considering Eq. 1, 2, and Eq. 6, we have:

$$\underline{f} = \int_v \underline{a}^T \underline{D} \underline{a} dv \underline{u} = \underline{K} \cdot \underline{u} \quad (7)$$

while,

$$\underline{K} = \int_v \underline{a}^T \underline{D} \underline{a} dv \quad (8)$$

5. Solution

Using COMSOL software a 3-D model for simulating the test is performed. A block of rock mass is considered. The length, height, and thickness of the block are 20m, 9m, and 7m respectively. A D-shape tunnel (gallery) is excavated in the model. The height is 3 m and the width considered as 2 m. Load is applied on two opponent walls of the tunnel in a circular area with the diameter of 0.915 m. As it was not possible to apply load only on a portion of the wall, two cylinders with the diameter of 0.915 m were considered in the walls. The material of the cylinders is the same as walls. Figure 4 shows the model used in COMSOL software.

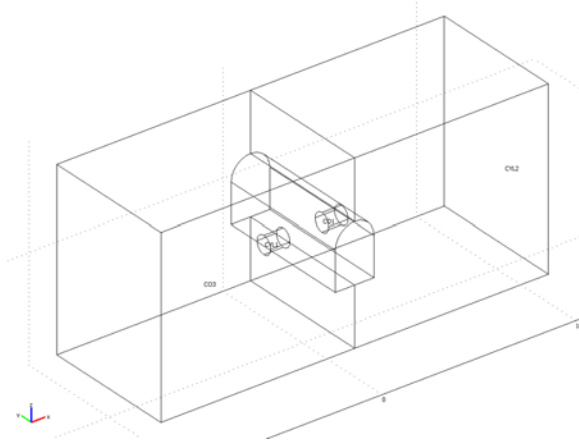


Fig.4 – The geometry of the model used in COMSOL software

As displacement in opponent walls in PLT test and therefore modulus are different, model is divided into two equal halves.

The mode which is used in the model is “Static analysis Elasto-Plastic material”. Input data were Young’s modulus, Poisson’s ratio, thermal expansion coefficient, and Density.

While Young’s modulus are different in the tests, other parameters considered constant in all tests.

Poisson’s ratio considered as 0.3, thermal expansion coefficient as 0, and density as 2650 Kg/m³.

Using formula 4 and 8, we can conclude that Δ and stiffness (and therefore Young’s Modulus) are linearly proportional (reverse relation). Therefore, one can use the output of the model for all other tests by multiplying or dividing the ratio of displacements.

6. Comparison between COMSOL output and other methods

As mentioned before, displacement in each individual wall is measured using multiple point extensometers. There are 69 data available. All tests performed using 20 MPa pressure with 0.915 diameter bearing plate. Knowing real displacement measured in the field and displacement resulting from running the model, Young’s modulus was calculated for all tests. COMSOL results along with ASTM and Unal results are presented in Table 1.

Table 1. Rock mass modulus results using ASTM, Unal, and COMSOL

	Displacement (mm)	ASTM	Unal	COMSOL
1	0.9022	16	14	18
2	8.92325	1.6	1	2
3	1.39859	13	14	11
4	1.75057	11	10	9
5	0.65635	20	22	24
6	1.25913	13	12	13
7	2.60705	6	5	6
8	1.98209	6	6	8
9	3.30697	4	3	5
10	2.16548	7	6	7
11	3.9124	3	1	4
12	9.15488	6	4	2
13	3.9209	5	3	4
14	2.49799	5	4	6
15	5.17724	5	3	3
16	3.75737	6	5	4
17	5.1913	4	3	3
18	2.18641	8	6	7
19	0.41507	46	35	38
20	0.5416	29	36	29
21	0.90202	18	20	18
22	0.56116	25	26	28
23	0.57231	26	30	28
24	0.45455	30	39	35
25	0.92442	20	23	17
26	0.79693	19	24	20
27	0.55617	31	34	29
28	0.6924	26	28	23
29	0.55878	29	28	28
30	0.82102	21	27	19
31	1.63973	13	9	10
32	1.06136	20	13	15
33	1.89794	10	8	8
34	1.17801	16	13	14
35	0.71312	22	18	22

	Displacement (mm)	ASTM	Unal	COMSOL
36	0.7231	21	17	22
37	1.48237	13	11	11
38	1.61439	12	11	10
39	1.77752	12	9	9
40	3.27053	6	4	5
41	0.7844	15	19	20
42	1.4536	8	8	11
43	2.61034	7	5	6
44	4.76521	5	3	3
45	2.43342	8	7	7
46	2.18729	8	6	7
47	4.66047	4	3	3
48	28.2502	1	0.5	1
49	2.44956	6	8	6
50	2.65729	4	7	6
51	6.49495	3	2	2
52	5.60267	3	3	3
53	2.05428	7	6	8
54	2.46817	7	5	6
55	0.61864	17	21	26
56	0.9139	14	17	17
57	1.986	7	7	8
58	1.79886	10	8	9
59	0.40323	26	30	39
60	0.78344	14	13	20
61	0.60131	18	23	26
62	19.97523	2	1	1
63	0.73078	22	20	22
64	0.89537	21	23	18
65	3.28524	7	5	5
66	6.02092	4	3	3
67	7.64737	3	2	2
68	2.08915	10	7	8
69	3.62358	7	5	4

Table 2 shows the statistical analysis of the test results.

