

# The Viscosity of Steam and of Nitrogen at Atmospheric Pressure and High Temperatures

By CHARLES F. BONILLA,<sup>1</sup> ROBERT D. BROOKS,<sup>2</sup> AND PHILIP L. WALKER, JR.<sup>3</sup>

## NOMENCLATURE

The following nomenclature is used in the paper.

- $A, B$  = constants in Equation [23]  
 $a, b$  = constants in van der Waals' equation  
 $b'$  = constant in Equation [19]  
 $C$  = Sutherland constant, °K (Equation [12])  
 $D$  = Capillary internal diameter, ft  
 $D_v$  = diffusion coefficients, sq ft/sec  
 $d$  = diameter of helix of capillary axis, ft  
 $E'$  = streamline flow correction factor independent of flow rate  
 $E''$  = streamline flow correction factor dependent on flow rate  
 $E$  = product of  $E'$  and  $E''$   
 $e$  = 2.71828...  
 $K$  = apparatus dimension ratio at 0° C ( $\pi D_o^4/128 L_o V_o$ )  
 $k$  = constant in Sutherland equation (Equation [12])  
 $L$  = length of capillary, ft  
 $L'$  = mean free path of molecules, ft  
 $P$  = pressure, lb/sq ft absolute  
 $P_1$  = initial reservoir pressure, lb/sq ft absolute  
 $P_2$  = instantaneous final reservoir pressure, lbs/sq ft absolute  
 $P_3$  = steady state final reservoir pressure, lb/sq ft absolute  
 $\Delta P$  = error in reading reservoir gage pressure, lb/sq ft  
 $p, q$  = constants in equation  $C = p + qT$   
 $R$  = gas law constant, ft lb/slug  $\times$  °K  
 $Re$  = Reynolds' Number of flow in capillary  
 $T$  = absolute temperature, °K = 273.1 + °C  
 $t$  = temperature, °C  
 $V_1$  = volume of manometer at  $P_1$  from initial level to manometer valve  
 $V_2$  = volume of manometer at  $P_3$  from final level to manometer valve  
 $V_1$  = hot volume of entrance tube and half of capillary, assumed at reservoir pressure and capillary temperature  
 $V_o$  = reservoir volume at 0° C, cu ft  
 $V_r$  = reservoir volume at reservoir temperature, cu ft  
 $w$  = mass flow rate, slugs/sec  
 $x, y$  = constants in Equation [8]  
 $y'$  = volume or mole fraction of water vapor in moist air  
 $Z$  = compressibility factor, dimensionless

## Greek Letters:

- $\alpha$  = mean coefficient of thermal expansion of capillary from 0° C to capillary temperature (Varies linearly for platinum from 8.83 to  $9.72 \times 10^{-6}$  as  $t_c$  varies from 0 to 1600 C)  
 $\beta$  = mean coefficient of thermal expansion of reservoir from 0° C to reservoir temperature ( $1.89 \times 10^{-5}$  for red brass)  
 $\theta$  = duration of run, seconds  
 $\Delta\theta$  = absolute error in duration of run, seconds

- $\mu$  = viscosity, slugs/ft sec  
 $\mu'$  = viscosity, micropoises  
 $\rho$  = density, slugs/cu ft

## Subscripts:

- $c$  = of the capillary  
 $m$  = of the manometer  
 $o$  = at 0° C.  
 $r$  = of the reservoir  
 $a$  = at atmospheric pressure  
 $P$  = at  $P$  atmospheres absolute

## INTRODUCTION

For calculations relating to high speed rocket flight, jet engines, and other applications, it is desirable to know the viscosity of air, nitrogen, water vapor, carbon dioxide and their mixtures, at high temperatures and in the range of atmospheric pressure. When this work was undertaken no values for air and nitrogen above 1100 deg C were available, nor for steam above 650 deg C. Shortly thereafter, however, Vasilescu's determinations (25)<sup>4</sup> for air, nitrogen, carbon dioxide, and argon up to about 1600 deg C appeared.

Although many values for steam have been reported, the agreement among them is poor, as shown in Fig. 1. Accordingly it was decided to construct an apparatus suitable for relative determinations up to high temperatures, to check the apparatus and Vasilescu's results mutually by runs with nitrogen, and then to carry out runs on steam to as high a temperature as feasible.

## APPARATUS

The method selected was that of capillary efflux from a reservoir at low temperature. This is simpler than Vasilescu's constant flow rate method, and for equal accuracy more convenient than Boelter and Sharp's falling rate method employing a reservoir at test temperature (2). The reservoir was constructed by silver-soldering 1/4-in. thick brass disks to both ends of a section of 5 1/2-in. brass pipe. By filling with water and weighing on a large precision beam balance this was found to have a volume of 172.4 cu in. It was mounted in a box and insulated with 2 in. of vermiculite. A copper-constantan thermocouple was attached at the top and another at the bottom. An open manometer 24 in. high made of 1/4-in. glass tube in an oil bath was attached to the reservoir by means of a short 3/16-in. OD copper tube. A thermocouple was attached to the tube and three others dipped into the oil bath to different levels. The manometric fluid was dibutyl phthalate. An attached steel scale indicated its level. Hole 1/8-in. high-pressure Monel needle valves with deep Teflon packing were used in the manometer line, and in the vacuum and gas line to the reservoir.

From the other end of the reservoir a similar short line went to the test capillary, located inside a Burrell high-temperature Globar furnace. The capillary was made of smooth platinum

<sup>1</sup> Columbia University, New York, N. Y.

<sup>2</sup> General Electric Co., Schenectady, N. Y.

<sup>3</sup> Pennsylvania State College, State College, Pa.

<sup>4</sup> Numbers in parentheses refer to the Bibliography at the end of the paper.

