■ Define Functions to Plot Stereographic and Cyclographic Projections

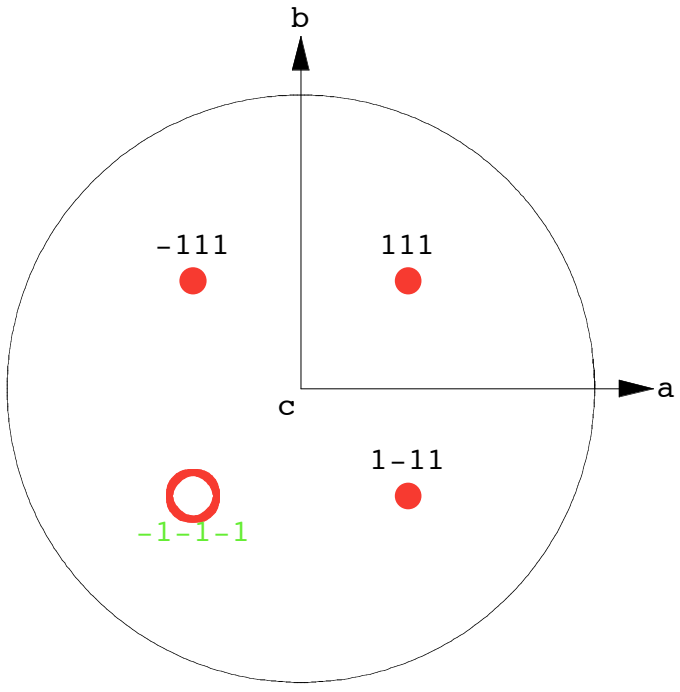
```
<< Graphics`Arrow`

stereographicProjection[{h_, k_, l_}] := Module[{n = {h, k, l} / Sqrt[{h, k, l}.{h, k, l}], θ, r, ϕ},
        θ = ArcCos[n[[3]]];
        r = Tan[θ/2];
        ϕ = ArcTan[n[[2]]/n[[1]]] - π/2 (Sign[n[[1]] - 1) - π/2 (Sign[n[[1]] + 1) (Sign[n[[2]] - 1);
        If[θ ≤ π/2, {{PointSize[0.04], RGBColor[1, 0, 0], Point[r {Cos[ϕ], Sin[ϕ]}]},
    Text[StringJoin[ToString[h], ToString[k], ToString[l]], r {Cos[ϕ], Sin[ϕ]}, {0, -2}]},
   {{Thickness[0.01], RGBColor[1, 0, 0], Circle[Tan[(π - θ)/2] {Cos[ϕ], Sin[ϕ]}, 0.08]},
    Text[StyleForm[StringJoin[ToString[h], ToString[k], ToString[l]], FontColor → RGBColor[0, 1, 0]],
     Tan[(π - θ)/2] {Cos[ϕ], Sin[ϕ]}, {0, 2}]}]]

cyclographicProjection[{h_, k_, l_}] := If[h == 0 && k == 0,
  {Thickness[0.01], RGBColor[1, 0, 1], Circle[{0, 0}, 1]}, Module[{n = {h, k, l} / Sqrt[{h, k, l}.{h, k, l}], θ, r, ϕ},
        θ = ArcCos[n[[3]]] - π/2;
        ϕ = ArcTan[n[[2]]/n[[1]]] - π/2 (Sign[n[[1]] - 1) - π/2 (Sign[n[[1]] + 1) (Sign[n[[2]] - 1);
   {{Thickness[0.01], RGBColor[1, 0, 1], Line[Table[{Sin[ψ] Sin[θ], Cos[ψ]}.{{Cos[ϕ], Sin[ϕ]}, {-Sin[ϕ], Cos[ϕ]}}, {ψ, 0, π, π/30}]]},
    {Thickness[0.01], RGBColor[1, 0, 1], Dashing[{0.04, 0.02}],
     Line[Table[{Sin[ψ] Sin[θ], Cos[ψ]}.{{Cos[ϕ], Sin[ϕ]}, {-Sin[ϕ], Cos[ϕ]}}, {ψ, π, 2 π, π/30}]]}}]]
```

