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Air at 70°F, and atmospheric pressure was passed through beds of anthracite - the samples ranging in size from chestnut through buckwheat No. 5 and representing the four major anthracite fields.

Data on pressure drop versus gas flow rate show that for the smallest size of anthracite, buckwheat No. 5, completely viscous flow exists up to the lifting velocity. For the other sizes of anthracite a considerable portion of the flow is in the transition region intermediate between fully developed viscous and turbulent flow.

The effect of changes in void volume in different sizes of anthracite on pressure drop can be satisfactorily explained by the relationship proposed by Leva.

The effect of variation in the particle to column diameter ratio on pressure drop is extensively examined. It is found that the pressure drop is not only affected by the above ratio but also by the absolute magnitude of the two diameters.

The lifting pressure drop is found to be essentially independent of particle size and dependent on the bulk density of the bed,

## INTRODUCTION

With the use of anthracite in fixed bed operations such as gasifiers, blast furnaces, and cupolas on the increase, it was thought desirable to examine the flow of air through anthracite beds in some detail. This work is meant to augment the general correlations of flow through packed beds as advanced first by Chilton and Colburn (5) and later by Ergun (8) by looking in greater detail at some of the finer points of this unit operation.

Chilton and Colburn showed that the gas flow data through broken solids of Blake (2), Burke and Plummer (3), Furnas (10), as well as their own, could be correlated by dimensional analysis. By plotting a modified friction factor, f, equals

versus a modified Reynolds number

the data roughly could be represented by two straight lines intersecting at a modified Reynolds number of 40. Viscous flow occurred below a value of 40 and turbulent flow above a value of about 80.

Later Ergun tied in the results of Chilton and Colburn with those of Kozeny (11) and Leva (12) to show that the data of gas flow through broken solids could be better correlated if the fractional void volume in the bed was taken into account. He plotted a modified friction factor, f'.

$$\left(\begin{array}{cc} g & d & \Delta & P \\ \hline L & \varrho & U^2 & W \end{array}\right) \qquad \left(\begin{array}{c} \epsilon^3 \\ \hline 1 - \epsilon \end{array}\right)$$

versus a modified Reynolds number

$$\left(\frac{\mathrm{d}PU}{\mu}\right) \quad \left(\frac{1}{1-\epsilon}\right)$$

The following comprehensive equation was found by Ergun to be applicable to all types of flow:

$$\frac{\Delta P}{L}g = 150 \frac{(1-\epsilon)^2}{\epsilon^3} \frac{\mu U}{d^2} + 1.75 \frac{1-\epsilon}{\epsilon^3} \frac{GU}{d} (1)$$

## EXPERIMENTAL

Samples Studied - Samples of several prepared sizes ranging from chestnut through buckwheat No. 5 were obtained from each of five anthracite collieries. These collieries represent the four major fields, one each being in the northern (N-1), western middle (WM-1) and southern (S-1) fields and two from the eastern middle fields (EM-I and EM-2).

The square mesh screen analyses of the samples as received from the collieries are shown in Table I. Such physical constants as apparent specific gravity, average square mesh screen size, equivalent spherical diameter, and shape factor

This paper is a condensed version of the M.S. thesis in Fuel Technology of Kenneth M. Barclay. The thesis is deposited in the University library for anyone wishing more details on this work.

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are given in Table II. The constants were determined in the following manner. The apparent specific gravity was measured by water displacement. The average square mesh size was determined by making a screen analysis on square mesh screens and plotting the results on arithmetic probability paper using a method described by Austin (1). The equivalent spherical diameter of each sample was determined by the method of Furnas (10). This method consists of counting and weighing 500 pieces of the sample picked at random. From their weight and specific gravity, the equivalent spherical diameter may be calculated. The method, however, is not applicable to very fine materials so that the values for the buckwheat No. 5 samples had to be calculated as noted in Table II. The shape factor, as defined by Furnas, is the ratio of the equivalent spherical diameter to the average square mesh size. The greater the deviation of the shape factor from one the more particle irregularity is indicated.

TABLE 1. — Square Mesh Screen Analyses of Anthracites Tested.