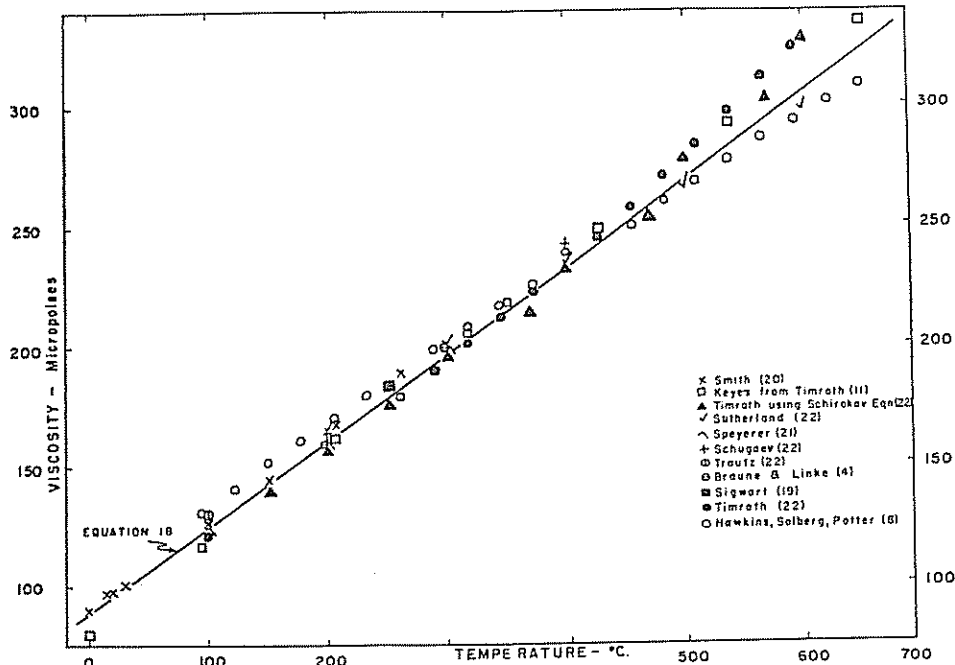


FIG. 1 DATA ON THE VISCOSITY OF ATMOSPHERIC PRESSURE STEAM IN THE LITERATURE

tubing with 0.008 in walls. The inlet and outlet tubes were 1/8 in. ID, 9 in. long, had conical reducing inner ends, and extended to about 1 in. from the center of the furnace. Between them was the capillary proper, approximately 0.030 in. ID and 30.4 in. long; tightly coiled into 7.5 turns on a 1.125 in. diameter helix, with two approach curves of similar diameter. The helix was supported on alundum rods stretching between triple alundum radiation shields at each end of the furnace. Platinum-platinum 10 per cent rhodium thermocouples were attached at the middle and at each end of the capillary, and at the outer ends of the inlet and outlet tubes. Calculations showed that the 1/8-in. ID inlet tube was long enough to heat the gas substantially to capillary temperature, and it and the outlet tube were large enough to cause negligible pressure drop. At the downstream end of the capillary a small quick-acting 1/4-turn plug cock was employed.

EXPERIMENTAL PROCEDURE

Before carrying out a run with nitrogen, the furnace was heated to temperature, the manometer valve was opened, the efflux cock closed, and the gas was passed into the reservoir from a cylinder until a pressure near the maximum of the manometer was obtained. After the manometer and all temperatures had become constant they were recorded, and the manometer valve was closed. The efflux cock was then rapidly opened and simultaneously a 1/10 second stop-watch started. After an interval of 1 to 6 min, the cock was closed and the stop-watch simultaneously stopped. The manometer valve was then opened, and within a few seconds the pressure had become constant and all observations were again recorded. The three capillary temperatures were averaged with the middle one weighted double. The average deviation amounting to about 4 deg C.

It can be shown by minimizing the error in viscosity for an absolute error ΔP in the initial and final manometer reading and an absolute error Δθ in time, that the greatest accuracy for a given initial pressure will be obtained with runs such that

$$\left( \frac{K\Delta\theta}{\mu} \cdot \frac{P_1 - P_a}{\Delta P} \right) = \left[ \left( \frac{P_1 - P_a}{P_2 - P_a} \right)^2 \left( \ln \frac{P_1 - P_a}{P_2 - P_a} \right) - 1 \right]^{1/2} \dots \dots \dots [1]$$

This corresponds to a decrease during any run to about 1/3 of the initial reservoir gage pressure. However, substantially full accuracy is obtained in spite of moderate departures from these conditions. Although the higher the initial pressure the greater is the expected accuracy, two or three runs were usually carried out on the same charge for convenience and to have runs at different flow rates. Results at high and low rates checked well, indicating that adiabatic cooling of the gas was negligible and that the kinetic corrections applied were adequate.

In the runs on steam, the reservoir, manometer, and all connecting tubes were wrapped with asbestos and resistance wire and heated to above 250 deg F to prevent condensation. The reservoir was first heated and evacuated, then steam admitted from a flask with water boiling under slight gage pressure.

