APPLICATION OF THE JONES/NELSON NONLINEAR STRESS-STRAIN LAW TO REENTRY GRAPHITE SURVIVAL PREDICTIONS

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Introduction

Reentry vehicle graphite nosetips encounter a severe thermostructural loading condition. Accurate prediction of material deformation response requires a set of constitutive relationships to account for the unique mechanical response of aerospace graphites demonstrated in laboratory studies by Jortner (Ref. 1).

This behavior, termed "biaxial softening," is the observation of increased strain and compliance in biaxial stress states over that accounted for by classical elastic-plastic deformation theories (Fig. 1). The data indicates the need for a new material model that can account for coupling between multiaxial stresses other than, and in fact opposite to, normal Poisson effects.

Jones and Nelson (Ref. 2) have developed a material model based on a strain energy scalar as the state variable controlling the constitutive equations of material response. The adaptation of this model for practical application and its use with a previously developed statistical failure model for prediction of graphite failure is the subject of this paper.

Modeling Approach

The method developed by Jones and Nelson uses the strain energy density function

$$U = (\sigma_r \epsilon_r + \sigma_z \epsilon_z + \sigma_\theta \epsilon_\theta + \tau_{rz} \gamma_{rz})/2$$

normalized to a reference value U_0 as a scalar quantity to vary the material properties as a function of stress state. The equation selected by Jones and Nelson is

$$\text{Material Property} = A[1-B(U/U_0)^C]$$

where the constants A, B, and C are determined from test data. In theory, the above equations are used in an iterative scheme to determine the secant moduli and Poisson ratios which simultaneously satisfy the individual stress-strain relationships, as well as the total strain energy to stress state dependence. Figure 2 shows the functional relationship of the variables for a uniaxial stress state. An inherent problem with the Jones/Nelson equation is that it does not extrapolate well beyond the data to which it was fit. A typical fit of the model to graphite data gives a stress-strain curve that rolls over and becomes asymptotic to zero stress at large strains. While this causes no problems for uniaxial strains within the failure limits of graphite, the total strain energy for a multiaxial stress state can be great enough to operate on the extrapolated stress-strain curve well beyond the failure point. The net result is a solution which is difficult or impossible to converge upon, as well as being a poor representation of material behavior.

A solution to this problem has been developed in which a two part stress-strain curve is used. The Jones/Nelson equation is used to describe the uniaxial stress-strain relation up to the failure threshold and a linear equation of the form

$$\sigma = \sigma_0 + E_\infty \epsilon$$

is used thereafter. The constants σ_0 and E_∞ are determined so that the line passes through the mean failure stress and strain point and the 95 percent survival stress and strain values from available uniaxial test data. The Jones/Nelson curve is constrained to intersect and be tangent to the linear portion at the 95 percent survival level. This technique correlates the stress and strain failure statistics, as well as accurately modeling observed stress-strain curves.

The resulting stress-strain model is shown in Figure 3. The linear relationship was reformulated so that the secant modulus is calculated by

$$E_{sec} = 8UE_\infty^2 / \left(-\sigma_0 + \sqrt{\sigma_0^2 + 8UE_\infty}\right)^2$$

which is compatible with the strain energy formulation used by the Jones/Nelson equation.

The amount of data available for the dependence of Poisson ratios on strain energy was considered insufficient to develop a valid model, so constant values are assumed. It was also found that interpolating values of the constants A, B, and C between temperatures gave erratic results. This was remedied by interpolating the values of the calculated secant moduli.

Results

Material models using the modified Jones/Nelson formulation have been developed for several aerospace grade graphites. They have been used in conjunction with a stress based statistical failure criterion (Ref. 3) and comparisons with laboratory test data for uniaxial and biaxial failure strains are presented in Figures 4 and 5. Additional correlation with predicted strains for full scale ground tests of ATJS nosetips (Ref. 4) provides further verification of the method.

Conclusion

An analysis method for thermostructural evaluation of graphites has been developed which accounts for the deformation and fracture behavior particular to graphitic materials. The practical application of the model has been demonstrated and its predictions are consistent with all known laboratory failure data.

