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Technical Note

Wedge Stability in the Roof of a Circular Tunnel: Plane Strain Condition

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NOMENCLATURE

Note: All forces are per unit length of excavation

- A: Anchor force, positive upwards.
- F: Factor of safety.
- N_0, N: Normal force acting on wedge face. Initial (prior to wedge displacement) and at any arbitrary displacement V, respectively.
- S_0, S: Shear force acting on wedge face. Initial (prior to wedge displacement) and at any arbitrary displacement V, respectively.
- P: Principal field stress magnitude.
- V: Wedge vertical displacement.
- W: Wedge weight.
- a: Excavation radius.
- i: Discontinuity dilation angle
- k_s, k_n: Discontinuity shear and normal stiffnesses.
- r: Radius to any point of interest ($r > a$).
- u, v: Discontinuity shear and normal displacements.
- z: Overburden depth.
- α: Wedge-semi-apical angle.
- ϕ: Discontinuity angle of frictional resistance.
- $\theta, \bar{\theta}$: Semi-included angle of wedge and angle between the vertical and any radial ray.
- σ_s, σ_n: Shear and normal stresses at any location.
- γ: Unit weight of rock.

INTRODUCTION

The estimation of support requirements to stabilize potentially removable rock blocks surrounding underground openings is of prime importance for rational design of such structures. Common practice has been to ignore the effect of *in situ* stresses in the evaluation of block stability. Consequently, support systems designed to carry the full dead weight of potentially free-falling blocks in roofs may be considerably overdesigned. This conservatism is especially pronounced where favourable block geometries are encountered comprising narrow apical angles. The interaction between stress redistribution and joint dilation during failure of tapered wedges is a complex problem that is neither well documented nor understood. It is evident that stability of parallel sided blocks around underground excavations will be enhanced by the restriction of joint dilation. What is not so well understood is the interaction between joint normal stress and displacement when only partial restriction to dilation is imposed. This concept is illustrated in Fig. 1 and is of importance in determining the limiting apical angle of a tapered wedge at which the geometry will dictate that the block is inherently unstable rather than stable.

To investigate the effect of *in situ* stresses on the stability of tapered blocks, a two dimensional solution developed by Bray [1] has been amalgamated with a solution for stresses adjacent to a circular cylinder in a plane hydrostatic stress field. This procedure is presented in the following.

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STABILITY ANALYSIS

Bray [1] proposed an analysis of a symmetric wedge present in the roof of an underground opening. The material comprising the body of the wedge is assumed rigid, all deformations being restricted to the bounding discontinuity planes. The geometry of this wedge is shown in Fig. 2. At some equilibrium position, normal and shear forces (N, S) act on the wedge, as does the wedge weight (W) and any anchor force (A). The discontinuity planes exhibit purely frictional shear strength with a geometric dilational component added in this revised treatment of the problem. The discontinuity planes have shear and normal stiffnesses k_s and k_n respectively.

Assuming the block to be rigid with a shear strength criterion characterized by an angle of internal friction (ϕ) and a geometric component (i), resolving vertically yields,

$$W - A = 2S\cos\alpha - 2N\sin\alpha$$

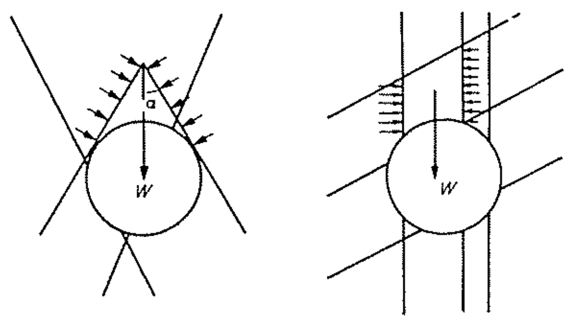

Fig. 1. Possible block geometries adjacent to a circular cross-section underground excavation. (a) Partial restriction of discontinuity dilation. (b) Full restriction of discontinuity dilation.

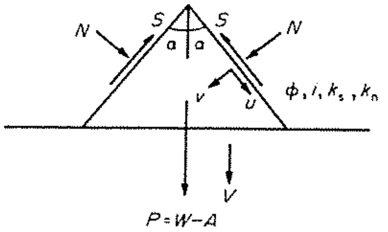

Fig. 2. Wedge geometry proposed by Bray [1].

or,

$$W - A = 2N\left(\tan(\phi + i)\frac{\cos\alpha}{F} - \sin\alpha\right) \quad (1)$$

since

$$S = \frac{N\tan(\phi + i)}{F}. \quad (2)$$

The factor of safety (F) corresponds to the amount by which the shear strength must be uniformly reduced to bring the block into a condition of limiting equilibrium.

