PENN STATE UNIVERSITY

Coupled Hydro-Mechanical Response of dual porosity coal seam

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ABSTRACT

A dual porosity model capable of accommodating the evolution of stresses, porosities, permeabilities, and gas pressures is applied to represent the coupled hydro-mechanical behavior in coal seams. This model involves two overlaying continua: a macroscopic cleat system or fracture system and a less permeable microscopic matrix system. The bases of this model are the physics of flow into the dual porosity concept and the mathematical theory of homogenization. The formulation leads to a coupled system of three nonlinear partial differential equations which was solved numerically using COMSOL Multiphysics. This paper details the computational procedure, simulation solutions, validation, and parametric studies for this coupled behavior.

INTRODUCTION

The significance of coupling behavior between gas flow and solid deformation has received considerable attention in physical processes of gas outbursts during coal mining and CO2 geological sequestration in coal seams [1-7]. Gas flow within coal seams differs from liquid flow due to the large gas compressibility and Klinkenberg effects [8]. Coal is a naturally fractured dual-porosity reservoir, consisting of micro-porous matrix and macro-porous cleats. Gas is stored primarily by sorption into the micro-porous coal matrix [9-10].

Gas transport in coal seams is commonly understood as two hydrodynamic mechanisms by taking dual-porosity into account: laminar flow through the macroscopic cleat (Darcy's law) and diffusion through the coal matrix bounded by cleat (Fick's law) [11]. Figure 1 shows the migration process of methane in the coal seam. In additions, sorption or desorption-induced strain of the coal matrix can change the porosity and the permeability of the coal seam. The increase/decline of pore pressure due to sorption/desorption results in a concomitant increase/decrease in effective stress, which consequently reduce/ increase the stress-dependent permeability of the cleat system. In contrast, the sorption-induced swelling/desorption-induced shrinkage of coal matrix widens/narrows the cleats and enhances/decreases permeability. The net change of permeability accompanying gas sorption or desorption is thus controlled by the competitive effects of change in pore pressure and change in coal matrix deformation, also dependent on the mechanical boundary conditions applied to the coal seam [12-13]. Figure 2 shows schematic of inter-relations between matrix and fracture system.

Numerical simulations of gas diffusion, gas flow, and the coupled hydro-mechanical response have been widely studied, in mass transport in porous media[14], in gas sorption effect on mass storage [15], and in gas diffusion effect [16]. For a coal seam media, in which the gas in the matrix blocks and in the cleats are considered as separate continua, the response of this coupled process can be represented by dual porosity models, related interactively through a transfer function [16-17], by dual permeability or multiple permeability models representing the porosity and permeability for all constituent components [18-20]. Such models have been applied to represent the response of coupled slightly compressible fluid flow and solid deformation systems, and also compressible gas flow with sorption mechanism [21].

In this paper, a dual porosity elastic model [21] was applied to represent the complex hydro-mechanical coupled behaviors in a coal sample.