Prepared	Screen	Colliery				
Size	Size	N-L	WM-1	EM-1	EM-2	S-1
Chestnut	Plus 1¼ in.	26.5	24.9	24.3	18.0	30.6
	1 x 1%	26.5	24.0	31.9	37.9	38.6
	X х 1	31.4	32.5	30.8	34.1	21.8
	Minus 🛚	15.6	18.5	13.0	10.0	9.1
Pea	Plus %	0.0	0.0	0.6	0.0	0.0
	14 x 14	67.5	71.6	56.1	74.7	66.8
	14 x 14	31.7	26.1	42.8	25.1	32.7
	Minus ¼	0,8	2.3	0.5	0.2	0.4
Buckwheat	Plus 0,371 in.	31.0	36.1	24.4	43.9	26.4
No. 1	0.371 x % in.	44.6	44.6	58.4	46.I	52.5
	¼ in. x 4 mesh	21.6	14.9	15.8	8.6	19.3
	4 x 6 mesh	2.0	2.6	0.9	0.5	1.0
	Minus 6 mesh	0.8	1.8	0.4	1.0	0.8
Buckwheat	Plus ¼ in.	3.8	6.0	4.2	4.2	1.8
No. 2	X x 4 mesh	46,0	41.7	56.2	67.3	44.5
	4 x 6 mesh	39.3	34.7	32.4	24.6	45.0
	6 x 8 mesh	8.5	10.4	5.5	2.8	7.4
	8 x 10 mesh	1.2	2.7	0.8	0.5	0.6
	Minus 10 mesh	1.3	4.5	0.8	0.6	0.6
Buckwheat	Plus 4 mesh	1.2	1.1	1.1	2.8	1.0
No. 3	4 x 6 mesh	23.3	17.5	18.3	29.2	10.8
	6 x 8 mesh	32.8	31.4	36.9	37.0	41.8
	8 x 10 mesh	17.4	16.8	17.8	16.0	24.1
	10 x 14 mesh	14.2	16.1	13.9	8.5	15,8
	Minus 14 mesh	11.1	17.1	12.0	6.5	6.5
Buckwheat	Plus 6 mesh	0.0	0.4	8.2	0.3	0.1
No. 4	6 x 8 mesh	0.1	1.2	17.0	9.2	3.3
	8 x 10 mesh	0.4	3.7	11.1	17.9	6.8
	10 x 14 mesh	10.1	25.2	27.0	34.5	37.0
	14 x 20 mesh	78.4	58.4	30.3	31.0	43.8
	Minus 20 mesh	11.0	11.2	6.3	7.1	8.9
Buckwheat	Plus 10 mesh	0.1	0.3	0.4	0.1	0,8
No. 5	10 x 20 mesh	12.5	24.3	19.0	33.1	33.2
	20 x 30 mesh	43.6	39.1	21.6	35.8	33.7
	30 x 40 mesh	25.7	18.3	9.3	14.9	18.8
	40 x 60 mesh	15.0	10.7	9.4	11.3	10,9
	Minus 60 mesh	4.1	7.3	9.5	4.9	2.6

#### **PROCEDURE**

In experimentally studying the flow of air through anthracite, the usual method for such investigations was used. This method is the determination of pressure drop versus flow rate data. A flow diagram of the test system used in the

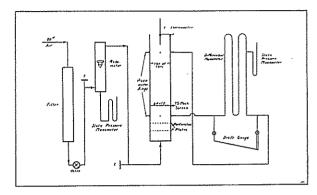


Fig. 1. -- Flow Sheet of AP Apparatus.

general air flow study of 6-inch diameter beds is shown in Figure 1. For the study of wall effect the system was altered so that the air could pass through any one of four rotameters with any one of thirteen tubes in the system or a rotary displacement meter. The range of air rates that could be measured was 0.001 to 200 cubic feet per minute. The diameters of tubes available were 4, 4, 4, 1, 2, 3, 4, 5, 6, 9, 12, 18 and 24 inches. Such devices as perforated plates, packs of small tubes, and baffles, were used in the large tubes to secure good air distribution before the air entered the test bed.

Generally, the tubes were packed according to the method recommended by Furnas for securing normal packing. This method consists of slowly pouring the material from a height of approximately one foot above the bed. The resulting bed has a percentage of voids that is reproducible. The only instances in which packing the beds in this manner gave inconsistent results were in the wall effect study of the buckwheat No. 4 and No. 5 sizes. The variations in voids with change in diameter of tube in those cases may have been due to the fact that the two smallest sizes of anthracite have a wider size consist than most of the larger sizes. Because of the great effect of change in the percentage voids on pressure drop, it was necessary to control the percentage of voids when the No. 4 and No. 5 sizes were being tested for wall effect. An examination of the data on amount of voids in the wall effect study of the large sizes with normal packing showed very little change of percentage voids with change in

tube diameter. Thus it appeared best to hold the percentage voids constant for each sample of buckwheat No. 4 or No. 5 studied. This was done in the following manner. Knowing the volume of a foot high section of each tube and the specific gravity of the sample, the weight of sample necessary to give an arbitrary amount of voids at a height of one foot was calculated. The sample was then poured into the

tube at a fast rate. This method of pouring gave a high percentage of voids and the resulting bed was more than one foot in height. The tube was then tapped and the bed densed until the height was just one foot, producing the desired percentage of voids.

