

Here is a more complete (and more correct?) analysis of the relationship between mean speed \bar{v}_1 vs. mean relative speed \bar{v}_{12} (or \bar{c} vs. \bar{c}_{rel}): $\boxed{\bar{c}_{rel} = \sqrt{2}\bar{c}}$

* Atkins-8: "When the molecules are moving in the same direction, $\bar{c}_{rel} = 0$; it is $2\bar{c}$ when they are approaching each other ... A typical mean direction of approach is... the most characteristic, so the mean speed of approach is $\sqrt{2}\bar{c}$."

90°

(Pythagoras, 45°)

* An alternative, and more detailed, explanation is available in Silbey et al., PChem-4 (2005):

- average collision is at 90° (*)

$$\langle v_{12} \rangle = \bar{c}_{rel} = \left(\frac{8k_B T}{\pi \mu} \right)^{0.5}$$

$$\mu \equiv \frac{1}{\frac{1}{m_1} + \frac{1}{m_2}} \quad (\text{equal by def.})$$

$$\Rightarrow \langle v_{12} \rangle^2 = \frac{8k_B T}{\pi} \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = \langle v_1 \rangle^2 + \langle v_2 \rangle^2 \quad (\langle v \rangle \equiv \bar{c})$$

∴ For collision of identical particles

$$\langle v_{11} \rangle^2 = 2 \langle v_1 \rangle^2$$

$$\Rightarrow \langle v_{11} \rangle = \sqrt{2} \langle v \rangle$$

$$\text{i.e. } \bar{c}_{rel} = \sqrt{2} \bar{c} \quad (\text{qed})$$

Example: $\bar{c}_{rel}(\text{H}_2/\text{O}_2) = 1824 \frac{\text{m}}{\text{s}}$ at 298K (Check!) (See Exercise 1.)