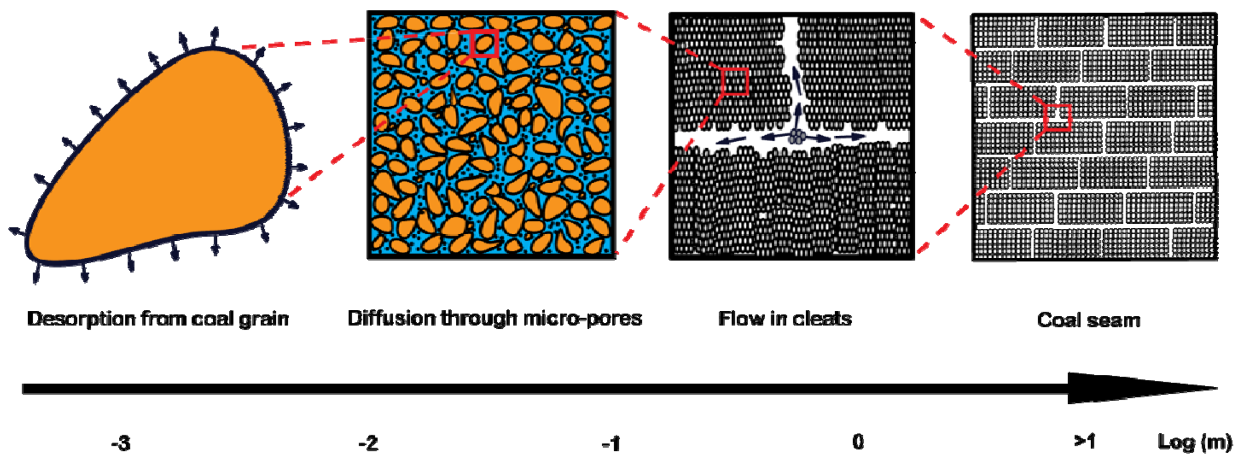

Figure 1 Migration process of gas in dual porosity coal seams

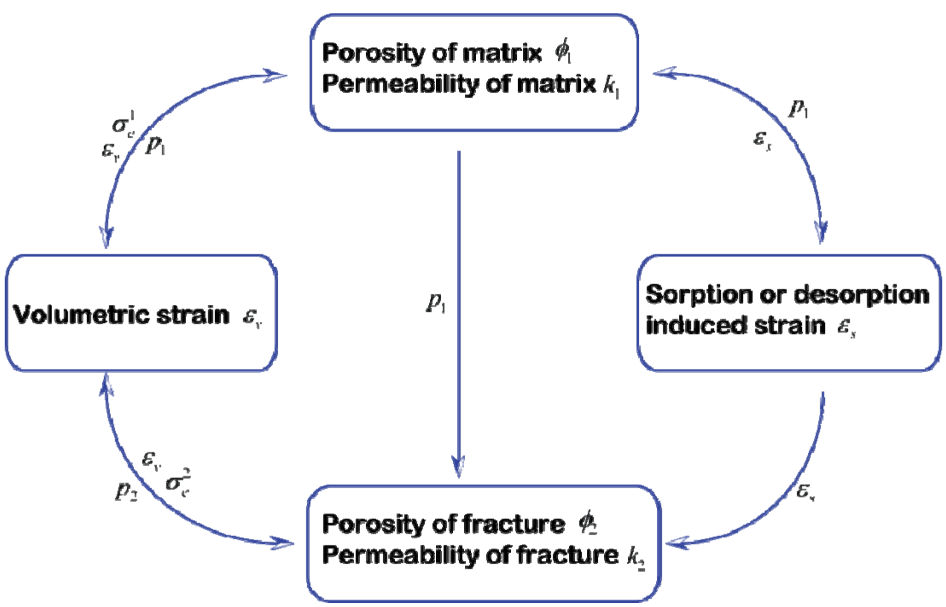

Figure 2 Schematic of inter-relations between matrix and fracture system

GOVERNING EQUATIONS

The governing equations for the behavior of a dual-porosity medium are developed in the following section for a three-dimensional (3-D) case. Equations for solid deformation, gas pressure responses due to Darcy's law are coupled.

Mechanical equation

Mechanical equilibrium of the solid phase is governed by the balance of linear momentum.

$$\sigma_{ij,j} + b_i = 0 \tag{1}$$

where σ_{ij} is the component of the total stress tensor and b_i is the component of body force.

Constitutive equation for isotropic linear poroelastic medium with dual porosities

$$\sigma_{ij} = 2G\varepsilon_{ij} + \frac{2G\upsilon}{1-2\upsilon}\varepsilon_{kk}\delta_{ij} - (\alpha_1 p_1 + \alpha_2 p_2)\delta_{ij} \tag{2}$$

where subscripts 1 and 2, represent the matrix and fractures, respectively; G is shear modulus of fractured porous media (FPM); ε_{ij} is the strain tensor; υ is the Poisson ratio of FPM; ε_{kk} represents volumetric strain with the summation involved; δ_{ij} is the Kronecker delta; α is the pressure ratio factor, compatible with Biot's coefficient [22]; p is the gas pressure.

For the physical process of gas flow in coal seam, taking gas sorption or desorption induced strain into account, equation (2) becomes [21]

$$\sigma_{ij} = 2G\varepsilon_{ij} + \frac{2G\upsilon}{1-2\upsilon}\varepsilon_{kk}\delta_{ij} - (\alpha_1 p_1 + \alpha_2 p_2)\delta_{ij} - K^{\#}\varepsilon_s\delta_{ij} \tag{3}$$

where $K^{\#}$ is the bulk modulus of FPM (coal seam); ε_s is the gas sorption or desorption induced strain.

The elastic parameters for equation (3) can be defined as

$$C_1 = \frac{1}{E}, \quad C_2 = \frac{1}{K_n \cdot a}, \quad D_{12} = \frac{1}{C_1 + C_2} \tag{4}$$

$$G = \frac{D_{12}}{2(1+\upsilon)}, \quad K^{\#} = \frac{D_{12}}{3(1-2\upsilon)} \tag{5}$$

$$\alpha_1 = 1 - \frac{K^{\#}}{K_s}, \quad \alpha_2 = 1 - \frac{K^{\#}}{K_\mathrm{n} \cdot a} \tag{6}$$

where, E is elastic modulus of FPM; K_n is fracture normal stiffness; C is the compliance tensor; D_{12} is the elastic stiffness tensor of dual-porosity medium; K_s is bulk modulus of solid grains.

Based on the Langmuir equation, gas sorption or desorption induced strain can be defined as [23-24]

$$\varepsilon_s = \varepsilon_L \frac{p_1}{p_1 + p_L} \tag{7}$$

where ε_L is the Langmuir volumetric strain, a constant representing the volumetric strain at infinite pore pressure; p_L is the Langmuir pressure constant, representing the pore pressure at which the measured volumetric strain is equal to 0.5 ε_L .

The strain-displacement relationship is defined as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{8}$$

where u_i is the component of displacement.