THEORY OF THE METHOD

Poiseuille's law for a perfect gas in streamline flow in a tube may be written

$$\mu = - \frac{\pi D^4 P}{128 w RT_c dL} \dots \dots \dots [2]$$

On integrating from 0 to L and from reservoir pressure P<sub>r</sub> to atmospheric pressure P<sub>a</sub>, the flow equation for variable density is obtained

$$\mu = \frac{\pi D^4 (P_r^2 - P_a^2)}{256 RL w T_c} \dots \dots \dots [3]$$

The instantaneous value of w is given by

$$w = \frac{d(P_r V_r)}{d\theta} = \frac{V_r}{R} \frac{d(P_r/Z_r T_r)}{d\theta} = \frac{V_r}{R} \left( \frac{1}{Z_r T_r} + \frac{P_r d \left( \frac{1}{Z_r} \right)}{Z_r dP_r} + \frac{P_r}{T_r} \frac{d \left( \frac{1}{T_r} \right)}{dT_r} \right) dP_r \dots \dots \dots [4]$$

If the reservoir is at constant temperature, such as near room temperature for nitrogen, and the compressibility factor Z is

unity, the last two terms of Equation [4] drop out. On substituting for  $w$  into Equation [3] and integrating over a run from time  $\theta$  to  $\theta$  and reservoir pressure  $P_1$  to  $P_2$ , there is obtained

$$\mu = \frac{\pi D^4}{128 L V_r} \cdot \frac{T_r}{T_c} \cdot \frac{P_a \theta}{\ln \left[ \frac{(P_2 + P_a)(P_1 - P_a)}{(P_1 + P_a)(P_2 - P_a)} \right]} \dots [5]$$

$D$ ,  $L$ , and  $V_r$  are not constant, but functions of temperature. Adding also, in this order, White's correction (26) for coiling (when  $Re (D/d)^{1/2} > 11.6$ ), Schiller's correction (18) for end effects, and a correction for slippage (25)

$$\mu = \frac{\pi D_s^4}{128 L_o V_o} \cdot \frac{P_a \theta}{\ln \frac{(P_2 + P_a)(P_1 - P_a)}{(P_1 + P_a)(P_2 - P_a)}} \cdot \frac{T_r}{T_c} \cdot \left( \frac{1 + 3 \alpha t_c}{1 + 3 \beta t_r} \right) \cdot \left[ 1 - \left( 1 - \left( \frac{11.6}{Re} \sqrt{\frac{d}{D}} \right)^{0.45} \right)^{2.22} \right] \cdot \left( \frac{1}{1 + \frac{w}{22.4 \mu L}} \right) \cdot \left( 1 + \frac{7.98 L'}{D} \right) = K \cdot \frac{P_a \theta}{\ln \frac{(P_2 + P_a)(P_1 - P_a)}{(P_1 + P_a)(P_2 - P_a)}} \cdot \frac{T_r}{T_c} \cdot E \dots [6]$$

The dimensions of the capillary were such that under most conditions the correction terms were small. Schiller's and White's corrections in particular, which vary with flow rate, were almost negligible and could be computed at the average flow rate of each run. However, if they are significant and vary over a run, as in the present apparatus with nitrogen at room temperature, it is preferable to include them in the final integration. Calling  $E'$  the product of the first and fourth correction terms in Equation [6], and  $E''$  the product of the second and third terms, it is readily shown that

$$K = \frac{2\mu}{\theta E' T_r} \cdot \frac{T_c}{T_r} \int_{P_2}^{P_1} \frac{dP}{E'' (P^2 - P_a^2)} = \frac{\mu}{\theta E' P_a T_r} \left[ \ln \frac{(P_2 + P_a)(P_1 - P_a)}{(P_1 + P_a)(P_2 - P_a)} + 2P_a \int_{P_2}^{P_1} \left( \frac{1}{E''} - 1 \right) \frac{dP}{P^2 - P_a^2} \right] \dots [7]$$

The second integral may be evaluated and plotted against  $P$  for a given capillary, gas, and capillary temperature. The correction can then be applied quickly by reading the values of the integral at  $P_1$  and  $P_2$  and subtracting.

With steam, a drift in reservoir temperature and a slight variation in compressibility factor might be expected over a run, and the complete form of Equation [4] would be required. However, only the runs obtained at constant  $T$ , are reported herein. From Fig. 2 (11) it is evident that we may assume

$$1/Z_r = x + yP \dots [8]$$

Equation [4] becomes

$$w = \frac{V_r}{RT_r} \left( \frac{1}{Z_r} + yP_r \right) \frac{dP_r}{d\theta} = \frac{V_r}{RT_r} (x + 2yP_r) \frac{dP_r}{d\theta} \dots [9]$$

Substituting for  $w$  into Equation [3] and integrating

$$\mu = \frac{KE\theta T_r/T_c}{\left( 2y + \frac{x}{P_a} \right) \ln \frac{P_1 - P_a}{P_2 - P_a} + \left( 2y - \frac{x}{P_a} \right) \ln \frac{P_1 + P_a}{P_2 + P_a}} \dots [10]$$

When a run is started, no appreciable error occurs because the outlet tube is at  $P_1$  instead of at  $P_a$ . However, the observed manometer pressure after the run,  $P_3$ , may differ significantly from the desired reservoir pressure,  $P_2$ , at the instant the run ends. By setting up a material balance equating the weight of the gas under these two conditions it is seen that  $P_2$  may be computed by

$$P_2 = \frac{P_3 V_3 - P_1 V_1}{T_m \left( \frac{V_r}{T_r} + \frac{V_3}{T_c} \right)} \dots [11]$$

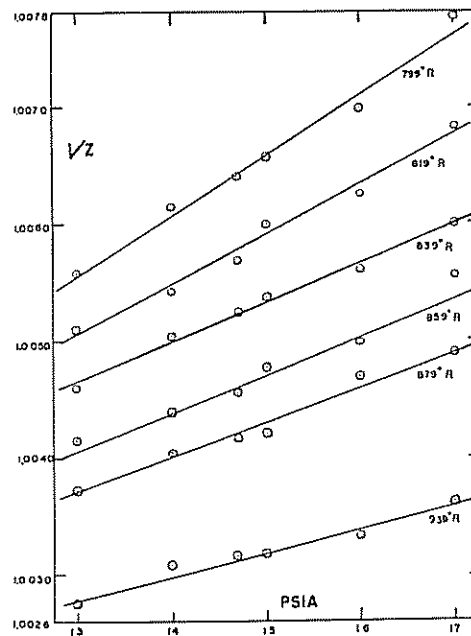


FIG. 2 RECIPROCAL OF THE COMPRESSIBILITY FACTOR OF SUPERHEATED STEAM,  $Z$ , VS PRESSURE, LB/IN.<sup>2</sup> ABS (For 340, 380, 400, 420, and 480 deg F. Calculated from Keenan and Keyes (11).)