After each test bed was packed, air was passed through the system at varying rates and the cor-

TABLE II. - Physical Constants of the Anthracites

COLLIERY DESCRIPTION	Size	Apparent Specific Gravity, g./cc.	Equivalent Spherical Diameter, inches	Average Square Mesh Size, inches	Shape Factor	Fractional Normal Voids on Packing
N-1 WM-1 EM-1 EM-2 S-1	Chestnut	1.49 1.54 1.64 1.50 1.62	0.96 0.92 0.93 0.99 1.05	1.05 1.01 1.03 1.04 1.13	0.91 0.91 0.89 0.95 0.93	0.479 0.518 0.489 0.452 0.476
N-1 WM-1 EM-1 EM-2 S-1	Pea	1.52 1.47 1.62 1.56 1.61	0.57 0.53 0.55 0.61 0.57	0.56 0.59 0.52 0.58 0.55	1.02 0.90 1.06 1.05 1.04	0.458 0.474 0.460 0.457 0.442
N-1 WM-1 EM-1 EM-2 S-1	Buckwheat No. 1	1.59 1.47 1.64 1.57 1.64	0.30 0.32 0.33 0.37 0.33	0.33 0.34 0.32 0.36 0.33	0.91 0.94 1.03 1.03 1.00	0.499 0.482 0.468 0.459 0.452
N-1 WM-1 EM-1 EM-2 S-1	Buckwheat No. 2	1.51 1.42 1.64 1.60 1.66	0.17 0.17 0.19 0.21 0.17	0.19 0.18 0.19 0.21 0.18	0.89 0.94 1.00 1.00 0.94	0.491 0.472 0.471 0.477 0.465
N-1 WM-1 EM-1 EM-2 S-1	Buckwheat No. 3	1.51 1.47 1.67 1.56 1.67	0.099 0.074 0.081 0.108 0.094	0.106 0.093 0.101 0.115 0.101	0.93 0.80 0.80 0.92 0.93	0.492 0.484 0.487 0.477 0.485
N-1 WM-1 EM-1 EM-2 S-1	Buckwheat No. 4	1.59 1.50 1.67 1.53 1.66	0.048 0.049 0.058 0.067 0.048	0.043 0.048 0.070 0.064 0.054	1.07 1.02 0.83 1.05 0.93	0.555 0.508 0.492 0.476 0.555
N-1 WM-1 EM-1 EM-2 S-1	Buckwheat No. 5	1.60 1.55 1.69 1.66 1.75	0.023° 0.025° 0.032° 0.029°	0.024 0.027 0.034 0.029 0.030	0.96°° 0.92°° 0.94°° 1.00°° 0.98°°	0.556 0.512 0.489 0.532 0.505

Calculated from shape factor and average square mesh size.

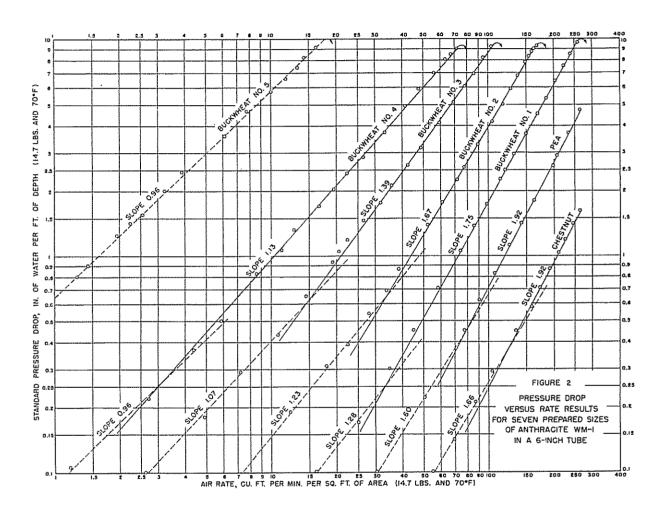
as Average of the shape factors of the other six prepared sizes from the same colliery.

responding pressure drops across the bed were determined. In the majority of the runs the pressure drop readings were made on an inclined draft gauge with graduations every 0.01 inch for differentials up to 3 inches and on a water filled manometer with graduations every 0.1 inch for readings greater than three inches. A few measurements were also made on a differential recording gauge which read to 0.001 inch with a range of 0 to 0.15 inch.

ried to the lifting velocity, which is indicated on the plot by an arrow pointing downward representing the decrease in pressure drop at that point.

Similar to the experience of previous investigators the present data of pressure drop versus air flow rate yield straight lines on a log-log plot giving an equation of the form

$$\Delta P = AU^{B}$$
.