Combining equations (1), (3), and (8), we have the following Navier-type equation

$$G\nabla u_i + \frac{G}{1-2\upsilon}u_{j,ji} = (\alpha_1 p_1 + \alpha_2 p_2)\delta_{ij} + K^{\#}\varepsilon_s\delta_{ij} + b_i \tag{9}$$

Gas flow equation

The mass balance equation for the gas phase is defined as

$$\frac{\partial m}{\partial t} + \nabla \cdot (\rho_g \mathbf{q}_g) = Q \tag{10}$$

where ρ_g is the gas density; $\mathbf{q}_g$ is the Darcy velocity vector; Q is the gas source; t is the time; and m is the gas content including free phase gas and adsorbed gas. In this study, gas sorption and desorption are assumed to take places only in the matrix. The gas contents in the matrix and the fractures, therefore, are defined as

$$m_1 = \rho_{g1}\phi_1 + \rho_{ga}\rho_c \frac{V_L p_1}{p_1 + p_L} \tag{11}$$

$$m_2 = \rho_{g2}\phi_2 \tag{12}$$

where m_1 and m_2 are gas contents in the matrix and fractures, respectively; ϕ_1 and ϕ_2 represent the porosities of the matrix and fractures; ρ_{ga} and ρ_c are the gas density at standard conditions and coal density, respectively; V_L and p_L represent the Langmuir volume constant and the Langmuir pressure constant.

According to the idea gas law, the gas density can be written as

$$\rho_g = \frac{M_g}{RT} p \tag{13}$$

where M_g is the molecular mass of the gas; R is the universal gas constant; and T is the absolute gas temperature.

Darcy velocity of the gas can be defined as the following, neglecting the gravitational term,

$$\boldsymbol{q}_g = -\frac{k}{\mu} \nabla \cdot p \tag{14}$$

where k the gas permeability of coal and μ is the dynamic viscosity of the gas.

Substituting equations (11)-(14) into equation (10), the gas flow governing equations for a dual-porosity medium are written as

$$\left[\phi_1 + p_a \rho_c \frac{V_L p_L}{(p_1 + p_L)^2} \right] \frac{\partial m}{\partial t} + p_1 \frac{\partial \phi_1}{\partial t} - \nabla \cdot \left(\frac{k_1}{\mu} p_1 \nabla \cdot p_1 \right) = \Gamma (p_2 - p_1) \tag{15}$$

$$\phi_2 \frac{\partial p_2}{\partial t} + p_2 \frac{\partial \phi_2}{\partial t} - \nabla \cdot \left(\frac{k_2}{\mu} p_2 \nabla \cdot p_2 \right) = -\Gamma (p_2 - p_1) \tag{16}$$

where p_a is standard atmosphere pressure, and ω represents the geometric leakage factor as a function of the shape factor $a^{\#}$ in Warren and Root's (1963) approach, expressed as

$$\Gamma = a^{\#} \frac{k_1}{\mu} \tag{17}$$

where, for a regularly spaced parallelepiped block-type matrix model, frequently referred to as the "sugar cube" model,

$$a^{\#} = \frac{60}{a^2} \tag{18}$$

In equations (15) and (16), ϕ_1, ϕ_2, k_1, and k_2 change with σ_e and ε_s.

Cross couplings

1. Porosity and permeability models for matrix

The porosity model for the matrix is given by [13, 21]

$$\phi_1 = \frac{1}{1+S}\left[(1+S_0)\phi_{10} + \alpha_1(S - S_0)\right] \tag{19}$$

where

$$S = \varepsilon_v + \frac{p_1}{K_S} - \varepsilon_s \tag{20}$$

$$S_0 = \frac{p_{10}}{K_S} - \varepsilon_L \frac{p_{10}}{p_{10} + p_L} \tag{21}$$

and ε_v is the volumetric strain, defined as

$$\varepsilon_v = \frac{\sigma_e^1}{K^{\#}} + \varepsilon_s \tag{22}$$

where σ_e^1 is the effective stress of matrix.

Considering the cubic law relation between permeability and porosity of the porous media, we have

$$\frac{k_1}{k_{10}} = \left\{\frac{1}{1+S}\left[(1+S_0)\phi_{10} + \alpha_1(S - S_0)\right]\right\}^3 \tag{23}$$

where the subscripts 0 and 1 represent the initial value of the variable and matrix.

2. Porosity and permeability models for fracture

For the REV cubic matrix, the porosity of a fracture system is given by [25]

$$\phi_2 = \frac{3b}{a} \tag{24}$$

The change in porosity is defined as

$$\Delta\phi_2 = \frac{3\Delta b}{a} - \frac{3b\Delta a}{a^2} = \phi_2\left(\frac{\Delta b}{b} - \frac{\Delta a}{a}\right) = \phi_2(\Delta\varepsilon_2 - \Delta\varepsilon_v) \tag{25}$$

where ε_2 and ε_v are the strain within the fracture and the volumetric strain of the matrix, respectively.

Substituting Eq. (22) into Eq. (25) yields,

$$\frac{\Delta\phi_2}{\phi_2} = \frac{\Delta\sigma_e^2}{Kn} - \frac{\Delta\sigma_e^1}{K^\#} - \Delta\varepsilon_s \tag{26}$$

The dynamic porosity in fracture can be expressed as

$$\phi_2 = \phi_{20}\exp\left[\frac{\sigma_e^2 - \sigma_e^{20}}{Kn} - \frac{\sigma_e^1 - \sigma_e^{10}}{K} - (\varepsilon_s - \varepsilon_s^0)\right] \tag{27}$$

where $\sigma_e^1 = \frac{\sigma_{kk}}{3} + \alpha_1 p_1$, $\sigma_e^2 = \frac{\sigma_{kk}}{3} + \alpha_2 p_2$.

