# Extrusion Process Modeling

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### Introduction

In the conventional processes of carbon and graphite,<sup>1,2</sup> manufacturing most artifacts are formed in an extrusion process. The anisotropy, one of the most important properties of the baked and graphitized products, is established in the extrusion operation. Besides the filler type and the ratio of the die to mud cylinder diameters (see Figure 1), the development of this anisotropy in an extruder is largely controlled by the rheology of coke-pitch mixes. In addition, the pressure required to extrude the artifacts, an important process parameter of the forming operation, is also directly related to the rheological behavior of coke-pitch mixes. The published investigations<sup>3-6</sup> suggested that the rheology of coke-pitch mixes can be modeled as Bingham bodies. However, all the experiments were conducted either in a capillary rheometer or in a parallel-plate rheometer, rather than in an extruder. This paper reports the results of modeling the rheological behavior of coke-pitch mixes in extruders. Both Newtonian fluid and Bingham plastic models were studied.



Figure 1. A Schematic Diagram of an Extruder.

#### Theory

#### Newtonian Fluids

As shown in Figure 1, an extruder can be modeled as a conical section connecting two straight pipes. For an incompressible Newtonian fluid with a slow flow rate, Bird<sup>7</sup> and Sutterby<sup>8</sup> suggested that a parabolic velocity profile can be assumed at each position along the Z-axis. Furthermore, the pressure gradient of a Newtonian fluid is related to the velocity profile. Hence, the pressure drop in each section, as shown in Equations (1)-(3), can be derived from its corresponding parabolic velocity profile.

Mud Cylinder

$$\frac{P_{o}-P}{A}=\zeta$$
 (1)

Cone Section

$$\frac{P_{0} - P}{A} = \frac{1}{3K} \left[ \frac{1}{(1 - K\zeta)^{3}} - 1 \right]$$
(2)

Die Section

$$\frac{P_{0} - P}{A} = \frac{\zeta}{B^{4}} - \frac{(1-B)^{2} \times (B^{2} + 2B + 3)}{3KB^{4}}$$
(3)

where: A =  $4 \mu v_0/R_1$ , while  $\mu$  is the Newtonian viscosity and  $v_0$  is the velocity at the center line of the mud cylinder, P<sub>0</sub> = the pressure at z = 0,  $\zeta$  =  $2/R_1$ , K = tan  $\alpha$ , B =  $R_2/R_1$ .

Using these equations, the relative pressure drop in each section can be calculated, even if the viscosity of the fluid is unknown. The validity of this model can then be checked by comparing the calculated relative pressure drops with the measured ones.

### Bingham Bodies

The flow of a Bingham body can be described by the Buckingham-Reiner equation.<sup>9</sup> This equation is first simplified by neglecting the higher order terms in the Taylor expansion series.<sup>10,11</sup> The pressure drop equations (Equations (4) and (5)) can then be obtained by integrating the simplified Buckingham-Reiner equation.

#### Straight Sections

$$\frac{B}{R^{5/2}}L + \frac{2\tau_0}{R}L = \Delta P \qquad (4)$$

Cone Section

$$\frac{2B}{3K} \frac{1}{R_1^{3/2}} \left[ \frac{1}{B^{3/2}} - 1 \right] + \frac{2\tau_0}{K} \ln \frac{1}{B} = \Delta P \quad (5)$$

where:  $B = (8\eta\tau_0/\pi)^{1/2} Q^{1/2}$ , while Q is

- the volumetric flow rate and  $\eta$  and  $\tau_0$  are the viscosity and the yield stress of a Bingham body, respectively,
- L = the length of the straight section, and
- R =the radius of the pipe, (=  $R_1$  or  $R_2$ ).

Equations (4) and (5) can be applied for a priori estimations of extrusion pressure, provided that the Bingham parameters (n and τ<sub>o</sub>) of mixes are coke-pitch known. Conversely, these rheological parameters can be calculated from the experimental data of extrusion pressures versus extrusion rates. In addition, since B is proportional to the square root of volumetric flow rate (or extrusion rate) for any given coke-pitch mix, a log-log plot of B versus extrusion rate should give a straight line with a slope of 0.5, regardless of the length of the die.

#### Experimental

The coke-pitch mixes were prepared from a petroleum coke and a coal tar pitch (Mettler softening point ~  $110^{\circ}$ C). The binder pitch contents in the experimental mixes were at 20-26 weight percent. Two extruders were investigated: the 152 mm extruder had a 254 mm diameter mud cylinder and a 152 mm diameter die, the 19 mm extruder had a 76 mm diameter mud cylinder and 19 mm diameter dies of different length (55 to 225 mm in length). The pressure measured by strain gauge transducers (Dynisco PT 420A and TPT 463E).

## Results and Discussion

In Table 1, the measured relative pressure drops in each section of the 152 mm extruder are compared with the model predicted values. The Newtonian model predicts much higher pressure drops in the die section, while substantially underestimating the pressure drops in the cone section, indicating that the rheological behavior of coke-pitch mixes in extruders cannot be adequately described as Newtonian fluids.

The predictions made on the basis of the Bingham plastic model are much closer to the experimental data than those predicted by the Newtonian model (see Table 1). A probable cause of the discrepancies between the predicted and measured values is the contribution of slip flow at low shear rates.<sup>12</sup> The Bingham parameters

Table 1. Relative Pressure Drops in the 152 mm

| Extruder                      |                  |            |        |
|-------------------------------|------------------|------------|--------|
|                               | ΔP<br>mud        | ΔP<br>cone | AP die |
| Experimental Data             | 21-43%           | 53-73%     | 5-10%  |
| Newtonian Model               | 22-27%           | 22-37%     | 41-51% |
| Bingham Model*                | 49-53%           | 20-26%     | 25-27% |
| * For Mix Formulati           | on A, $\tau_0 =$ | 13 psi and |        |
| η = 1.0 x 10 <sup>6</sup> poi | se.              |            |        |

of coke-pitch mixes (Mix Formulation B) are estimated to be τ<sub>0</sub> = 35 psi and  $n = 1.5 \pm 1.0 \times 10^6$  poise at an extrusion temperature of about 108°C. These values agree well with those reported in the literature. 4,6 As shown in Figure 2, the plot of the quantity B versus the extrusion rate on a log-log scale indeed gives rise to a straight line. A slope of 0.71 of this straight line is slightly higher than the theoretical value of 0.5. However, the prediction that the data measured on a longer die would fall on the same straight line obtained from the data measured on a shorter die is in Figure 2. Hence, it can be confirmed concluded that the Bingham model (with further refinements) can adequately describe the rheological behavior of coke-pitch mixes in extruders.

As indicated earlier, further refinements of the Bingham model should incorporate the contribution of slip flow at low shear rates. In its present form, the model has difficulties in predicting extrusion pressure at low extrusion rates. As seen in Figure 2, the datum point corresponding to an extrusion rate of 0.74 ft/min falls much below the trend described by the straight line. Furthermore, the extrusion pressures measured at extrusion rates of ca. 0.3 ft/min are lower than pressure drops required by the yield stress of the coke-pitch mixes. These facts indicate that the contribution of a slip flow mechanism to the total extrusion flow is more significant at low extrusion rates.



Figure 2. Quantity B Versus Extrusion Rate on a 19 mm Extruder at about 108°C. Δ = Short Die; M = Long Die.

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