## RESULTS AND DISCUSSION

Pressure Drop Versus Air Flow Rate — Figure 2 shows a typical log-log plot of pressure drop versus air flow rate, for the seven prepared sizes of WM-1. Normal packing in the six-inch tube was used, and corrections were applied for pressure drop through the supporting screen when necessary. No correction for possible wall effect has been made. Usually for the samples of buckwheat No. 1 and smaller the tests have been car-

Over the flow rate range investigated, all the data except that of buckwheat No. 5 show a relatively sharp change in slope at some point which necessitates representing each set of data by two equations. Undoubtedly this change in slope is caused by a change in the nature of the flow occurring.

Theoretically, the pressure drop through a granular bed is proportional to the flow rate when in viscous flow and to the square of the flow rate when in turbulent flow. From Figure 2 it is seen that the flow through buckwheat No. 5 is ideally

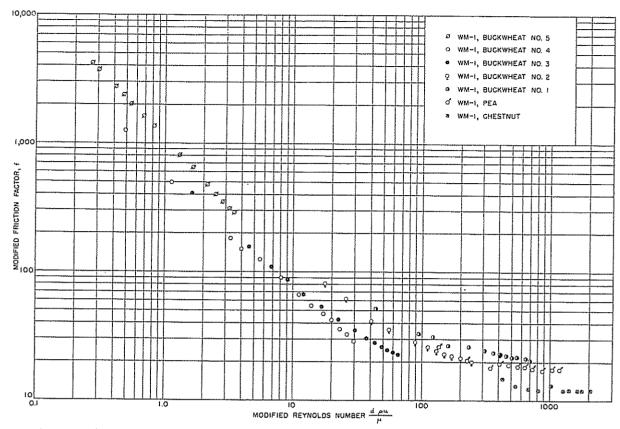


Fig. 3. - Modified Reynolds Numbers vs. Modified Friction Factors as Calculated from Data of Figure 2.

viscous over the entire flow-rate range. Furthermore, for this small size of anthracite only viscous flow can be realized under the present circumstances since the lifting velocity of the bed is reached prior to the development of some turbulent-flow character. At the other extreme, it is seen that a slope of two is closely reached in the larger sizes at higher flow rates indicating nearly ideal turbulent flow.

Of particular interest are the data for the intermediate sizes of anthracite which show prolonged regions of linearity with slopes intermediate between one and two. It would be expected that if the flow rates were lowered sufficiently and if the small pressure drops could be determined that the slopes would tend to the limiting value of one. However, it is seen that these flow rates would be too low for commercial interest, so for all intents the slope of the pressure drop versus flow rate plot increases with increasing size of anthracite. It is furthermore seen that the slope at the point of lifting of the bed also increases with increasing size of anthracite. Essentially similar data on these points were obtained for the other anthracites investigated.

Pressure drop variation with intermediate powers of flow rate between one and two has also

been observed by White (14) in studying the flow of gas through spirals of copper pipe. He found that in the upper portion of the viscous region the pressure drop varied roughly as the 1.8 power of gas velocity for a 15 to 1 ratio spiral. In order to check this result further, the authors bent a 10-foot length of copper tubing, 5/32 inch in diameter, so that one turn was made for every fifteen tube diameters in length. Pressure drop on the curved tube varied as the 1.33 power of gas velocity even though the calculated Reynolds numbers were well in the viscous region. A theoretical explanation of this behavior for viscous flow through curved pipes has been given by Dean (6). He concluded that this higher power in the supposed viscous region is caused by eddy current flow perpendicular to the main direction of flow. This is undoubtedly partly accountable for the intermediate values of slope in the present work.

In order to examine the present data in the light of Chilton and Colburn's results, modified friction factors and Reynolds numbers have been calculated from the data in Figure 2 and are shown in Figure 3. In a manner similar to Chilton and Colburn's original correlation, the data can be represented by two straight lines inter-

secting at a Reynolds number of 40. It is seen, however, that the Reynolds number must be well away from this value to give true viscous or turbulent flow. The data for the pea and chestnut anthracites supposedly lie well in the turbulent region (that is, at a modified Reynolds number greater that 80), but as seen in Figure 2, the slopes are less than 1.7 over the lower range of flow rates. Furthermore, even though the data for the buckwheat No. 4 anthracite is supposedly all in the viscous region the slope over most of the plot in Figure 2 is 1.13.