Table. 2 . Statistical information of 35 PLT tests.

	ASTM	UNAL	COMSOL
Min.	1	0.5	0.6
Max.	46	39	39.0
Mean	12.6	12.4	12.6
Median	10	8	9.0
Mode	7	3	6.0
STD.	9.1	10.2	9.9
Skewness	1.1	0.9	0.9
Count	69	69	69

Figures 5 shows the relation between Young's modulus calculated from COMSOL software and ASTM and Unal method.

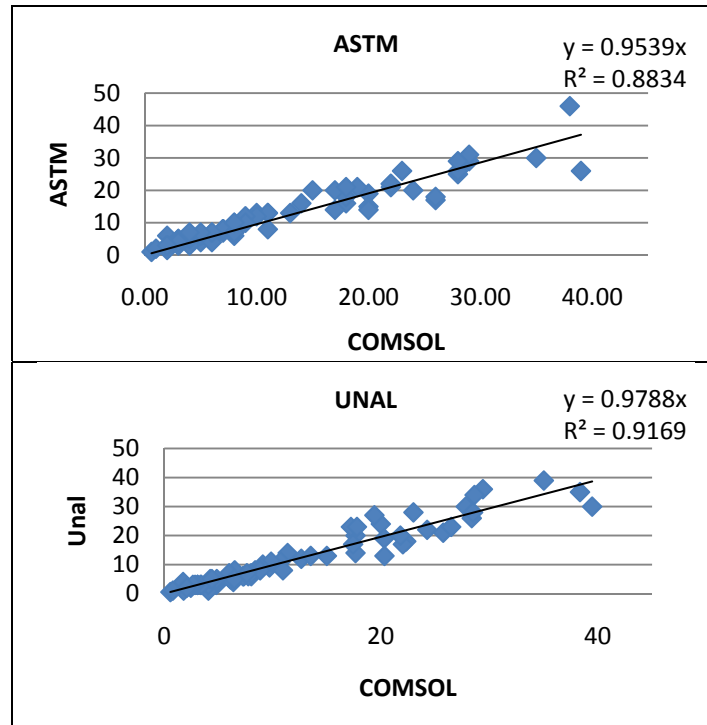


Figure 5. Relation between COMSOL and ASTM and Unal formulas

It can be seen that the values of all methods are in good consistent to each other. But in general, it can be seen that the values from COMSOL are a little greater than the values from ASTM or Unal formula.

7. Parametric Study

To evaluate the effect of other parameters on the results, sensitivity analysis is performed. As mentioned before, input parameters except Young's modulus were Poisson's ratio, thermal expansion coefficient,

and density. Among them Poisson's ratio is the only parameter that is used in governing equation and has the effect on the output. Therefore, only Poisson's ratio was changed. Three different scenarios were considered. Poisson's ratio was considered as 0.2, 0.25, and 0.35 in these scenarios. The statistical results are presented in table 3.

Table 3. Statistical values of different models using different Poisson's ratio

	$\nu= 0.2$	$\nu= 0.25$	$\nu= 0.3$ (Base Case)	$\nu= 0.35$
Min.	0.6	0.6	0.6	0.5
Max.	42.1	40.9	39.0	37.7
Mean	13.5	13.1	12.6	12.1
Median	9.4	9.2	9.0	8.4
Mode	8	5	6.0	6
STD.	10.5	10.3	9.9	9.4
Skewness	0.9	0.9	0.9	0.9
Count	69	69	69	69

As can be seen, by increasing the Poisson's ratio, Young's modulus will decrease. But, it should be noted that the change in Young's modulus has not the same magnitude of Poisson's ratio and can be neglected in practice.

8. Conclusion

Using COMSOL Multiphysics 3.5 software, Plate Load Test which was performed in Bakhtiary dam was simulated. Young's modulus for 35 different tests were calculated and compared with their equivalent values calculated by ASTM and Unal formula. Results show very good consistency between different methods.

To evaluate sensitivity of results regarding input variable, three different values (except base case) were considered in calculations. Results do not show significant difference in values.

9. References

1. Unal, E. 1997. Determination of in situ deformation modulus: New approaches for Plate-Loading Tests, *Int. J. of Rock Mech. Min. Sci.* 34: 6, 897-915.
2. ASTM D-4394-08. 2008. Standard Test Method for Determining In Situ Modulus of Deformation of Rock Mass Using Rigid Plate Loading Method.
3. Elsworth, D. 2009. Lecture notes.