■ List of Poles

```
poles = {{1, 1, 1}, {1, -1, 1}, {-1, 1, 1}, {-1, -1, -1}};
Show[Graphics[{Circle[{0, 0}, 1], Arrow[{0, 0}, {1.2, 0}], Arrow[{0, 0}, {0, 1.2}], Text["a", {1.2, 0}, {-1, 0}],
    Text["b", {0, 1.2}, {0, -1}], Text["c", {0, 0}, {1, 1}], Map[stereographicProjection[#] &, poles]}],
  AspectRatio → 1, PlotRange → All, TextStyle → {Font → "Times", FontSize → 18}];
```

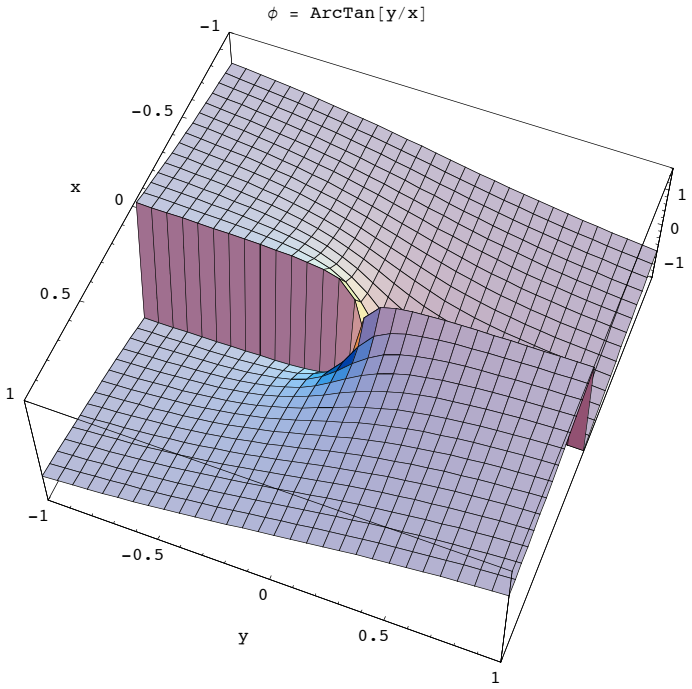


■ Spherical to Cartesian Transformation and Branch Cuts

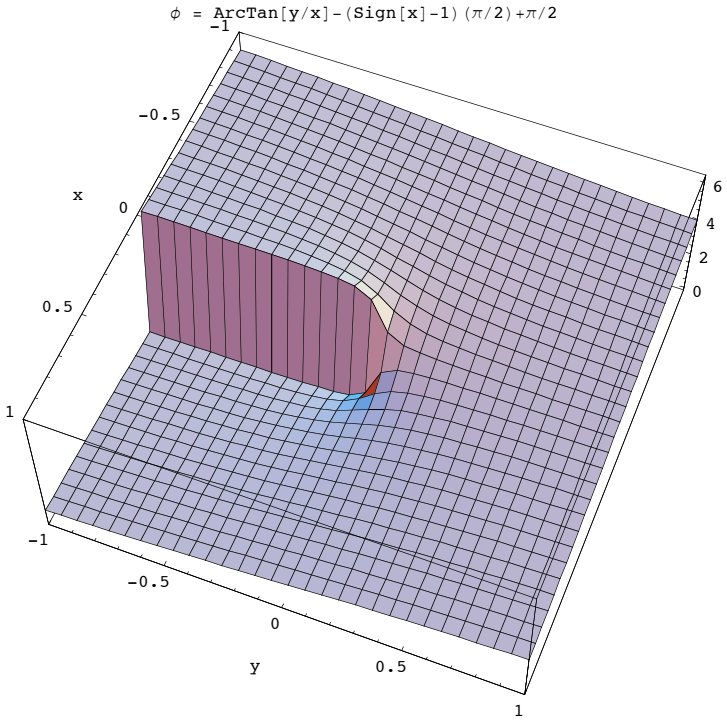
$r = \sqrt{x^2 + y^2 + z^2}$
$\theta = \cos^{-1}(\frac{z}{r}) = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$
$\phi = \tan^{-1}(\frac{y}{x})$