Film Concept of Flow Through Packed Beds Qualitatively the value of B can be related to the existence of a film around each particle in which the flow is viscous. It is pictured that the thickness of this film, for a given gas, varies with gas velocity and viscosity, but not with particle size. Hence, the volume of viscous flow film in the voids of the bed for a given gas velocity and viscosity will vary roughly as the amount of surface area per unit-volume of bed. For large particles the proportional amount of void space that

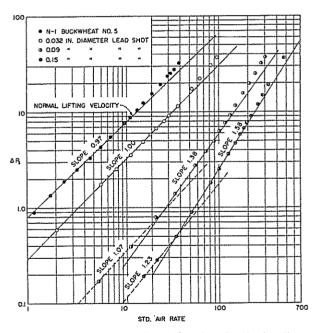


Fig. 4. — Pressure Drop Versus Flow Rate for Varying Sizes of Lead Shot and N-I Buckwheat No. 5.

is occupied by films is small, and the majority of the flow is turbulent giving a value of B approaching two. As the size of the particle is decreased, the proportion of void space occupied by the film will increase with a consequent decrease in B. Finally one would expect to reach a limiting size below which no turbulent flow

would occur prior to the lifting velocity. Such was the case in the present work for buckwheat No. 5.

TABLE III. — Results of Furnas for the Effect of Change of Temperature on the Constant B for Air Flow Through a Bed of Iron Ore.

Average	
Temperature,	
°C.	В
36	1.67
216	1.56
330	1.48
<del>44</del> 5	1.48
554	1.40
660	1.35

To find what gas velocity is necessary through buckwheat No. 5 to have the flow deviate from completely viscous, a 70 mesh wire was clamped on top of the bed to hold it in place when the normal lifting velocity was exceeded. A test was then made in which the maximum velocity reached approximately three times the normal lifting velocity. The results are shown in Figure 4. In correcting the data the pressure drop through the screen has been considered. It is seen that B becomes greater than one at a point just above the normal lifting velocity and increases rapidly until a slope of about 1.3 is reached at the highest velocity used.

In order to investigate further the thinning of films with increase in velocity, tests were made on three sizes of lead shot. The 0.032 inch lead shot was small enough so that the void space was completely occupied by films at normal velocities and B equals one. In a similar manner to buckwheat No. 5, a velocity was reached where B became greater than one. The other two sizes of shot were large enough so that the void space was not completely occupied by films even at the low velocity. Furthermore, for these two samples three changes in slope are observed over the range of flow velocity used.

The film concept holds that the film thickness should vary as some direct relation with the gas viscosity. The effect of viscosity can be studied by varying the gas temperature. Furnas (10) studied the pressure drop when air was passed through Mesabi iron ore at temperatures ranging from 26°C. to 660°C. The results presented in Table III show that B decreases with increasing temperature. Since the viscosity increases with temperature, the film thickness also increases with temperature. Thus there is a smaller proportion of the void space in which turbulent flow can occur and B is decreased.

Effect of Change in Void Volume on Pressure Drop — Kozeny (11) presented the emperical relationship that for viscous flow the pressure drop varies as

$$\frac{(1-\epsilon)^2}{\epsilon^3}$$

Leva (12) showed that the pressure drop for turbulent flow varies as

$$\frac{1-\varepsilon}{\varepsilon^3}$$

but stated that the pressure drop actually varies as

$$\frac{(1-\epsilon)^{3-n}}{\epsilon^a}$$

for any type of flow. The term n was said to vary from one for completely viscous flow to two for completely turbulent flow. Thus n is synonymous with B discussed in the present work.

In order to examine the effect of voids in beds of anthracite, tests were made on chestnut and buckwheat Nos. 1, 4 and 5. Each size was tested at the minimum and maximum amount of voids that could conveniently be obtained. The results of these tests are shown in Figure 5.

Comparisons between measured and calculated results are shown in Table IV. In general, the agreement is quite satisfactory, buckwheat No. 5 being the exception. Differences in orientation of the particles caused by variation in packing density may account for some of the deviation.

Another fact which should be noted in Figure 5 is the low value of pressure drop to which the data were carried for the buckwheat No. 1 and chestnut samples. Despite this, B is still considerably greater than one for both samples, again indicating the difficulty of obtaining true viscous flow for the larger sizes of anthracite.

