ELASTIC CONSTANTS OF GRAPHITE*

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The elastic constants can be obtained by direct mechanical measurements, resonant methods, or from sonic velocity measurements. Generally, it is a project requiring careful planning, sensitive equipment, skilled testing and knowledgable evaluation to derive the elastic constants for an anisotropic material such as graphite. The purpose of this paper is to describe a fairly simple procedure to easily and accurately obtain the elastic constants of graphite using sonic velocity measurements. This is accomplished by noting an additional degree of symmetry in both molded and extruded textures.

Sonic velocity measurements directly yield the stiffness values, C_{ij}; but the compliances, S_{ij}, must be calculated. Using different transmitting crystals, both the longitudinal and the two shear wave mode velocities can easily be measured. The general method is to measure the velocities in the z direction, the axis of symmetry, and one of the normal directions, x or y. This will directly yield four of the five elastic constants used to describe elastic deformation of a body with hexagonal symmetry with one isotropic plane. In the past, it was considered necessary to determine the velocities in one intermediate direction to evaluate the fifth constant (C_{13}); however, it will be shown and demonstrated that only the two normal directions are required. This was found possible by observing that the shear wave velocity associated with the C_{44} did not vary with the angle of measurement.

When plane waves are propagated through a solid, the Cristoffel velocity equations for the three transmitted modes in an orthogonal coordinate system are as follows:[1]

$$\begin{vmatrix} \Gamma_{11} - \rho v^2 & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{13} & \Gamma_{22} - \rho v^2 & \Gamma_{23} \\ \Gamma_{23} & \Gamma_{23} & \Gamma_{33} - \rho v^2 \end{vmatrix} = 0 \qquad (1)$$

where ρ = density of the solid, v = velocity of the wave propagation, and the Γ_{ij} are the Cristoffel stiffnesses. For a wave propagating in a direction whose cosines are ℓ, m, n with reference to a Cartesian coordinate system, the Γ_{ij} take on the following form for a crystal of hexagonal (D_{6h}) symmetry:

$$\Gamma_{11} = \ell^2 c_{11} + m^2 c_{66} + n^2 c_{55}$$

$$\Gamma_{12} = \ell m\ (c_{12} + c_{66})$$

$$\Gamma_{13} = n\ell\ (c_{13} + c_{55})$$

$$\Gamma_{23} = mn\ (c_{13} + c_{44})$$

$$\Gamma_{22} = \Gamma_{11}$$

$$\Gamma_{33} = m^2 c_{44} + n^2 c_{33}$$

with $c_{55} = c_{44}$ and $c_{66} = 1/2(c_{11} - c_{12})$.

Equations (1) diagonalize when the wave propagates in the z-axis direction or in the x-y plane. For the first case, $n = 1$, $m = \ell = 0$, and

$$\begin{vmatrix} c_{44} - \rho v_z^2 & 0 & 0 \\ 0 & c_{44} - \rho v_z^2 & 0 \\ 0 & 0 & c_{33} - \rho v_z^2 \end{vmatrix} = 0$$

with the obvious solutions

$$\rho\left(v_z^z\right)^2 = c_{33}\ ; \quad \rho\left(v_z^{xy}\right)^2 = c_{44} \qquad (2)$$

where v_z^z is a plane wave traveling in the z direction and polarized in the z direction (*i.e.*, longitudinally), and v_z^{xy} is traveling in the z direction but polarized transversely (*i.e.*, shear wave).

In the second case, in which the wave velocity is in the x-y plane, because of cylindrical symmetry, we may take $n = m = 0$ and obtain

$$\begin{vmatrix} c_{66} - \rho v_x^2 & 0 & 0 \\ 0 & c_{11} - \rho v_x^2 & 0 \\ 0 & 0 & c_{44} - \rho v_x^2 \end{vmatrix} = 0$$

with the obvious solutions

$$\rho\left(v_x^x\right)^2 = c_{11}\ ; \quad \rho\left(v_x^z\right)^2 = c_{44}\ ;$$
$$\rho\left(v_x^y\right)^2 = c_{66} = \tfrac{1}{2}(c_{11} - c_{12}) \qquad (3)$$

with the same notation as before.

Equations (2) and (3) therefore yield the stiffness constants c_{11}, c_{33}, c_{44}, and c_{12} from the relevant velocity measurements. It remains to determine c_{13}.

Consider a wave propagating in the direction r not lying in the x-y plane. Then m may be taken as zero and the secular determinant becomes

$$\begin{vmatrix} \ell^2 c_{11} + n^2 c_{44} - \rho v_r^2 & 0 & n\ell(c_{13} + c_{44}) \\ 0 & \ell^2 c_{66} + n^2 c_{44} - \rho v_r^2 & 0 \\ n\ell(c_{13} + c_{44}) & 0 & \ell^2 c_{44} + n^2 c_{33} - \rho v_r^2 \end{vmatrix}$$

One solution factors out:

$$\rho\left(v_r^{xy}\right)^2 = \ell^2 c_{66} + n^2 c_{44} \qquad (4)$$

representing a shear wave. The remaining two solutions are a shear wave v_r^{rz} and a longitudinal wave v_r^r obtained from the quadratic