If, following excavation of the opening, the wedge is displaced vertically an amount V, normal and tangential joint displacements of v and u will be induced where

$$u = V\cos\alpha$$
$$v = V\sin\alpha - u\tan i = V(\sin\alpha - \cos\alpha\tan i). \quad (3)$$

In the derivation of (2) it is assumed that the vertical displacement (V) is not sufficiently large that one of the "sawteeth" comprising the idealized joint model is fully overriden. Similarly no failure of the teeth is considered.

Under the displacements u and v, the normal and shear stresses will be changed from initial values (S_0, N_0) prior to displacement to some values S and N.

$$N_0 - N = k_n v = k_n V(\sin\alpha - \cos\alpha\tan i) \quad (4)$$
$$S - S_0 = k_s u = k_s V\cos\alpha.$$

Substituting (4) into (2) yields

$$S_0 + k_s V\cos\alpha = (N_0 - k_n V(\sin\alpha - \cos\alpha\tan i)) \times \frac{\tan(\phi + i)}{F}. \quad (5)$$

Rearranging for V gives

$$V = \frac{N_0\tan(\phi + i) - S_0 F}{k_s\cos\alpha F + k_n(\sin\alpha - \cos\alpha\tan i)\tan(\phi + i)}. \quad (6)$$

Equations (1) and (6) constitute two independent equations in 3 unknowns (N, F, V), rendering the problem indeterminate in the absence of any simplifying assumptions. It is assumed in the following that the initial forces acting on the wedge (N_0, S_0) may be estimated from the elastic stress distribution resultant of excavation. The force required to yield the block (negative A) may be deduced from the available equations since, at failure, F is equal to unity. The only additional information required are the magnitudes of the initial shear and normal forces acting on the block prior to block displacement. The determination of these quantities will be discussed in the following.

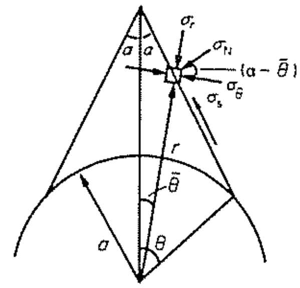

Fig. 3. Wedge geometry in the roof of a circular tunnel.

INITIAL ELASTIC STRESSES

Previous considerations of the problem [2–4], all drawing heavily on the original analysis by Bray, idealized the stress regime in a horizontal roof as one of homogeneous, uniaxial stress in the plane of the roof. This idealization is reasonable for large span openings but may no longer be sufficient to describe the stress state adjacent to an opening of more limited dimensions. It is with this factor in mind that a solution is presented for the forces acting on a wedge in the roof of a circular cross section opening as illustrated in Fig. 3. The preexcavation stress regime is assumed to be hydrostatic and conditions of simple plane strain are deemed applicable.

Thick walled cylinder theory is used to determine the elastic stresses acting on the faces of a wedge superimposed on the existing cylinder geometry as shown in Fig. 4. The stress regime at any location along the wedge limb may be determined.

Stress transformation equations are invoked to evaluate the resultant components of these stresses acting normal and tangential to the wedge plane. The normal and shear stress distribution may then be summed analytically to yield the resultant forces that may then be used in the previously discussed rigid wedge analysis.

It is advantageous to express the radius of interest (r) in terms of the cavity radius (a), the wedge semi-apical angle (α) and the angles θ and $\bar{\theta}$ defined as shown in Fig. 3. Thus

$$r = a\frac{\sin(\alpha + \theta)}{\sin(\alpha + \bar{\theta})}. \quad (7)$$

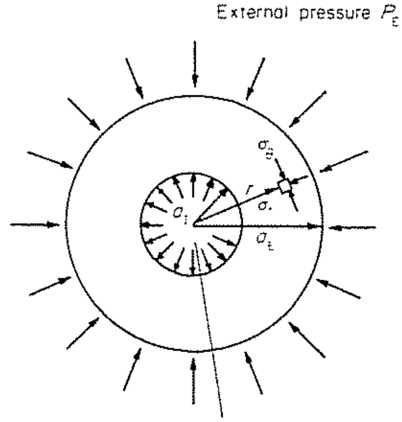

Fig. 4. Field distribution around the excavation.