Wall Effect — Due to the influence of the wall on particle arrangement in a bed of broken material, a greater amount of void space is present

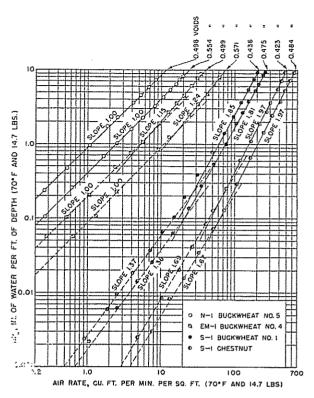


Fig. 5. — Effect of Voids on Pressure Drop Versus Air Rate for Different Sizes of Anthracite.

next to the wall than in the central portion of the bed. This high void section at the wall has a lower resistance than the center of the bed with the result that a greater proportion of the gas flowing through the bed passes through the section adjoining the wall. This uneven proportioning results in a lower pressure drop than would be the case if the amount of voids at the wall were the same as in the center of the bed. This is the phenomenon that is generally known as wall effect. The wall, however, has other effects;

TABLE IV.- Comparison Between Measured and Calculated Values of Pressure Drop as Affected by Per Cont Voids

Аптивасіте	Type or Frow	Air Rate, efm.	Minimum Voids	MEASURED P AT MIN. Voids	Maximum Voids		Calculated P at Max, Voids	Percentage Deviation from Measured
S-1, Chestnut	Viscous <sup>1</sup>	30	0,423	0.032	0.484	0.051	0.053	+ 4%
	Turbulent <sup>2</sup>	300	0.423	2,20	0.484	3.70	3.45	43
S-1, Buckwheat	Viscous	10	0.436	0.039	0.475	0.061	0.058	- 5%
No. 1	Turbulent	200	0.436	6.00	0.475	8.80	8.50	- 4%
N-1, Buckwheat	Viscous	11	0.499	0.093	0.571	0.020	0.019	- 5%
No. 4	Turbulent	30	0.499	3.45	0.571	7.40	7.30	- 13
N-1, Buckwheat No. 5	Viscous	10	0.498	4.20	0.554	8.40	7.40	-12%

Modified Reynolds number less than 40.
Modified Reynolds number greater than 80.

namely it also affects the packing and orientation of particles in the central portion of the bed and it causes pressure loss due to friction between it and the gas stream. All three effects should be grouped together in defining wall effect. Since the individual effects tend to vary with change in the container diameter to particle diameter ratio, the net effect should also vary with that ratio.

One main reason for lack of agreement between the data and equations of different investigators has been the confusion concerning wall effect. While there have been many conjectures about the significance of wall effect there have been few actual experimental data gathered on the subiect. Also, wall effect has been considered from two different viewpoints; one, the effect of change of the container diameter to particle diameter ratio with normal packing in each tube, letting the voids change as they will, and the other, the effect of changing the ratio of diameters with a constant amount of average voids in the various tubes studied. Since the amount of average voids itself has a great effect on the pressure drop the two viewpoints can give entirely different results.

Furnas (10), on the basis of data on air flow through two sizes of iron ore in three different diameter tubes and on one size of lead shot in four different diameter tubes with a constant amount of voids for each particle size studied, worked out an evaluation of wall effect. Even though his results were based mainly upon theory with but few experimental data, they have been generally used since that time in considerations of wall effect. When Furnas' wall effect evaluations are plotted on logarithmic paper, straight line relationships are obtained between the wall effect factor W (the fractional part that the pressure drop is of what it would be were there no wall effect) and the ratio of the container diameter to the particle diameter, D/d. The results show wall effect to be negligible when the D/d ratio becomes greater than roughly 50:1.

Other workers including Carman (4), Diepschlag (7), Fehling (9), and Rose (13) also discussed wall effects.

Because of the widely conflicting opinions concerning the significance of wall effect, experimental data have been gathered on the subject in beds of anthracite. In studying wall effect, thirteen tubes were used varying in diameter from ½ to 24 inches. The diameter of the tubes available necessarily limited the wall effect study to the five buckwheat sizes since high enough values of D/d to define the wall effect could not be reached with the pea and chestnut sizes.

The buckwheat No. 1, No. 2 and No. 3 sizes were studied using normal packing. For the buckwheat No. 4 and No. 5 sizes, however, seemingly due to the wide size consist there was no consistency in the amount of voids obtained by normal packing. Hence, it was necessary to pack the beds by the methods described previously so that a constant amount of voids was obtained for the various tubes utilized. The fractional amount of voids arbitrarily used was 0.489 for buckwheat No. 4 and 0.496 for buckwheat No. 5, those being approximately the lowest amount of voids that could be obtained with close packing. The amount of voids varied somewhat with change in the size of tube for the other sizes studied with normal packing. The over-all average of the voids was 0.443 for buckwheat No. 1, 0.448 for buckwheat No. 2 and 0.450 for buckwheat No. 3.

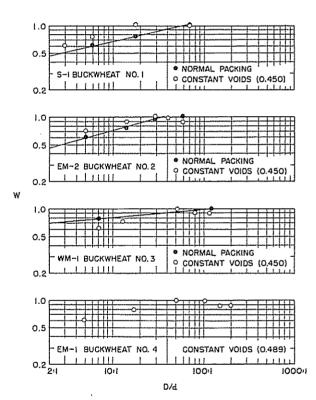


Fig. 6. - Wall Effect Data, Turbulent Region.