For the fracture system with orthogonal fractures, the cubic law for fracture permeability can be defined as

$$k_2 = \frac{b^3}{12a} \tag{28}$$

The change in permeability of the fracture system then can be expressed as

$$\Delta k_2 = \frac{3b^2\Delta b}{12a} - \frac{b^3\Delta a}{12a^2} = k_2\left(\frac{3\Delta b}{b} - \frac{\Delta a}{a}\right) = k_2(3\Delta\varepsilon_2 - \Delta\varepsilon_v) \tag{29}$$

Substituting Eq. 22 into Eq. (29), yields,

$$\frac{\Delta k_2}{k_2} = 3\frac{\Delta\sigma_e^2}{Kn} - \frac{\Delta\sigma_e^1}{K^\#} - \Delta\varepsilon_s \tag{30}$$

The porosity in fracture can be rewritten as

$$k_2 = k_{20}\exp\left[3\frac{\sigma_e^2 - \sigma_e^{20}}{Kn} - \frac{\sigma_e^1 - \sigma_e^{10}}{K} - (\varepsilon_s - \varepsilon_s^0)\right] \tag{31}$$

where k_{20} is the initial porosity of the fracture system at the initial effective stress σ_e^{20}.

$$\phi_{20} = \frac{3b_0}{a_0} \tag{32}$$

$$k_{20} = \frac{b_0^3}{12a_0} \tag{33}$$

Therefore, the general porosity and permeability model for a dual-porosity medium is defined by Eqs. (19), (23), (27), and (31).

Coupled field equations

Governing equation for coal deformation:

$$Gu_{i,kk}+\frac{G}{1-2\upsilon}u_{k,ki}=\alpha_1 p_{1,i}+\alpha_2 p_{2,i}+K\varepsilon_L\frac{p_L}{(p_1+p_L)^2}p_{1,i}-b_i \tag{34}$$

Gas flow equation in matrix:

$$\delta_{ts}\frac{\partial p_1}{\partial t}-\frac{k_1}{\mu}\nabla^2 p_1=\Gamma(p_2-p_1) \tag{35}$$

where $\delta_{ts}=\phi_1+p_a\rho_c\frac{V_L p_L}{(p_1+p_L)^2}+\frac{(\alpha_1-\phi_1)p_1}{(1+S)K_S}-\frac{(\alpha_1-\phi_1)\varepsilon_L p_L p_1}{(1+S)(p_1+p_L)^2}+\frac{(\alpha_1-\phi_1)p_1}{(1+S)}\frac{\partial\varepsilon_v}{\partial p_1}$

Gas flow equation in fracture:

$$\phi_2\left(1+\frac{\alpha_2 p_2}{K_n}\right)\frac{\partial p_2}{\partial t}-\frac{k_2}{\mu}\nabla^2 p_2=-\Gamma(p_2-p_1)+R \tag{36}$$

where $R=\phi_2\left(\frac{\alpha_1 p_2}{K}+\frac{\varepsilon_L p_L p_1}{(p_1+p_L)^2}\right)\frac{\partial p_1}{\partial t}-\frac{p_2\phi_2}{3}\left(\frac{1}{K_n}-\frac{1}{K}\right)\frac{\partial\sigma_{kk}}{\partial t}$

Hence, Eqs. (34)-(36) define a model for coupled coal deformation and gas flow in dual-porosity medium.

BOUNDARY AND INITIAL CONDITIONS

For the Navier-type equation, the displacement and stress conditions on the boundary are given as

$$u_i=\tilde{u}_i(t),\ \sigma_{ij}n_j=\tilde{F}_i(t)\ \text{on}\ \partial\Omega \tag{37}$$

where $\tilde{u}_i(t)$ and $\tilde{F}_i(t)$ are the components of prescribed displacement and stress on the boundary $\partial\Omega$, respectively; and n_j is the direction cosine of the vector normal to the boundary. The initial conditions for displacement and stress in the domain Ω are described as

$$u_i(0)=u_0\quad \sigma_{ij}(0)=\sigma_0\ \text{in}\ \Omega \tag{38}$$

Here, u_0 and σ_0 are initial value of displacement and stress in the domain Ω. For the gas flow equations, the Dirichlet and Neumann boundary conditions are defined as

$$p_1=\tilde{p}_1(t)\quad \vec{n}\cdot\frac{k_1}{\mu}\nabla p_1=\tilde{Q}_s^1(t)\ \text{on}\ \partial\Omega \tag{39}$$

$$p_2 = \tilde{p}_2(t) \quad \vec{n} \cdot \frac{k_2}{\mu} \nabla p_2 = \tilde{Q}_s^2(t) \text{ on } \partial\Omega \tag{40}$$

Here, $\tilde{p}(t)$ and $\tilde{Q}_s(t)$ are the specified gas pressure and gas flux on the boundary. The initial conditions for gas flow are

$$p_1(0) = p_{10} \quad p_2(0) = p_{20} \text{ in } \Omega. \tag{41}$$

Formulation

In order to investigate the dual poroelastic response of a coal seam to CO2 injection, a simulation model was built as shown in Figure 3. It was a cylinder specimen with 1 inch in diameter and 2 inches in length. Axial symmetric structural mechanic model was used. The model has 1016 elements in total and the number of degrees of freedom is 8436 comprising two displacements and two pressures (matrix and fracture) at each node.