ANALYSIS OF VASILESCO'S RESULTS

The Sutherland equation

$$\mu = \frac{kT^{3/2}}{T + C} \dots [12]$$

generally holds for high temperatures. For instance, statistical analysis (10) of all of Vasilescu's data above 0 deg C shows that a  $T^2$  term in the denominator of Equation [12] is not justifiable, i.e., its coefficient is not significantly different from zero. This equation is generally satisfactory down to room temperature or 0 deg C, but Keyes (13) has shown that a third constant is necessary to fit data down to 80 deg K.

However, Vasilescu, employing the value of  $C = 110.0$  deg K for nitrogen and computing  $k$  from viscosity data at 0 deg C, found that he had to increase the value of  $C$  as temperature was raised, in order for his results to agree with Equation [12]. He thus concluded that, over his whole range above 0 deg C,  $C$  varied with temperature. From an inspection of his plot of  $C$  vs temperature

TABLE 1 RESULTS OF STATISTICAL ANALYSIS OF PREVIOUS HIGHEST TEMPERATURE VISCOSITY DATA  
(*C* and *k* values are for Sutherland's Equation [12] with units of deg K and micropoises)

Gas	Reference	Range, deg C	Linear Regression of <i>C</i> vs <i>T</i>		Designation equation	Range, deg C	Linear Regression of ( <i>T</i> <sup>1/2</sup> / <i>μ</i> ) vs (1/ <i>T</i> )		Reference
			<i>C</i>	<i>K</i>			<i>C</i>	<i>K</i>	
Nitrogen	(25)	0-1420	106.9	14.002	[14]	227-1420	109.08	14.050	(10)
Air	(25)	0-1560	110.9	14.835	[15]	16-1560	109.58	14.600	(10)
Argon	(25)	0-1595	137.2	19.137	[16]	298-1595	135.38	19.071	(10)
Carbon Dioxide	(25)					280-1413	219.44	15.237	[17a]
Oxygen	(24)					0-827	126.8	17.00	[17b]

it is evident that a straight line,  $C = p + qT$ , satisfactorily describes the variation over the whole range. But if this is so, substitution of this relation for *C* into Equation [12] yields

$$\mu = \frac{k}{(1 + q)} \frac{T^{3/2}}{\left(T + \frac{p}{1 + q}\right)} \dots \dots \dots [13]$$

It is evident that the results above 0 deg C do fit Equation [12], but with a different value of *C*. Computing the values of *p* and *q* by linear regression of Vasilescu's values of *C* vs. *T* gives the new values of *C* and *K* listed in Table 1. The standard deviations of *C* values from the regression relations were all under 2 per cent. Linear regression of Vasilescu's results above 0 deg C was also carried out for (*T*<sup>1/2</sup>/*μ*) against (1/*T*), the 0 deg C points not being included in this case because of lack of information on their proper relative weight. With each gas the results by his methods I and II proved to be identical at the 95 per cent confidence level. The values of *k* and *C* from all of his data by this analysis are also given in Table 1, and may be slightly preferable for extrapolating his results to higher temperatures. All of these values of *k* are based on 172 micropoises for air at 0 deg C, the viscosity accepted by Vasilescu. If instead, Bearden's (1) value of 181.920 micropoises at 0 deg C is corrected to 0 deg C with *C* = 110.9, 172.06 is obtained, and *k* should accordingly be increased by 1 part in about 3,000.

Literature data on oxygen were also analyzed. The only high temperature data are by Trautz (24) and extend to 827 deg C. The coefficients yielded by linear regression of *T*<sup>1/2</sup>/*μ* against 1/*T*, Rigden's (17) relative viscosity of oxygen to air of 1.11647 at 17 deg C, and Bearden's absolute value for air are also listed in Table 1. Values calculated from Equations [14, 15, 16, 17a and 17b] in Table 1 are given in Table 2.

EXPERIMENTAL RESULTS WITH NITROGEN

Table 3 gives the runs carried out on nitrogen above room temperature. Equation [14] was assumed to be correct, and for each run *K* was calculated by Equation [6]. In the first place it is evident that there is no trend in *K* with temperature. To show this quantitatively, the relation between *K* and *t* in deg C was found by Least Squares to be:  $K = 3.49 \times 10^{-12} - 0.69 \times 10^{-18}t$ . Using the "t-test," the coefficient of *t* at the 95 per cent confidence level is between  $+63 \times 10^{-18}$  and  $-82 \times 10^{-18}$ . These limits, when multiplied by the highest temperatures of Vasilescu's data give about  $10^{-13}$  for the last term of the equation, which is thus only of the order of magnitude of  $\pm 3$  per cent of *K*. Actually, *K* must be considerably more constant than the uncertainty at the 95 per cent level, and the probable effect of the complete temperature range as given by the above equation and by comparison with Vasilescu's runs is under 0.5 per cent. Since Vasilescu's results and the present ones agree closely as to the effect of temperature, it is a reasonable presumption that both are reliable. This is also strong evidence that Vasilescu's results on the other gases are reliable, also that the present apparatus is suitable for accurate work. The percentage standard deviation of the values of *K* in Table 1 about their mean,  $3.486 \times 10^{-12}$ , is

1.43 per cent, and the probable error is  $\frac{1.43 \times 0.0745}{\sqrt{22}} = 0.2$  per cent.