References

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2. R. M. Jones, D. A. R. Nelson, Jr., "A New Material Model for the Nonlinear Biaxial Behavior of ATJ-S Graphite," J. Composite Materials, Vol. 9, January 1975.
3. J. G. Crose, J. D. Buch, E. Y. Robinson, "A Fracture Criterion for Anisotropic Graphites in Polyaxial Stress States," Proc. 12th Conf. on Carbon, 1975.
4. J. D. Buch, J. G. Crose, R. L. Holman, T. E. Mack, "Correlation of a Physically Based Statistical Theory of Fracture...," Proc. 13th Conf. on Carbon, 1977.

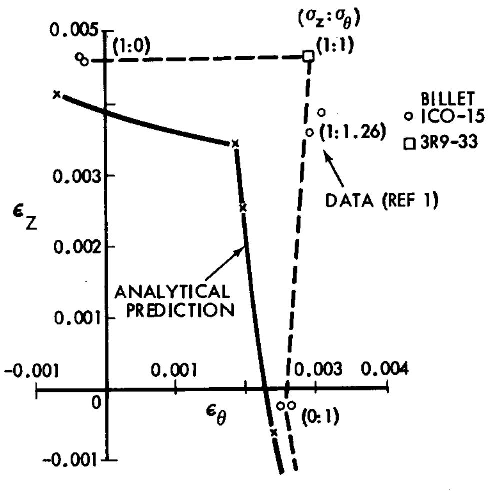

Fig. 1. ATJS Biaxial Stress Response at 70°F for 3550 psi Maximum Principal Stress Elastic-Plastic Theory Prediction and Data

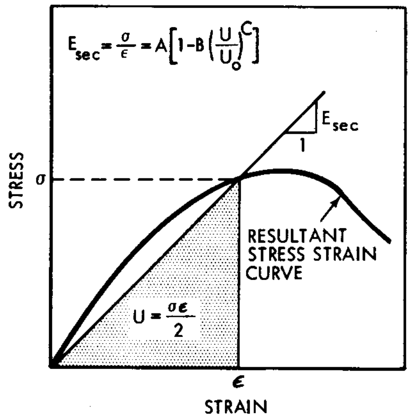

Fig. 2. Jones/Nelson Model of Uniaxial Stress-Strain Behavior

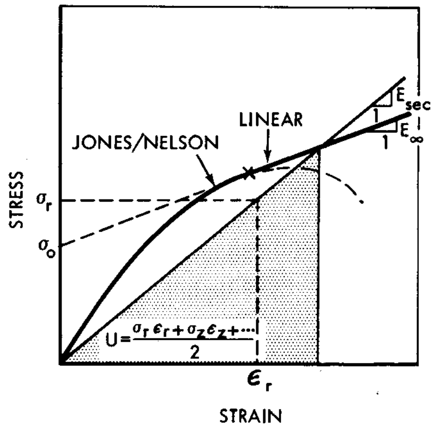

Fig. 3. Modified Jones/Nelson Model of Multiaxial Stress-Strain Behavior

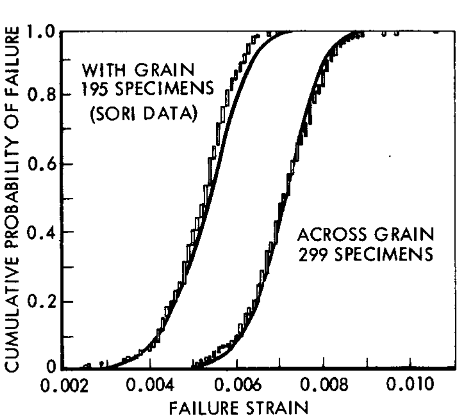

Fig. 4. 994-2 Graphite Failure Strain at 2000°F, Predictions and Data

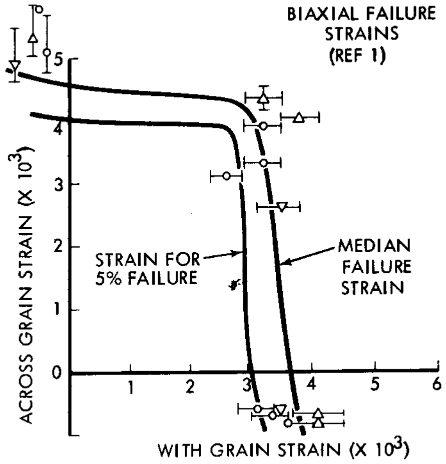

Fig. 5. ATJS Biaxial Failure Strains at 2000°F, Predictions and Data