The results of the study of wall effect in beds of anthracite are shown in Figures 6 and 7 in terms of logarithmic plots of W versus D/d. Here the wall effect factor W is defined as the fractional part the pressure drop is of the maximum obtained for a given gas velocity through a given size of coal.

In expressing the data in Figures 6 and 7 it has been assumed that B does not change for a

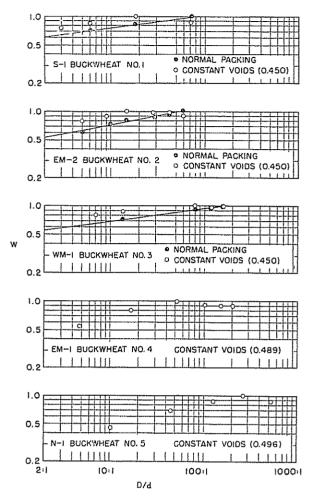


Fig. 7. - Wall Effect Data, Viscous Region.

given sample with change in the size of the tube, or in other words, with change in wall effect. Although there was some variation of B with change in tube size it was not great and usually there was no particular trend. The one notable exception was in the case of the buckwheat No. 5 study where B increased slightly with decrease in the size of the tube. Assuming B to be constant for a given sample with change in the size of the tubes, allows the simple representation of the data as shown in Figures 6 and 7. The assumption is further supported by the fact that Furnas' data on wall effect show B to be a constant with change in D/d.

W was obtained by plotting the data for the various tubes used on logarithmic paper and picking off the pressure drop for each tube at some arbitrary rate, the rate being chosen so that it was either well in the viscous or turbulent region.

The relative values were then calculated using maximum pressure drop as unity. Throughout, d is the equivalent spherical diameter of the sample. Using the average square mesh screen size, however, should give nearly as good an evaluation, since it varies little from the equivalent spherical diameter.

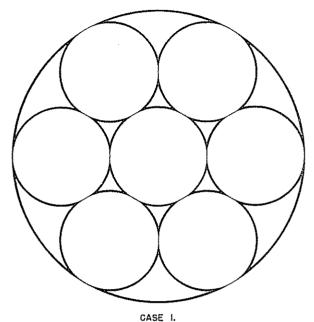
The data on the three largest buckwheat sizes have been shown in Figures 6 and 7 in two ways, as experimentally determined for normal voids and calculated to a constant value of 0.450 voids, using the relationships for effect of voids previously given. The data on buckwheat No. 4 and No. 5 have been shown as experimentally determined at a constant amount of voids.

It will be noted that there is little difference between the results in the viscous and turbulent regions. There is, however, a great difference between the results for normal voids and for constant voids. The data for normal voids show the relationship usually attributed to wall effect in that W increases with increase in D/d until it becomes unity in the range of D/d ratios of between about 50:1 and 100:1. Although not enough data were obtained at higher D/d ratios and normal voids to unmistakably define the further trend of W with D/d, it apparently remains constant at unity for further increase in D/d. The data for constant voids, on the other hand, show an increase in W with D/d until the critical D/d (arbitrarily defined here as the lowest D/d at which W is one) is reached. With further increase in D/d above the critical value, W then begins to decrease. Also, it has been found that the wall effect and the critical value of D/d increase markedly with decrease in particle size.

It is felt that the increase in wall effect and the critical D/d with decrease in particle size may be satisfactorily explained on the basis of the film concept previously introduced. It is pictured that the flow through a film is inhibited while the flow through space unoccupied by film is uninhibited. Relatively speaking, there will be less pressure drop through the space unoccupied by film and thus a greater proportion of the gas will preferentially flow through that space. The flow inhibiting power of the film seems to be present whether the flow is nominally viscous or turbulent.

Cross sections of the bed for two cases of flow through spheres packed in a cylinder have been shown in Figure 8. The film has been represented by a wide dark line on all the surfaces exposed in the bed. The dimension d is small enough so that for spheres of that diameter the films occupy all of the void space in the central portion of the bed. It can be seen that in Case 1

The criterion of viscous and turbulent region was a modified Reynolds number below 40 and above 80, respectively, as suggested by Chilton and Colburn.



DIAMETER OF SPHERE = 6 d



CASE 2

DIAMETER OF SPHERE = d + OF CONTAINER = 3 d

Fig. 8. — Use of Film Concept to Qualitatively Explain the Importance of the Magnitude of Particle and Container Diameters on Wall Effect.

the spheres are relatively large and that there is a large amount of space for uninhibited flow in the central portion of the bed as well as at the wall. In Case 2, however, the spheres are small while the film remains the same thickness as in Case 1. The flow through the central portion of the bed is then entirely inhibited while part of the flow at the wall is uninhibited. It thus seems apparent that in Case 2, at low D/d ratios, the wall effect will be greater than in Case 1. It also seems reasonable that a higher D/d ratio must be reached in Case 2 than in Case 1 before the maximum pressure drop is attained.