■ $x - y$ Plane ($z = 0$)

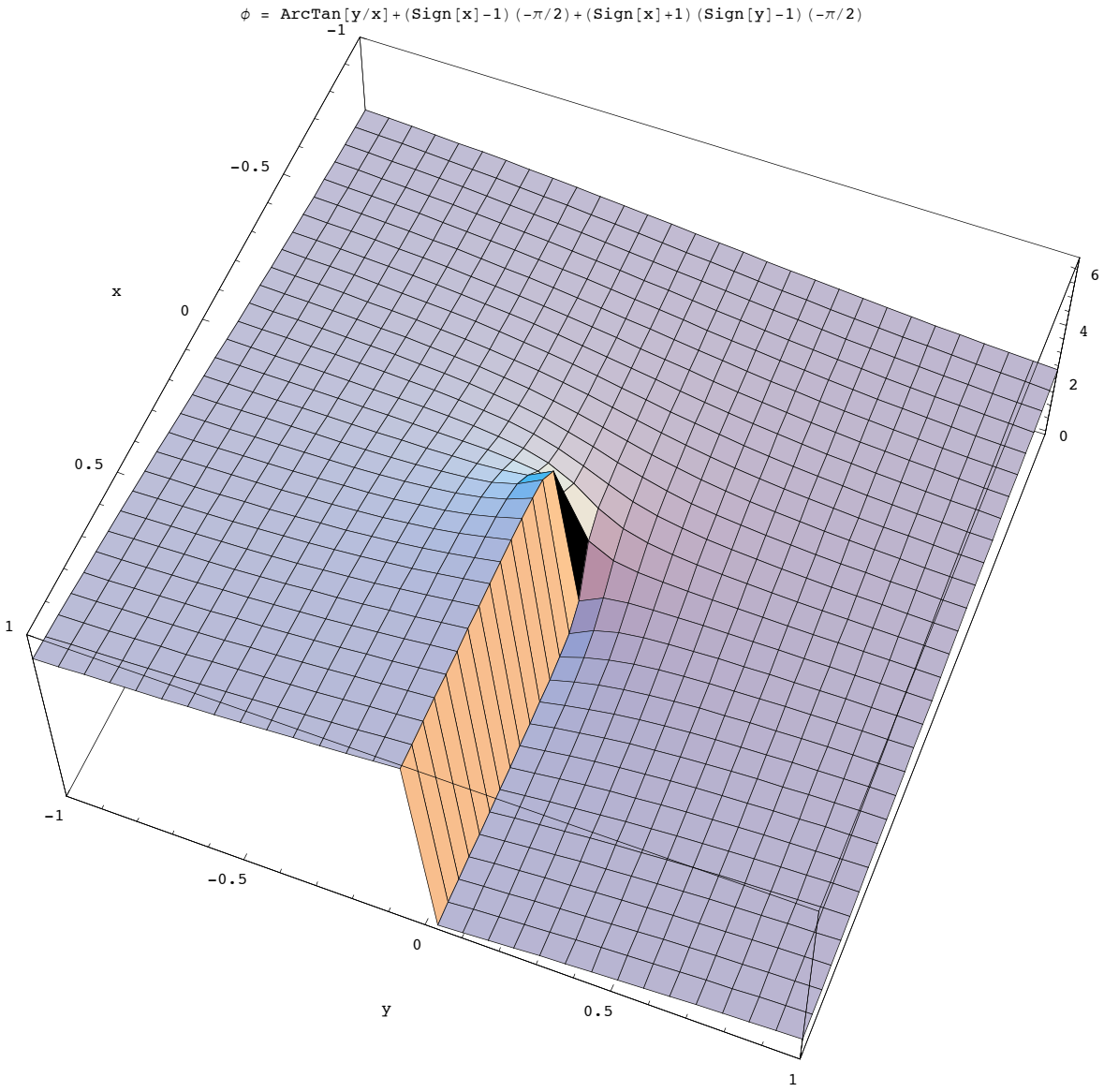
```
Plot3D[ArcTan[y/x], {x, -1, 1}, {y, -1, 1}, PlotPoints → 30,
  AxesLabel → {"x", "y", None}, PlotLabel → "ϕ = ArcTan[y/x]", ViewPoint → {2.342, 0.925, 3.885}];
```




```
Plot3D[ArcTan[y/x] - 1/2 (Sign[x] - 1) π + π/2, {x, -1, 1}, {y, -1, 1}, PlotPoints → 30, AxesLabel → {"x", "y", None},
  PlotLabel → "ϕ = ArcTan[y/x]-(Sign[x]-1)(π/2)+π/2", ViewPoint → {2.342, 0.925, 3.885}];
```



```
Plot3D[ArcTan[y/x] + 1/2 (Sign[x] - 1) (-π) + 1/2 (Sign[x] + 1) (Sign[y] - 1) (-π), {x, -1, 1}, {y, -1, 1}, PlotPoints → 30,
  AxesLabel → {"x", "y", None}, PlotLabel → "ϕ = ArcTan[y/x]+(Sign[x]-1)(-π/2)+(Sign[x]+1)(Sign[y]-1)(-π/2)",
  ViewPoint → {2.342, 0.925, 3.885}];
```