Nine additional runs carried out at 22 to 24 deg C give an average *K* of  $3.469 \times 10^{-12}$  with a standard deviation of 0.96 per cent. These runs, which were analyzed by Equation [7], lasted only 60 seconds each, and it is not felt that they should alter the previous value of *K*. They would decrease its apparent dependence, on temperature, however.

TABLE 2 VISCOSITY OF AIR, NITROGEN, ARGON, CARBON DIOXIDE, AND OXYGEN AT ATMOSPHERIC PRESSURE

Temperature, degrees	Viscosity, micropoises						Carbon dioxide
	C	K	F	Air	Nitrogen	Argon	
-73.1	200	-99.6	133.1	129.0	160.5	147.1	
-23.1	250	-9.6	160.3	155.1	195.4	178.3	
0.0	273.1	32	(172.0)	166.3	210.5	191.8	
25.0	298.1	77.0	184.2	177.9	226.3	205.9	
26.9	300	80.4	185.1	178.8	227.4	207.0	
126.9	400	260.4	229.2	221.0	285.0	258.1	
226.9	500	440.4	267.8	257.9	335.8	303.2	236.8
326.9	600	620.4	302.6	291.1	381.5	343.7	273.3
426.9	700	800.4	334.3	321.4	423.3	380.8	306.9
526.9	800	980.4	363.5	349.4	462.0	415.0	338.2
626.9	900	1160.4	390.9	375.5	498.2	447.0	367.5
726.9	1000	1340.4	416.6	400.0	532.2	477.0	395.1
826.9	1100	1520.4	440.9	423.3	564.3	505.5	421.3
926.9	1200	1700.4	464.1	445.4	594.9	532.6	448.2
976.9	1250	1790.4	475.3	456.0	609.7	545.6	458.2
1026.9	1300	1880.4	486.2	466.5	624.1	558.4	470.2
1126.9	1400	2060.4	507.4	486.7	652.1	583.2	492.2
1226.9	1500	2240.4	527.8	506.2	679.1	607.0	514.8
1326.9	1600	2420.4	547.5	525.0	705.0	630.0	535.9
1426.9	1700	2600.4	566.5	543.2	730.1	652.2	556.1
1476.9	1750	2690.4	575.7	552.0	742.4	663.0	565.2
1526.9	1800	2780.4	584.9	560.7	754.4	673.7	576.2
1626.9	1900	2960.4	602.6	577.7	777.8	694.4	595.4
1726.9	2000	3140.4	620.1	594.4	800.9	714.8	614.3
2226.9	2500	4040.4	700.7	671.4	907.1	808.9	700.4

VISCOSITY OF STEAM

The results that could be located in the literature on the viscosity of steam at atmospheric pressure, most of which were discussed by Hawkins (7), are plotted in Fig. 1. There is evidently a spread of some 10 per cent, and in addition some workers found an upward concavity and others the common downward concavity when temperature is the abscissa. It is difficult to weigh these results as to reliability, and the Least Squares straight line, which would be close to the arithmetic mean line, has been plotted. Its equation is

$$\mu' = 88.4 + 0.361 t \dots \dots \dots [18]$$

Experimental results with the present apparatus are given in Table 4, and plotted in Fig. 3 as deviations from equation [18]. The mean deviation of each point from the average of its group is 0.64 per cent. It is seen that five to seven runs were carried out at or near a given temperature, starting at 450 deg C. At this temperature the perfect gas law is followed closely enough in the capillary that Equation [10] is entirely adequate. The temperature was raised in 150-deg increments to 1200 deg C. Unfortunately, at 1350 deg C the alundum rods yielded and the capillary constant *K* decreased by some 20 per cent, due to pinching at one point. It is planned to repair the apparatus and re-1500 deg C as well as obtain data below 450 deg C.

It was found that Equation [12] did not fit the steam data ve

TABLE 3 DATA FROM HIGH TEMPERATURE RUNS ON NITROGEN

Run no.	P <sub>1</sub> - P <sub>a</sub> gage	Pressures, lb/ft <sup>2</sup>		Time θ sec	Capillary temperature, mean deg C	Correction Terms			Apparatus constant K × 10 <sup>12</sup>
		P <sub>1</sub> - P <sub>a</sub> gage	P <sub>a</sub> abs			Metal expansion	Slippage	Flow losses	
42	105.4	56.7	2105	120	367.4	1.010	1.0016	1.001	3.47
43	58.7	30.8	2105	120	372.7	1.010	1.0016	1.001	3.50
44	30.8	17.3	2105	120	373.7	1.010	1.0016	1.001	3.31
45	92.7	53.0	2105	120	409.1	1.011	1.0018	1.000	3.49
46	53.0	30.5	2105	120	411.4	1.011	1.0018	1.000	3.48
47	30.5	17.3	2105	120	413.8	1.011	1.0018	1.000	3.54
48	98.8	63.5	2105	120	527.2	1.014	1.0025	1.000	3.47
49	63.5	41.1	2105	120	527.8	1.014	1.0025	1.000	3.47
50	41.1	26.5	2105	120	526.3	1.014	1.0025	1.000	3.44
51	96.4	64.5	2080	120	568.5	1.0143	1.0024	1.000	3.48
52	64.5	43.1	2080	120	569.9	1.0143	1.0024	1.000	3.52
53	43.1	28.9	2080	120	569.1	1.0143	1.0024	1.000	3.51
54	88.6	65.5	2086	120	743.2	1.019	1.0029	1.000	3.49
55	65.5	48.2	2086	120	748.4	1.019	1.0029	1.000	3.59
56	48.2	30.0	2086	120	750.5	1.019	1.0029	1.000	3.50
57	98.1	76.0	2086	120	841.1	1.022	1.0036	1.000	3.53
58	76.0	46.2	2086	240	857.1	1.022	1.0036	1.000	3.47
59	86.9	54.7	2080	220	870.3	1.022	1.0036	1.000	3.56
60	54.7	29.9	2080	300	873.2	1.022	1.0036	1.000	3.47
61	96.8	66.6	2130	240	1094.2	1.028	1.0040	1.000	3.47
62	66.6	38.4	2130	360	1096.7	1.028	1.0040	1.000	3.46
63	38.4	24.4	2130	300	1102.2	1.028	1.0040	1.000	3.46