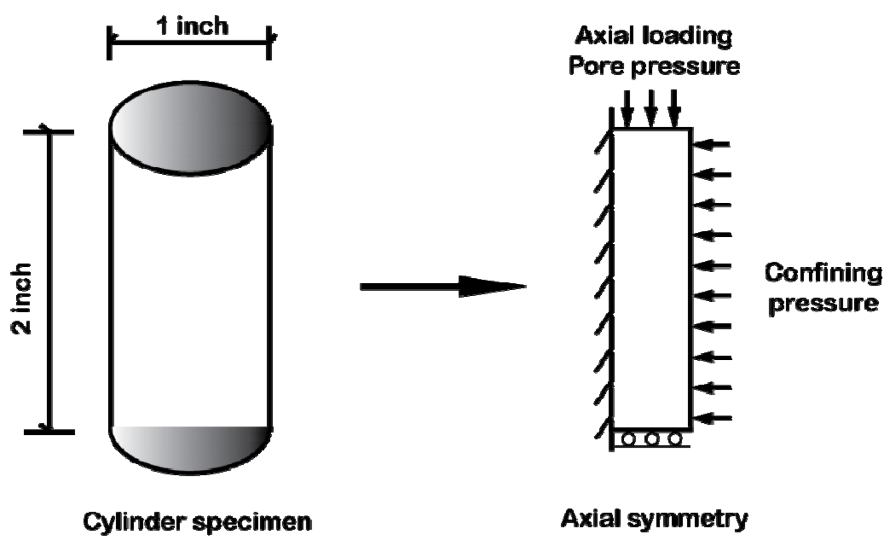

Figure 3 Model geometry of CO_2 injection to a coal seam

For the coal deformation model, the left boundary is symmetric plane and bottom boundary is rollered and in situ stresses are applied to the top and the right side. For gas flow, a constant pressure of 8MPa is applied on the top boundary. No flow conditions are applied to all the other boundaries. An initial pressure of 0.5MPa is applied in the model. Input properties are listed in Table 1. The values of these properties were chosen from the literature with the initial porosity and permeability of the fracture system calculated from Eq. (27) and Eq. (31).

Table 1 Property parameters of the model

Parameter	Value
Young's modulus of coal, E (GPa)	2.7
Young's modulus of coal grain, E_s (GPa)	8.1
Possion's ratio of coal, v	0.339
Density of coal, ρ_c(kg/m^3)	1400
Dynamic viscosity of CO_2,μ(Pa s)	1.84×10^{-5}
Lanmuir pressure constant, P_L(MPa)	6.109
Lanmuir volume constant, V_L(MPa)	0.015
Lanmuir volumetric strain constant, ε_L	0.02295
Initial porosity of matrix,φ_{m0}	0.02
Initial permeability of matrix, k_{m0}(m^2)	10^{-18}
Fracture aperture, b_0(m)	1×10^{-4}
Matrix size, a_0(m)	0.01

SIMULATION RESULTS

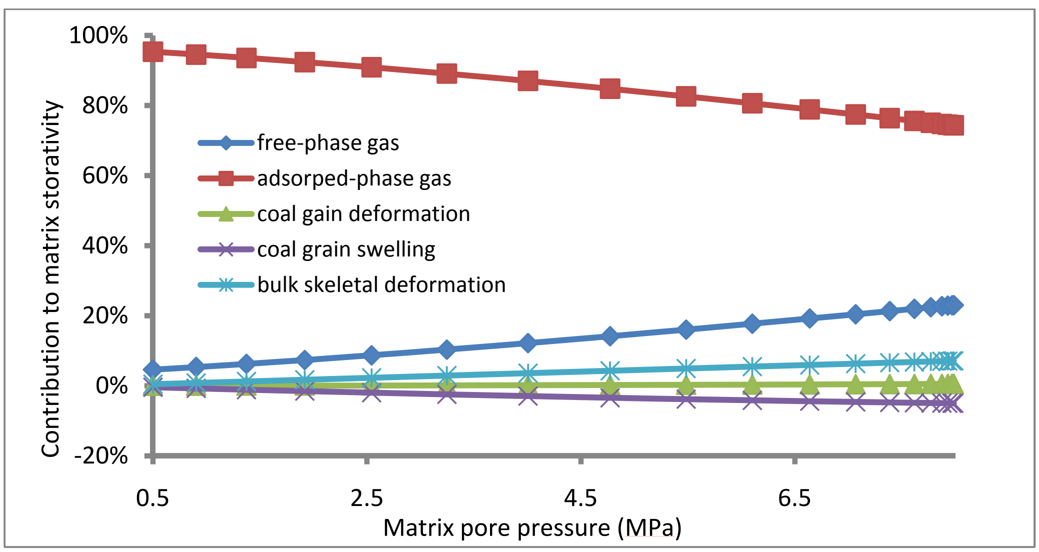

Figure 4 Contributions of each mechanism to gas storativity in the matrix

There are five contributing mechanisms to the storativity: free gas compression, gas absorption, coal grain deformation, coal shrinking or swelling, and coal skeletal deformation. As the matrix pore pressure increases, the volume of gas sequestered from the adsorbed-phase gas contributes about 95.35%-74.36% to the total gas storativity. The volume of gas released from the free-phase contributes 4.61%-23.02% to the total gas storativity, and that from bulk deformation contributes 0.45%-7.10% to the total gas storativity. The contributions from the other mechanisms are less than 6% in total. These results indicate that gas sorption is the primary mechanism for gas production or sequestration.