So the spherical to cartesian conversion for ϕ with the desired branch cut is: $\phi = \tan^{-1}(\frac{y}{x}) - \frac{\pi}{2}(\text{Sign}(x) - 1) - \frac{\pi}{2}(\text{Sign}(x) + 1)(\text{Sign}(y) - 1)$
Similarly, for the x-z plane ($y = 0$), $\phi = \frac{\pi}{2}(1 - \text{Sign}(x))$ and for the y-z plane ($x = 0$), $\phi = \frac{\pi}{2}(2 - \text{Sign}(y))$.

■ $x - z$ Plane ($y = 0$)

```
Plot3D[ArcTan[y/x] /. y → 0, {x, -1, 1}, {z, -1, 1}, PlotPoints → 30,
  AxesLabel → {"x", "z", None}, PlotLabel → "ϕ = ArcTan[y/x]", ViewPoint → {2.342, 0.925, 3.885}];

Plot3D[π/2 (1 - Sign[x]) /. y → 0, {x, -1, 1}, {z, -1, 1}, PlotPoints → 30, AxesLabel → {"x", "z", None},
  PlotLabel → "ϕ = π/2(1-Sign[x])", ViewPoint → {2.342, 0.925, 3.885}, PlotRange → {0, π}];
```

$y - z$ Plane ($x = 0$)

```
Plot3D[ArcTan[y/x] /. x → 0, {y, -1, 1}, {z, -1, 1}, PlotPoints → 30,
  AxesLabel → {"y", "z", None}, PlotLabel → "ϕ = ArcTan[y/x]", ViewPoint → {2.342, 0.925, 3.885}];

Plot3D[π/2 (2 - Sign[y]) /. x → 0, {y, -1, 1}, {z, -1, 1}, PlotPoints → 30, AxesLabel → {"y", "z", None},
  PlotLabel → "ϕ = π/2(2-Sign[y])", ViewPoint → {2.342, 0.925, 3.885}, PlotRange → {π/2, 3π/2}];
```