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$$(\ell^2 c_{11} + n^2 c_{44} - \rho v^2)(\ell^2 c_{44} + n^2 c_{33} - \rho v_p^2) = n^2 \ell^2 (c_{13} + c_{44})^2 \quad (5)$$

In general these solutions will depend upon the directional cosines ℓ and n. In the present case (see Fig. 1), the shear wave v_p^{pz} is found empirically not to depend on the cosines. For the special cases $\ell = 0$, v_p^{pz} becomes v_z^{xy}, and for the case n = 0, v_p^{pz} becomes v_x^z, both of which satisfy

$$\rho(v_z^{xy})^2 = \rho(v_x^z)^2 = c_{44}$$

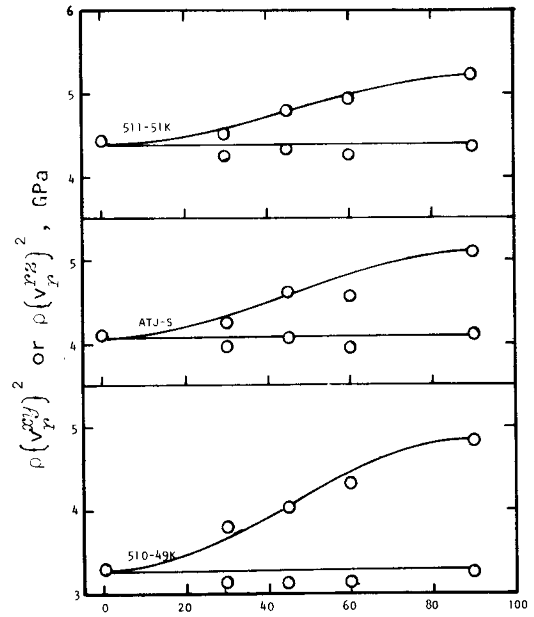

Fig. 1. Shear Velocity Measurements as a Function of the Angle of Rotation.

Therefore we may conclude on the basis of the data for any ℓ and n

$$\rho(v_p^{pz})^2 = c_{44} \quad (6)$$

and the associated longitudinal wave becomes

$$\rho(v_p^p)^2 = (c_{11} - c_{33})\ell^2 + c_{33} \quad (7)$$

which depends quadratically on the directional cosine ℓ.

Substituting Eq. (6) into Eq. (5) and using $n^2 + \ell^2 = 1$, we obtain

$$c_{13} = (c_{11} - c_{44})^{\frac{1}{2}}(c_{33} - c_{44})^{\frac{1}{2}} - c_{44}$$

Thus, the value of c_{13} can be calculated from the values of c_{11}, c_{33}, and c_{44}. And all five elastic constants can be obtained from measurements only in the axial and one transverse direction. This is further confirmed by comparing the calculated values of $\rho(v_p^p)^2$ to the measured values in Fig. 2. Additional confirmation is also shown by a comparison of sonically derived constants to values obtained by the strain gage techniques in Table I.

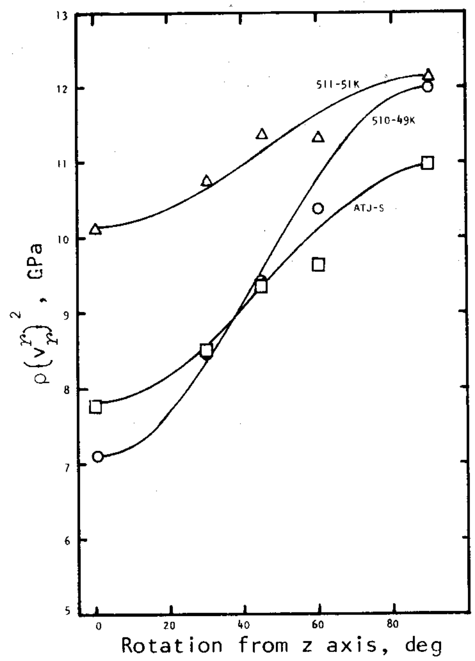

Fig. 2. Longitudinal Velocity Measurements as a Function of the Angle of Rotation.

Table I. A Comparison of Elastic Constants

	Graphite Grade					
	ATJ-S		H-451		AGOT	
	M[a]	S	M[b]	S	M[c]	S
$1/S_{11}$, MPa	11.44	10.76	7.86	8.55	11.95	12.07
$1/S_{33}$, MPa	8.00	7.67	7.17	7.54	5.05	5.17
$-S_{12}/S_{11}$	0.11	0.06	0.11	0.14	0.05	0.04
$-S_{13}/S_{11}$	0.13	0.13	0.11	0.12	0.11	0.12
$-S_{13}/S_{33}$	0.09	0.09	0.13	0.14	0.08	0.07

M — by direct mechanical test

[a]Southern Research Institute

[b]General Atomic Company

[c]Oak Ridge National Laboratory

S — by sonic techniques, Oak Ridge National Laboratory.

Reference

(1) R.F.S. Hearmon, p. 68 in *An Introduction to Applied Anisotropic Elasticity*, Oxford University Press, London, 1961.