No tenable explanation has been found for the decrease in W at high D/d ratios for the data at constant voids. The trend is present for both the calculated and the experimentally determined

data. At first it was thought that the trend might be a false one due to experimental errors. Extensive checking, however, with special emphasis being placed on experimenting with the distribution of the air before it entered the bed, uncovered no such errors.

The results obtained in the study of wall effect at constant voids make it clear why there has been disagreement between investigators concerning wall effect. If wall effect at constant voids is studied only at high D/d ratios, the data will show an increase in pressure drop for decrease in D/d, while if it is studied only at low D/d ratios, it will show a decrease in pressure drop for decrease in D/d. Some investigators have held that the former occurs, others that the latter occurs.

Furnas has shown that wall effect varies with the amount of voids in the bed and that such a variation should occur is logical on the basis of the film concept. In the present study of wall effect in anthracite beds no attempt has been made to study the effect of change of voids on wall effect, since the spread between minimum and maximum voids in anthracite is not great enough for the true trends to be isolated. For that reason the anthracite results are probably not ap-

TABLE V. - Lifting Velocity Data

		J.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,		
	LIFTING	Pressure		
	Velocity,	Drop,	Bulk	
	cu.ft./sq.ft./	lbs./sq.ft./	DENSITY,	
Sample	min.	ft. bed depth	lbs./cu.ft.	
Buckwheat No.	1			
N-1	260	51.0	49.8	
WM-1	270	52.0	47.6	
S-1	260	57.1	56.2	
Buckwheat No.	2			
N-1	185	51.0	48.1	
WM-1	175	50.0	46.9	
EM-1	200	57.1	54.3	
EM-2	200	57.1	52.4	
S-1	180	59.9	55.6	
Buckwheat No.	3			
N-1	115	51.0	48.0	
WM-1	110	50.0	47.5	
EM-1	110	57.1	53.5	
EM-2	125	54.7	51.0	
S-1	115	54.7	53.8	
Buckwheat No.	4			
N-1	62	44.6	44.2	
WM-I	75	48.8	46,2	
EM-I	94	54.7	53.0	
EM-2	85	52.0	50.2	
S-1	74	54.7	52.6	
Buckwheat No.	5			
N-1	23	46.7	44.5	
WM-1	18	49.4	47.3	
EM-1	19	57.1	54.0	
EM-2	25	51.0	48.6	

plicable to a material having voids which do not fall in the general range of 0.40 to 0.55 without some adjustment for the difference in voids.

Lifting Velocity - When the pressure drop per unit length through the bed becomes greater than the pressure caused by the weight of the bed per unit length, the bed should commence lifting. In most equipment in which fuel is used in a fixed bed the lifting velocity may not be exceeded. The lifting velocity is thus a limiting factor in determining the maximum capacity of fixed bed equipment and data concerning it should be useful in designing such equipment.

Table V presents data on the lifting velocity (for air at 70°F. and 14.7 lbs./sq. in.), lifting pressure drop, and bulk density for anthracite sizes buckwheat No. 1 through 5. The pressure caused by the bed weight per foot length of bed is identical to the bulk density figures if the small contribution of the air weight is not considered. It is seen that, as expected, the lifting pressure drop is closely related to the bulk density of the bed and, in turn, the pressure which it creates per foot of height. The lifting pressure drop is seen to be consistently somewhat higher than the pressure created by the bed, which gives an indication of the magnitude of frictional forces which must be overcome before moving the bed.

One can then, with this information on the upper limit of allowable pressure drop in the bed and the knowledge that the fractional void volume will be close to 0.5, use equation 1 previously presented to calculate the allowable flow rate of gas through a particular size of anthracite under given conditions of gas temperature and pressure.

## NOTATION

A = coefficient in equation  $\Delta P = AU^n$ 

B = slope in equation  $\Delta P = AU$ "

d = the equivalent spherical diameter of particles

D = column diameter

g = gravitational constant  $G = mass-flow rate of gas, G = \varrho U$ 

L = bed height

 $\Delta P = pressure drop$ 

U = superficial fluid velocity based on empty column cross section

W = wall effect factor, the fractional part that the pressure drop in a bed of a given diameter is of the maximum pressure drop at the same flow rate for any bed diameter

 $\varepsilon = \text{fractional void volume in bed}$ 

 $\rho = density of gas$ 

 $\mu = absolute viscosity of fluid$ 

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