As we can see in Figure 5 the permeability ratio decreases with an increase in the matrix pore pressure. The effective stress effect and the sorption effect are competing: an increase in the matrix pressure enhances the matrix permeability while an increase in the sorption reduces the matrix permeability. For this particular condition the resultant effect is a monotonic decrease in permeability with increasing pressure as the effects of sorption-induced swelling dominate.

Unlike the flow of slightly compressible fluids where the fracture permeability typically increases with an increase in pore pressure in the fracture, under this particular condition, injection-induced permeabilities within the fracture will initially increase and subsequently decrease as sorptive stresses build up, as shown in Figure 6.

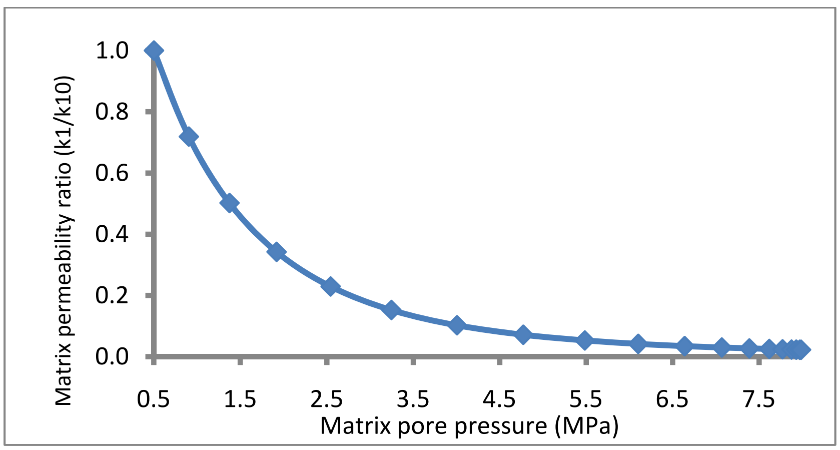

Figure 5 The relation between matrix permeability ratio and matrix pore pressure at the specific point of $x = 0.00635$ and $y = 0.0254$

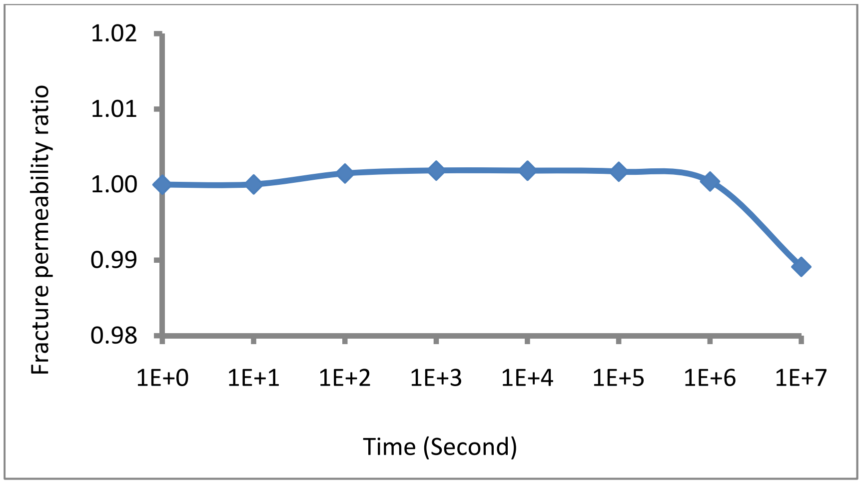

Figure 6 The relation between matrix permeability ratio and fracture pressure at the specific point of $x = 0.00635$ and $y = 0.0254$

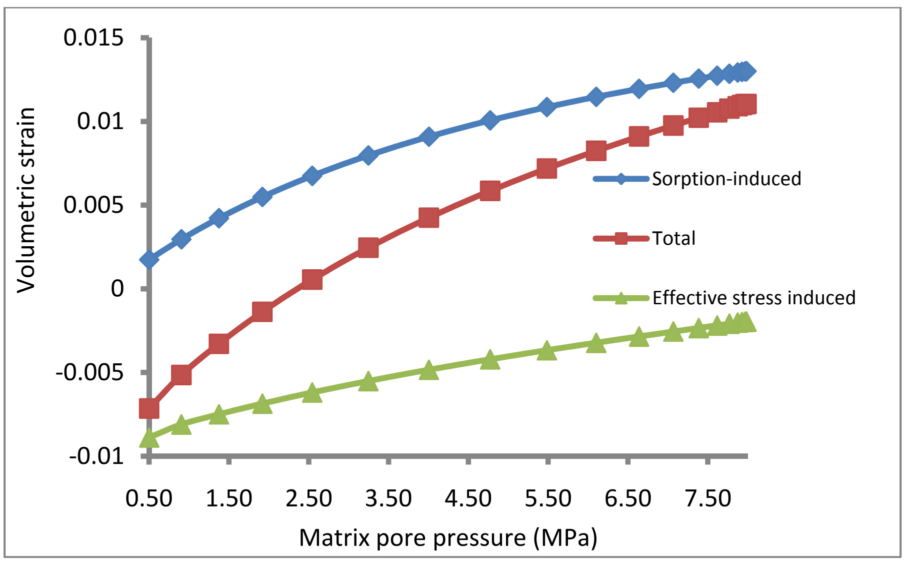

Figure 7 The relations between volumetric strains and matrix pore pressure

Results from Figure 7 also support that the effects of sorption-induced swelling is greater than effective stresses induced permeability increase.