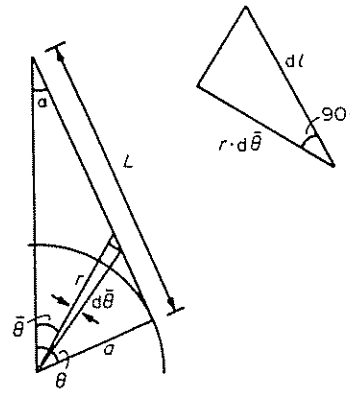

Fig. 5. Wedge geometry at a differential level.

Substitution of (7) into the expressions for radial and tagential stresses and appropriate transformation of axes may be used to give the normal stress acting at any point as,

$$\sigma_n = P\left\{1 + \frac{\sin^2(\bar{\theta}+\alpha)}{\sin^2(\theta+\alpha)}\cdot\cos 2(\bar{\theta}+\alpha)\right\}, \tag{8}$$

where P is the magnitude of the pre-excavation hydrostatic field stress. The total normal force (N_0) acting on the wedge limb may be determined as:

$$N_0 = \int_0^\theta \sigma_N\cdot \mathrm{d}l, \tag{9}$$

where l is the infinitesimal increment of wedge length as shown in Fig. 5. Therefore

$$\mathrm{d}l = \frac{r\cdot \mathrm{d}\bar{\theta}}{\sin(\bar{\theta}+\alpha)} = a\frac{\sin(\theta+\alpha)}{\sin^2(\bar{\theta}+\alpha)}\mathrm{d}\bar{\theta} \tag{10}$$

Substitution of (8) and (10) into (9), integration and rearrangement gives

$$N_0 = P\cdot a\left[\frac{\sin\theta\,\sin(\theta+2\alpha)}{\tan\alpha\,\sin(\theta+\alpha)}\right]. \tag{11}$$

Similar treatment for the shear stress gives

$$S_0 = \frac{P\cdot a}{2\sin(\theta+\alpha)}\cdot[\cos 2(\theta+\alpha) - \cos 2\alpha]. \tag{12}$$

Thus, the initial shear and normal forces acting on the rigid block may be evaluated from (11) and (12) above.

EVALUATION OF WEDGE WEIGHT (W)

From the wedge geometry given in Fig. 4, the wedge volume and hence weight may be calculated per unit length of section. The wedge weight is given by

$$W = \gamma a^2(\sin^2\theta(\cot\theta + \cot\alpha) - \theta), \tag{13}$$

where γ is the unit weight of rock.

RANGE OF SOLUTION VALIDITY

The analysis is restricted to wedges of semi-apical angle less than the combined residual friction angle and angle of dilation. At angles greater than this, wedges will be inherently unstable. No allowance is made for breaking of asperities on discontinuities, similarly it is assumed that the shear displacement is restricted to less than one half of the roughness wavelength. Clearly, however, any chosen geometric friction angle may be modified to account for breakage at an anticipated ambient stress level.

The stress analysis assumes that the medium is homogeneous, isotropic and linearly elastic. The presence of joints is assumed not to affect the initial stress field upon excavation. The effect of wedge self weight is not included within the initial stress analysis although this assumption is unlikely to yield significant error at the stress levels for which the analysis is considered valid.

No allowance for a stress free ground surface is made in the analysis. The ratio of cavity depth to radius (z/a) for which the analytical solution may be considered free from serious error in this respect is considered to be approximately 25. If the field stress intensity at any depth (z) is assumed of equivalent magnitude to the overburden loading then the dimensionless field stress ratio ($P/\gamma a$) may be considered equal to the ratio z/a. The planes comprising the wedge strike parallel to the cavity axis and are aligned parallel to the direction of the third principal field stress.

Despite these simplifying assumptions, the analysis method is believed to give useful insight into the stabilizing influence of *in situ* stress on tapered blocks.

SOLUTION PROCEDURE

The quantities involved are most conveniently dealt with in dimensionless form. The dimensionless anchor force ($A/2Pa$) required to bring any wedge to limiting equilibrium may be calculated as follows:

(i) For any given wedge geometry (α, θ), the initial normal (N_0) and shear (S_0) forces acting on the wedge may be evaluated from equations (11) and (12). The wedge weight may be evaluated from (13).

(ii) Application of a vertical force ($-A$) to yield the wedge will alter the initial forces (N_0, S_0) to (N, S). The revised normal force (N) is evaluated by substituting N_0 and S_0 into equations (6) and (4) where at yield, $F = 1$.