Average = 3.49

TABLE 4 VISCOSITY RUNS ON STEAM

Run no.	P <sub>a</sub> lb/sq ft abs	P <sub>1</sub> - P <sub>a</sub> lb/sq ft gage	P <sub>2</sub> - P <sub>a</sub> lb/sq ft gage	θ sec	E	Temperature, deg C			μ micropoises	
						Manometer	Reservoir	Capillary		
46	2101.30	90.94	16.27	180.5	1.0024	185.1	206.2	452.7	244.0	
47	2102.10	99.57	10.09	240	1.0027	184.3	205.9	454.9	242.9	
48	2101.30	93.58	5.30	300	1.0029	184.3	202.9	453.5	240.5	
49	2103.42	114.22	6.40	300	1.0025	185.3	204.9	453.3	241.2	
50	2105.54	91.98	5.00	300	1.0028	186.1	205.8	454.9	238.6	
									Average	241.4
39	2089.28	28.12	13.15	120	1.0080	185.7	207.8	604.0	303.4	
40	2089.28	115.33	36.28	180	1.0074	184.6	207.7	601.4	303.1	
41	2086.45	41.10	12.95	180	1.0077	184.6	207.5	600.3	300.4	
42	2099.18	108.26	34.94	180	1.0075	186.4	206.5	601.6	310.3	
43	2099.88	110.65	35.67	180	1.0076	186.0	207.7	606.1	309.4	
44	2099.88	41.21	12.90	180	1.0079	185.5	207.6	602.9	299.8	
									Average	304.4
82	2105.90	33.03	11.75	240	1.0135	159.0	194.8	754.1	375.3	
83	2106.60	102.09	35.70	240	1.0134	160.9	195.2	753.4	373.5	
84	2105.54	42.07	14.73	240	1.0135	162.4	195.4	756.7	369.6	
85	2106.25	85.60	30.00	240	1.0133	162.2	196.1	751.1	374.9	
86	2106.25	34.40	12.32	240	1.0133	162.4	196.3	751.4	377.9	
									Average	374.2
53	2097.76	123.72	55.86	240	1.0186	173.5	184.1	900.8	423.2	
54	2098.47	60.46	27.29	240	1.0187	173.8	184.1	900.3	419.2	
55	2097.06	120.69	54.02	240	1.0188	173.8	184.2	901.8	418.0	
56	2097.06	58.19	26.68	240	1.0190	175.6	184.0	903.4	426.2	
57	2097.76	103.06	46.97	240	1.0188	175.0	184.4	900.0	423.1	
58	2097.76	51.81	23.18	240	1.0188	174.9	185.5	900.5	417.8	
									Average	421.3
21	2118.42	52.90	29.20	197.5	1.0197	153.1	249.9	1050.6	470.8	
23	2099.81	70.76	37.05	240	1.0230	179.7	195.2	1047.7	473.1	
24	2101.58	44.71	23.39	240	1.0230	180.4	196.9	1053.7	470.4	
26	2118.63	104.20	54.34	240	1.0233	157.2	192.5	1054.4	471.9	
77	2118.27	59.20	31.01	240	1.0233	157.2	193.7	1054.4	471.1	
78	2116.86	34.23	17.85	240	1.0232	157.0	193.0	1053.0	467.1	
79	2117.92	44.75	23.28	240	1.0230	159.6	196.6	1051.8	470.7	
									Average	470.7
30	2100.59	114.39	56.42	300	1.0270	182.8	212.7	1201.0	508.4	
68	2112.62	67.89	39.51	240	1.0283	150.4	193.9	1206.3	518.6	
70	2112.62	98.59	57.31	240	1.0280	150.6	194.3	1198.8	520.1	
71	2112.62	61.18	35.45	240	1.0280	151.4	194.8	1194.0	509.5	
72	2113.32	37.67	21.87	240	1.0279	151.7	194.9	1197.7	508.2	
73	2110.49	95.96	55.66	240	1.0280	153.3	196.8	1205.6	511.3	
74	2111.20	58.96	34.30	240	1.0280	153.2	196.9	1203.7	510.6	
									Average	512.4

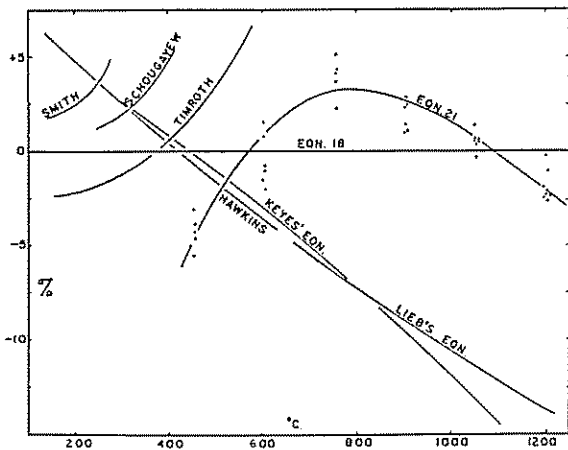


FIG. 3 PERCENTAGE DEVIATION OF THE VISCOSITY OF STEAM AT ATMOSPHERIC PRESSURE BY VARIOUS AUTHORS FROM THE STRAIGHT LINE RELATION EQUATION [18]

closely in spite of the high temperatures, and that a third coefficient would be required. Several variations of Equation [12] were tried which contained in the denominator an additional term which decreased with temperature increases, but the fit was not good. The following form was found reasonably satisfactory, however:

$$\mu' = \frac{kT^{3/2}}{c + T + b'T^2} \dots \dots \dots [19]$$

Rewriting it as follows:

$$\frac{T^{3/2}}{\mu'} = \frac{c}{k} + \frac{T}{k} + \frac{b'T^2}{k} \dots \dots \dots [20]$$

the best values of c/k, k and b'/k were found by multiple regression of all of the individual points as (T<sup>3/2</sup>/μ') against T and T<sup>2</sup>. The final equation obtained is

$$\mu' = \frac{39.37 T^{3/2}}{3315 - T + 0.001158 T^2} \dots \dots \dots [21]$$

for  $\mu'$  in micropoises and  $T$  in deg K. The estimate of error of Equation [2] is 3.1 micropoises at the 95 per cent confidence level. Recommended viscosity values are given in Table 4, Equation [22] being used above its intersection at 570 deg C with Equation [18], and the latter at lower temperatures.

A comparison of some of the other published results with Equation [18] is also given in Fig. 3. At low temperatures, the new data seem to agree in slope with Timroth's and with Schougan-yew's results, but lie some 6 per cent below Timroth's. The high temperature concavity, however, agrees with that of Hawkins (Fig. 1). If this apparatus should show a point of inflection below 600 deg C with lower temperature runs, discrepancies in slope of the earlier data would be partially reconciled. It is noted that the simple linear relation of Equation [18] differs from practically all of the available experimental data by less than 4 per cent over the range of 100 to 1300 deg C. This accuracy is adequate for most applications.

#### CORRECTION OF VISCOSITY TO LOWER PRESSURES

Approximate methods are available for correction to lower pressures, as encountered in the upper atmosphere, and to zero pressure, a frequent reference pressure. Steam values may be readily corrected by Keyes' formula (12), which gives

$$\mu'_a = \mu'_p = 1.51(1 - P) - 0.0059(1 - P^2) \dots [22]$$

for micropoises and atmospheres. Thus, the zero pressure viscosity is found to be 1.5 micropoises lower than that at atmospheric pressure.

For the other gases the method described by Othmer and Josefowitz (16) was employed. This is based on the isothermal relationship

$$\log \mu = A \log(ap + P/\rho) + B = A \log(RT + Pb + abP^2/R^2T^2) + B \dots [23]$$

where  $a$  and  $b$  are constants in van der Waal's equation of state and  $A$  is obtained (16) from high pressure viscosity data. The correction equations obtained for nitrogen, in terms of  $P$  in atmospheres and  $t$  in deg C, are as follows:

(a)  $P$  down to 0.2 atmospheres and  $t$  from 20 to 900 deg C:

$$(\mu_a/\mu_p) = 1.00212 + (0.00000922 \log t - 0.0000279)P - 0.0007 \log t \dots [24]$$

(b)  $P$  below 0.2 atmospheres and  $t$  from 20 to 500 deg C:

$$(\mu_a/\mu_p) = 1.00292 + (0.0000411 \log t - 0.000111)P - 0.001 \log t \dots [25]$$

(c)  $P$  below 0.2 atmospheres and  $t$  from 500 to 900 deg C:

$$(\mu_a/\mu_p) = 1.0029 - 0.000000209 t \dots [26]$$

Above 900 deg C the correction is trivial. The correction from atmospheric to zero pressure is obtained by subtracting 0.3 micropoises between 0 and 50 deg C, 0.2 micropoises between 50 and 300 deg, and 0.1 micropoise between 300 and 800 deg C. The same correction may be used for the other permanent gases in Table 2, though the calculated corrections are slightly smaller.

#### VISCOSITY OF MOIST AIR

It is frequently desired to obtain an approximate estimate of the viscosity of a binary gaseous mixture. However, theoretical equations for this purpose are very complicated (6). Empirical interpolation equations such as Trautz's (23) need data for several compositions, and others, such as Buddenberg and Wilke's (5) require additional physical properties. The simplest formulas, such as Mann's (15), only need the molecular weight, but are likely to

be less accurate. For instance, Mann's would show no change in viscosity on mixing ammonia and ethylene at 12 deg C, at which temperature the pure components have the same viscosity. Actually a 4-per cent increase occurs.

To investigate the possibility of a simple approximate graphical correction that could be applied to linearly interpolated viscosities, Fig. 4 was prepared for several convenient binary systems with a wide range of molecular weight ratios. It is seen that the farther from unity the molecular weight ratio of the pure components and possibly the closer to unity the viscosity ratio, the larger is the fraction by which the viscosity of the mixture exceeds that interpolated from the components. The peak of the excess viscosity plot also shifts toward the component of lower molecular weight. Considerable ranges of temperature are seen not to affect the correction appreciably, as in the nitrogen-hydrogen and carbon monoxide-hydrogen systems. While a high accuracy could not be expected in estimating a correction for a binary system from these curves, it seems likely that the result would not be off by more than several per cent, which is adequate reliability for most cases. A generalized curve or equation correlating these and other binary system data would seem desirable before application of these curves as a general method.

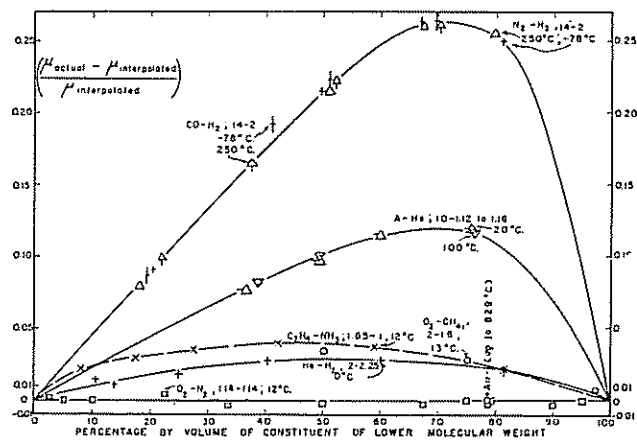


FIG. 4 DEVIATION FROM INTERPOLATED VISCOSITY FOR BINARY GAS MIXTURES

(First number following the symbols of the gases is their molecular weight ratio. Second number is the viscosity ratio of heavier over lighter gas.)