VALIDATION

The general physical description of dual-porosity behavior is shown in Figure 8. At the initial stages of pumping, fluid flow occurs mainly within the fractures. After exhausting the storage, flow begins to occur primarily between the matrix and fractures showing the reduction of pressure gradient. The model in this study shows the same characteristics as fracture pressure builds up quickly and then slows down when sorption starts with the matrix.

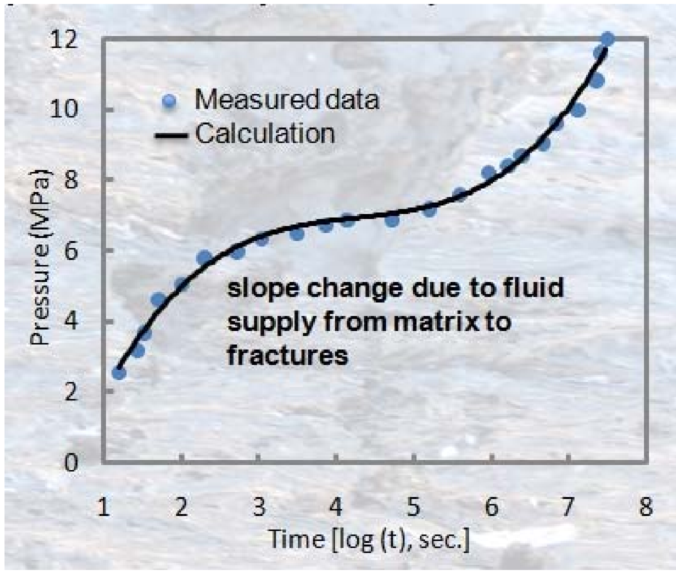

Figure 8 Pressure transient in a typical dual-porosity reservoir[20]

PARAMETRIC STUDY

A series of injection conditions as listed in Table 2 was simulated to investigate the mechanical responses of a dual porosity coal seam. Simulation results are presented in terms of (1) the impacts of modulus ratio, (2) the impacts of fracture spacing, and (3) the impacts of in situ stresses.

Table 2 Parametric study

Case 1 ratio of coal bulk modulus to gain modulus	$K^{\#}/K_s = 1/2$ $K^{\#}/K_s = 1/5$ $K^{\#}/K_s = 1/10$
Case 2 fracture spacing	$a = 0.01$ $a = 0.05$ $a = 0.10$
Case 3 in situ stresses	$\sigma_1 = -7.5\text{MPa},\ \sigma_3 = -5\text{MPa}$ $\sigma_1 = -15\text{MPa},\ \sigma_3 = -10\text{MPa}$ $\sigma_1 = -30\text{MPa},\ \sigma_3 = -20\text{MPa}$

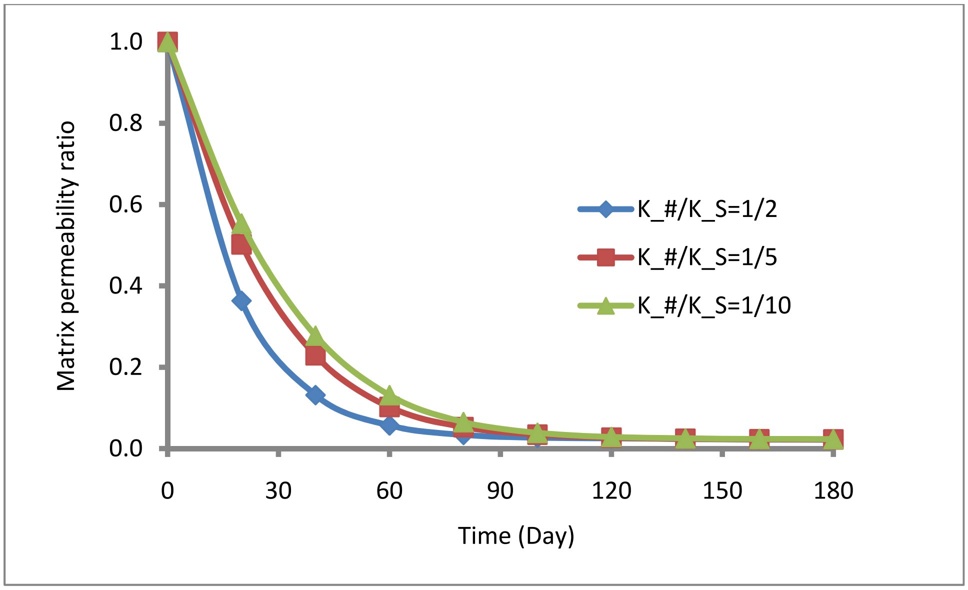

Figure 9 The impacts of ratios of the coal bulk modulus to the coal grain modulus