(iii) The yield force ($-A$) is then given by equation (1) where the factor of safety is equal to unity.

PARAMETRIC STUDIES

A number of studies are presented for wedge semi-apical angles of 10°, 20° and 30° with varying shear to normal stiffness ratios. Comparison of treatment of the problem as a rigid block failure mechanism with numerical simulation for a deformable wedge [2] indicated that reasonable agreement between the two methods existed for wedges initially close to limiting equilibrium following excavation.

The proposed solution procedure differs from that previously discussed [2] in that for semi-apical angles

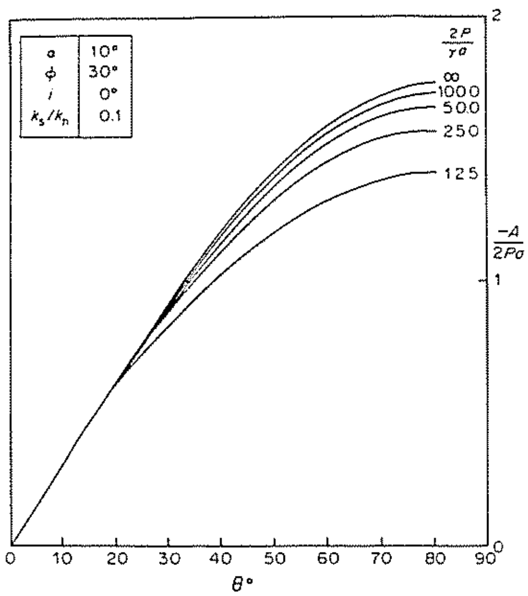

Fig. 6. Dimensionless wedge pull-out force ($-A/2Pa$) as a function of dimensionless field pressure ($2P/\gamma a$).

lower than the friction angle, the angle of intersection of the discontinuity planes with a circular excavation boundary may be less than the angle of stress obiquity required for stability. Hence, localized failure may be incurred at the excavation periphery. The assumption that the block behaves rigidly, however, negates the possibility of any relative displacements in this zone other than those compatible with the rigid body displacements.

The variation in dimensionless pull-out force ($A/2Pa$) as a function of wedge size and dimensionless field stress ($2P/\gamma a$) is shown in Fig. 6. At the ambient stress level for which the solution may be considered valid ($2P/\gamma a > 50$), the influence of wedge weight is shown to have negligible effect on the pull-out force.

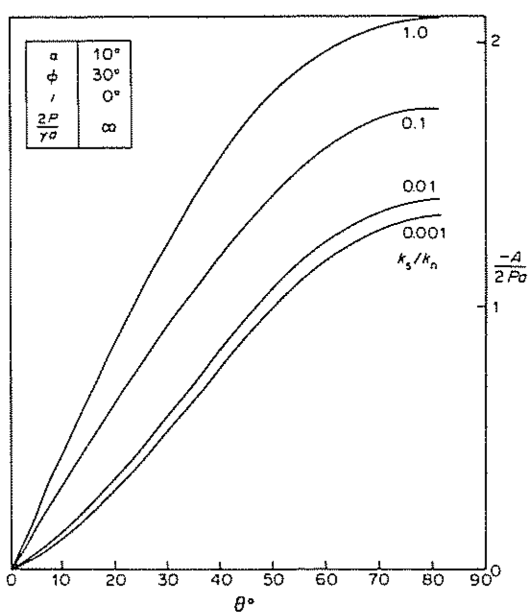

Fig. 7. Dimensionless wedge pull-out force ($-A/2Pa$) as a function of discontinuity shear stiffness to normal stiffness ratio (k_s/k_n).

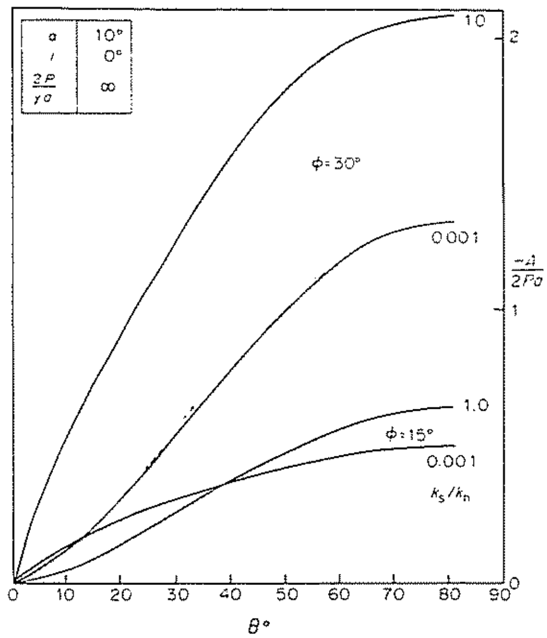

Fig. 8. Dimensionless wedge pull-out force ($-A/2Pa$) as a function of discontinuity friction angle (ϕ).