For the system air (considered as a single gas)—water vapor, the molecular weight ratio, 1.7, approximates that of ethylene-ammonia, 1.65, so the curve of the latter pair seems indicated for predicting the viscosity of moist air. As an illustration, the correction to be added to the viscosity of air to give that of air containing 5 per cent of moisture by volume has been computed by this method and listed in Table 5. The same correction has been recently obtained by calculation from assumed force laws between molecules by Hirschfelder (9). In Table 5, the discrepancy between these two methods is seen to range from 0 to 1 per cent. Thus, the much simpler method using the ethane-ammonia curve in Fig. 5 seems adequate for other moisture contents in most applications, particularly since Hirschfelder's simple force law assumptions can't agree entirely with the actual facts (13). Up to 55 per cent of moisture by volume the correction of Fig. 4 is given within 0.001 by the expression  $0.08\sqrt{y'(1-y')}$ , where  $y'$  is the volume fraction of water vapor.

The other recent method for computing the viscosity of gas mixtures which seems most reliable is that of Buddenberg and Wilke (5). Taking the diffusion coefficient for air and water vapor as 0.239 sq cm/sec at 8 deg C, ideal gas law values of density,  $a$

TABLE 5 VISCOSITY OF ATMOSPHERIC PRESSURE STEAM  
With Correction to Be Added to the Viscosity of Dry Air to Yield That of Air With 5 Moles Per Cent of Moisture

Temperature Deg C	Temperature Deg F	Steam micro- poises	Moist air Fig. 4 per cent	Moist air Hirschfelder (9) per cent	Moist air Buddenberg (5) per cent
0.0	32	88.4	-1.1	-1.2	-0.4
5.0	77	97.4			
26.9	80.4	98.1	-0.8	-1.0	-0.3
126.9	260.4	131.2	-0.5	-0.5	+0.1
226.9	440.4	170.3	-0.2	-0.3	+0.5
326.9	620.4	206.4	0.0	0.0	+0.8
426.9	800.4	242.5			
526.9	980.4	278.6	+0.4	+0.2	+1.4
626.9	1160.4	317.0			
726.9	1340.4	355.4	+0.8	+0.4	+2.1
826.9	1520.4	397.2			
926.9	1700.4	432.6	+1.3	+0.5	+2.6
976.9	1790.4	449.1			
1026.9	1880.4	464.6			
1126.9	2060.4	492.9	+1.6	+0.6	+3.2
1226.9	2240.4	517.4			
1326.9	2420.4	538.4	+1.5	+0.6	+3.0
1426.9	2600.4	555.5			
1476.9	2690.4	563.9			
1526.9	2780.4	570.9			
1626.9	2960.4	582.7			
1726.9	3140.4	592.1	+1.4	+0.9	+2.8
2226.9	4040.4	(611.1)			

viscosities from Table 2 for air and Table 5, for water vapor, the values of  $(\mu/D_r\rho)$  of 0.586 and 0.489 are obtained for air and water vapor, respectively. Assuming them independent of temperature and substituting into Buddenberg and Wilke's formula, air with 5 per cent moisture is obtained as having a viscosity of  $(\mu_{steam}/13.87) + (\mu_{air}/1.0427)$ , or the per cent deviation from the viscosity of air is  $7.2(\mu_{steam}/\mu_{air}) - 4.10$ . These values have been included in Table 5 for comparison. In view of the much better agreement between the two previous methods, it would seem that Buddenberg and Wilke's is less reliable for this system, at least at low moisture contents.

ACKNOWLEDGMENTS

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The Viscosity of Steam and of Nitrogen at Atmospheric Pressure and High Temperatures

By G. F. Bonilla, R. D. Brooks, and P. L. Walker, Jr.

General Discussion on Heat Transfer (London, Sept. 11-13, 1951)

Errata

<u>Page</u>	<u>Column</u>	<u>Line</u>	<u>Reads</u>	<u>Should Read</u>
1	1	D <sub>v</sub>	coefficients	coefficient
1	1	Second V <sub>1</sub>	V <sub>1</sub> = hot volume	V <sub>1</sub> = hot volume
1	2	7 from bot.	Hole	Hole
2	1	5 from bot.	amounting	amounted
2	1	Eqn. 1	(exponent =) 1/	(exponent =) 1/2
2	2	Eqn. 2	-126	126
2	2	Eqn. 4	$\frac{d(P_r V_r)}{d\theta}$	$\frac{d(\rho_r V_r)}{d\theta}$
2	2	Eqn. 4	$\left(\frac{1}{T_r}\right)$	$\left(\frac{1}{T_r}\right)$
3	1	Eqn. 7	Second integral sign applied to the whole fraction.	
3	1	Eqn. 8	y <sup>b</sup>	yP
4	1	7	K	k
4	1	bottom	K	k
4		Table 1 (two times)	K	k
4	1	19	at 0 deg C	at 20 deg C
4	2	1	$\sqrt{22}$	$\sqrt{22}$
6	1	2	Equation (2)	Equation (21)
6	1	4	Equation (22)	Equation (21)
6	1	Equation 22	$\mu_a^0 = \mu_p^0$	$\mu_a^0 = \mu_p^0$
7	1	bottom		The experimental work was carried out at the Department of Chemical Engineering, The Johns Hopkins University, Baltimore 18, Maryland.
7	2	Ref. 7	1940	1948