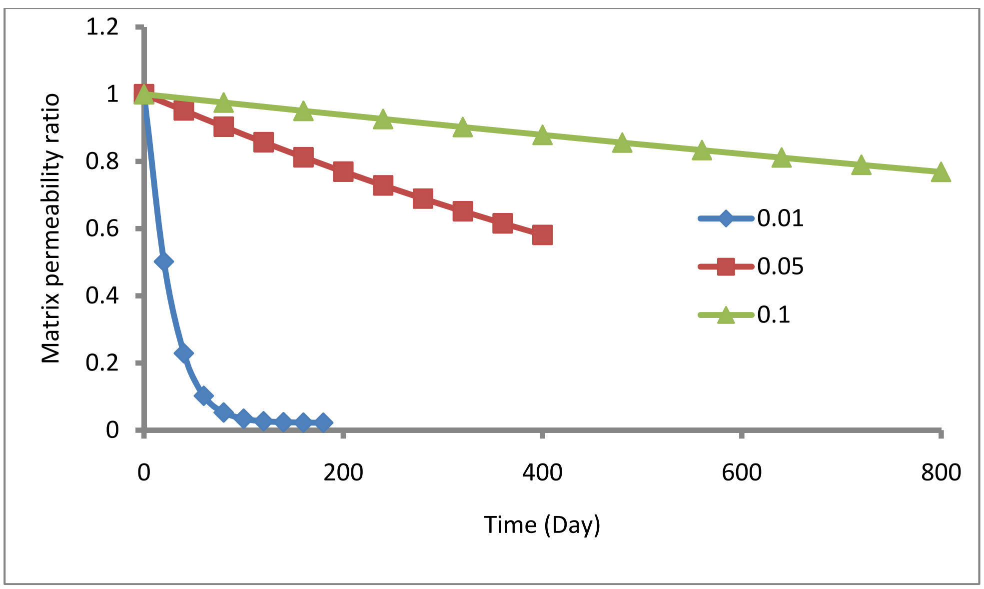

Figure 10 The impacts of in situ stresses

The relation between matrix permeability ratio and the timing of gas flow at a specific point is shown in Figure 9, Figure10, and Figure 11. The permeability ratio decreases with an increase in the matrix pore pressure. The greater the ratio of coal bulk modulus to coal grain modulus, the lower the in situ stresses, the smaller the fracture spacing, the more rapid the reduction in matrix permeability ratio.

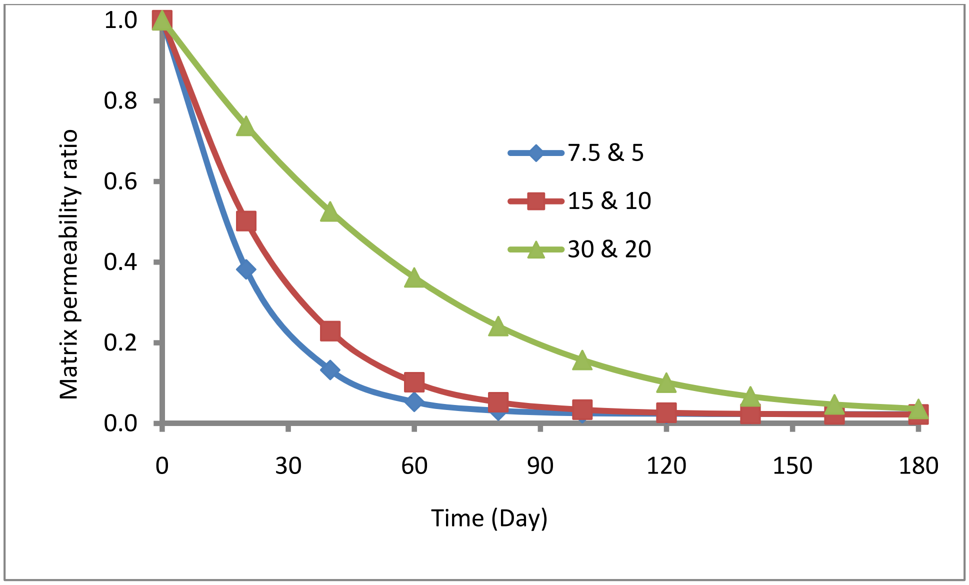

Figure 11 The impacts of fracture spacing

Conclusions

In this study, a dual-poroelastic model is applied to simulate the coupled behavior in the process of CO_2 injection into a coal specimen. This model is capable of simulating compressible gas flow and transport in matrix and fracture system, also taking account of the role of sorption-induced strain, for the dual porosity coal specimen. It can recover the evolution of porosity and permeability in both the coal matrix and the fracture network. It can represent the important non-linear responses due to the completion between effective stress and sorption-induced stress. From this study, the following conclusions are obtained. Gas sorption or desorption is the primary mechanism for either gas sequestration (sorption) or production (desorption). The greater the ratio of coal bulk modulus to coal grain modulus, the lower the in situ stresses, the smaller the fracture spacing, the more rapid the reduction in matrix permeability ratio. Injection-induced permeability within the fracture system initially increases and subsequently decreases as sorption induced stress builds up.

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