The effect of the ratio k_s/k_n to the pull-out resistance is illustrated in Fig. 7. As this ratio increases to unity, the pull-out resistance similarily increases. The influence of the k_s/k_n ratio is most pronounced for wedges bounded by discontinuities of high friction angle with respect to the apical angle. This effect is illustrated in Fig. 8 where the variation in pull-out resistance is shown for joint friction angles of 15° and 30°. The importance of joint rigidity is clearly illustrated by this figure. The force displacement characteristics of a joint may therefore assume similar importance to joint shear strength where wedge failure is controlled by transmission of external field forces to the wedges.

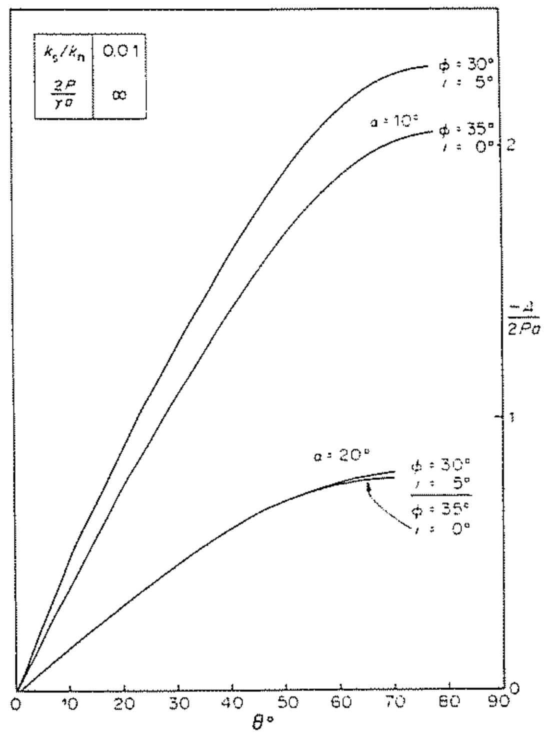

Fig. 9. Dimensionless wedge pull-out force ($-A/2Pa$) as a function of discontinuity dilation characteristics.

The role of dilation induced by shear displacements along the bounding discontinuities is illustrated in Fig. 9. The pull-out resistances of a wedge bounded by discontinuities of comparable shear strength with and without a geometric component are shown. The increase in stability afforded by the geometric component of friction is most pronounced where dilation is most severely restricted. Thus, wedges of narrow apical angle benefit most markedly from this phenomenon.

CONCLUSIONS

A simple model has been developed to evaluate the pull-out resistance of symmetric wedges adjacent to a circular cross section cavity. The requirement that the surrounding rock mass is assumed rigid and that the stress analysis procedure is only valid for high depth of cover situations limits the applicability of the analysis. Despite these shortcomings, however, the analysis is considered useful for parametric evaluation of wedge stability adjacent to underground openings.

Three main points emerge from these studies. These are as follows:

(i) For a given apical angle, an increase in wedge size (increasing θ) results in an increased pull-out force required to yield the block. Unlike the case of a wedge adjacent to a planar excavation face, doubling the wedge base linear dimension, does not double the pull-out capacity.

(ii) The effect of stress redistribution consequent of wedge displacement is shown to markedly affect the ultimate pull-out resistance of wedges. Discontinuities of high normal rigidity with respect to shear rigidity exhibit reduced pull-out resistance over wedges bounded by discontinuities of high relative shear stiffness. These load deformation properties may be as significant in evaluating the ultimate pull-out resistance of a wedge as the discontinuity friction angle.

(iii) The importance of restriction of normal displacement across discontinuities undergoing shear is highlighted. This effect is most pronounced for wedges of narrow apical angles.

Thus, it may be concluded that initial stresses contribute significantly to the stability of wedges adjacent to underground openings where the apical angles are such that field stresses may be transmitted to the isolated body. Ignoring the effect of *in situ* stresses may result in overdesign where wedges may be partially or fully self-supporting.

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