You have the entire class period to complete this closed-book examination. I give you all the equations and numbers that you will need, except those that I have asked you to memorize on the syllabus and the study guide. If you have any questions about the meaning of the exam questions, please come see me. Good luck.

- Poisson's relations: $TV^{\gamma-1} = \text{const.}$; $pV^{\gamma} = \text{const.}$; $p^{(1-\gamma)/\gamma}T = \text{const.}$, for $\gamma = c_p/c_v = 1.4$
- Specific heat capacity of dry air at 0°C: $c_v = 720 \text{ J kg}^{-1} \text{ K}^{-1}$; $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$
- Specific heat capacity of water vapor, 0°C: $c_v = 1390 \text{ J kg}^{-1} \text{ K}^{-1}$; $c_p = 1850 \text{ J kg}^{-1} \text{ K}^{-1}$
- Specific heat capacity of liquid water, 0°C: $c_w = 4218 \text{ J kg}^{-1} \text{ K}^{-1}$
- Specific heat capacity of ice, 0°C: $c_i = 2106 \text{ J kg}^{-1} \text{ K}^{-1}$
- $R_v = 46\gamma J \text{ kg}^{-1} \text{ K}^{-1}$; $R_d = 287 \text{ J kg}^{-1} \text{ K}^{-1}$

1. (25 points) Define and explain the following concepts in a few words, numbers, or diagrams.
   a. the equation for the Ideal Gas Law
      $$pV = nR*T \quad (p=\text{pressure}; V=\text{volume}; n=\# \text{ molecules}; R=\text{gas constant}; T=\text{temp})$$
      $$p = \rho R T \quad (\rho=\text{density}; R=\text{mass dependent gas constant})$$
   b. the equation for enthalpy
      $$H = U + pV \quad (U=\text{internal energy}; p=\text{pressure}; V=\text{volume})$$
   c. the First Law of Thermodynamics
      $$\frac{du}{dt} = Q + W \quad (\frac{du}{dt}=\text{change in } U; Q=\text{heating}; W=\text{working})$$
   d. four independent ways you can increase entropy
      1. increase volume
      2. increase temperature
      3. increase # of molecules
      4. mix two types of molecules
   e. definition for temperature in terms of molecular velocity
      $$\frac{1}{3} <mv^2> = kT$$
2. (10 points) Show with diagrams which environmental lapse rates cause dry air parcels to be statically stable and which caused dry air parcels to be statically unstable.

3. (10 points) How does a barometer work? Please draw a diagram and discuss the forces that are involved. You should end up with a very simple equation.

4. (10 points) Why physically is the specific heat capacity at constant pressure greater than the specific heat capacity at constant volume? Why physically is the specific heat capacity of water vapor greater than the specific heat capacity of dry air?

\[ C_v = \frac{\partial Q}{\partial T} \quad ; \quad C_p = \frac{\partial H}{\partial T} \]

a.) At constant pressure, some energy goes into doing work (\( W = -pdV/\partial t \)) of expanding the volume, while for constant \( V \), all energy goes into raising \( U \), so \( C_p > C_v \)

b.) \( C_w > C_d \) because 1 kg of \( \text{H}_2\text{O} \) is more molecules than 1 kg of air; specific heats are per unit mass and the heat capacity of a water molecule is greater than that of \( \text{Ne} \) or \( \text{O}_2 \) (six degrees of freedom compared to 5 for \( \text{Ne} \) or \( \text{O}_2 \))
5. (20 points) Suppose that a dry air parcel with a temperature of 290 K is at a pressure of 850 hPa. The air parcel’s volume is \(1 \times 10^7 \text{ m}^3\) at this pressure.

a. What is the air parcel’s potential temperature?

\[
\Theta = 290 \left( \frac{1000}{850} \right)^{286} \Rightarrow \Theta = 304 \text{ K}
\]

b. What is the air parcel’s density?

\[
p = \frac{P}{R_T T} = \frac{850}{287.290} \Rightarrow p = 1.02 \text{ kg m}^{-3}
\]

c. What is the air parcel’s temperature if it is raised adiabatically to a pressure level of 500 hPa?

\[
304 = T \left( \frac{1000}{500} \right)^{286} \Rightarrow T = 249 \text{ K}
\]

\[
\frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{1}{286}} \Rightarrow T = T_0 \left( \frac{p}{p_0} \right)^{286}
\]

\[
T_{500} = 290 \left( \frac{500}{850} \right)^{286} \Rightarrow T_{500} = 249 \text{ K}
\]

d. If heating occurs so that the air parcel absorbs \(10^8 \text{ J}\) of energy at constant pressure, what is the new potential temperature? (hint: Find the change in temperature for a constant pressure process and add that change to the temperature.)

\[
Q = \frac{dH}{dt} = \nabla \cdot \rho \frac{dV}{dt} + \frac{dP}{dt} \Rightarrow \int Q dt = c_p \Delta T
\]

\[
\Delta T = \frac{10^8}{c_p \cdot p \cdot V} \Rightarrow \Delta T = 9.8 \text{ K}
\]

The new temperature is \(T_{\text{new}} = 290 + 9.8 = 299.8 \text{ K}\)

\[
\Theta_{\text{new}} = 299.8 \left( \frac{1000}{850} \right)^{286} \Rightarrow \Theta_{\text{new}} = 314 \text{ K}
\]
6. (10 points) As air ascends adiabatically, which changes more, pressure or volume? (hint: Use Poisson’s Relation between pressure and volume.)

1. \( pV^\gamma \) = constant. Differentiate:
\[
0 = \frac{dp}{p}V^\gamma + p\gamma V^{\gamma-1}dV = \frac{dp}{p}. pV^\gamma + \gamma \frac{dV}{V}. pV^\gamma
\]
\[
\Rightarrow \frac{dp}{p} / \frac{dV}{V} = -\gamma = -1.4
\]
\[\text{Pressure change is greater.}\]

2. Suppose pressure changes a factor of 2. How much does volume change? \( pV^\gamma = \left(\frac{p_f}{p_i}\right)^\gamma V_f \), where \( p_f = \frac{p}{2} \)
\[V_f / V = (2)^{1/\gamma} = 1.6. \text{ The pressure change is greater.}\]

7. (15 points) Which has greater buoyancy at a pressure of 1000 hPa, an air parcel with a temperature of 315 K and a water vapor partial pressure of 10 hPa, or an air parcel with a temperature of 305 K and a water vapor pressure of 45 hPa?

- The densities of dry air and water vapor add together to give the total density. Density determines buoyancy.

\[ \rho = \frac{\theta}{R_A} + \frac{\theta_d}{R_v} \]

parcel 1: \( \rho_1 = \frac{(10000-1000)}{287.315} + \frac{1000}{462.315} = (1.095 + .007) \text{ kg m}^{-3} \)
\[\rho_1 = 1.10 \text{ kg m}^{-3}\]

parcel 2: \( \rho_2 = \frac{(10000-4500)}{287.305} + \frac{4500}{462.305} = (1.091 + .032) \text{ kg m}^{-3} \)
\[\rho_2 = 1.12 \text{ kg m}^{-3}\]

\[\rho_1 < \rho_2, \text{ so parcel 1 is more buoyant than parcel 2.}\]