COMPUTATIONAL GEOMECHANICS (GeoEE 557)

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Content: Modeling deformation, flow and transport using finite element, finite difference, and boundary element methods. Coupled processes of flow, deformation, and heat, mass and reactive transport.

Objectives: To gain an appreciation of numerical methods through exposure to finite element, finite difference, and boundary element codes. The course is designed for potential users and developers of domain and integral numerical methods that are ubiquitously applied in hydrology and atmospheric sciences; mining, civil, mechanical and petroleum engineering; and the geologic and materials sciences.

Prerequisites: Matrix algebra, elementary calculus, elementary programming in MatLab

Location: T-Th 4:40-5:55; 218 Hosler (3 credits)

1. Overview of Coupled Processes
   1.1. Uncoupled systems
   1.2. Coupled processes – Thermo-Hydro-Mechanical-Chemical (T-H-M-C)
   1.3. Balances of Mass, Momentum and Energy
   1.4. Interaction and Onsager Processes
   1.5. Building representative systems – incorporating couplings
   1.6. Preliminary examples of THM processes
   1.7. Key couplings
   1.8. Dependent variables – their selection and modes of coupling

2. Mechanical Behavior (M) Deformation
   2.1. Conservation of momentum and Hooke’s law
       2.1.1. Principle of virtual work
   2.2. Finite element representation
       2.2.1. 1-dimensional elements
       2.2.2. 2-dimensional behavior
           2.2.2.1. CST elements
           2.2.2.2. Isoparametric elements
   2.3. Variational procedures

3. Hydraulic Behavior (H) Flow
   3.1. Conservation of mass and Darcy’s law
   3.2. Steady behavior
       3.2.1. 1-dimensional elements
       3.2.2. 2-dimensional behavior – 2-D triangular, and 2-D isoparametric elements
   3.3. Transient behavior
       3.3.1. Time stepping methods
   3.4. Dual porosity flows

4. Hydro-Mechanical Coupling (H-M)
   4.1. Stiffness and conductance terms
   4.2. Coupling terms and symmetry
   4.3. Time stepping
4.4. 1-dimensional examples
4.5. Dual porosity behavior

5. Mass (Chemical) Transport (C)  
   5.1. Conservation of mass and Fick’s law  
   5.2. Steady behavior  
   5.3. Transient behavior  
   5.4. Considerations of local equilibrium

6. Hydro-Mechanical-Chemical Coupling (H-M-C)  
   6.1. Stiffness, conductance and advective terms  
   6.2. Coupling terms and symmetries

7. Thermal Transport (T)  
   7.1. Conservation of Energy and Fourier’s law  
   7.2. Steady behavior  
   7.3. Transient behavior

8. Thermo-Hydro-Mechanical Coupling (T-H-M)  
   8.1. Stiffness, conductance and advective terms  
   8.2. Coupling terms and symmetry  
   8.3. Time stepping  
   8.4. 1-dimensional examples

9. Thermo-Hydro-Mechanical-Chemical Coupling (T-H-M-C)  
   9.1. Stiffness, conductance and advective terms  
   9.2. Coupling terms and symmetries

10. Summary Behaviors

11. Boundary Element Methods – Introduction  
   11.1. Indirect method – General principles  
       11.1.1. Groundwater mechanics  
       11.1.2. Elasticity  
   11.2. Direct Method – General principles  
       11.2.1. Groundwater mechanics  
       11.2.2. Elasticity  
   11.3. Coupled FEM-BEM analysis

12. Overview and Summary

   Selected texts on reserve in the EMS library.
   Lecture Overheads and Course Notes available online.

   Grading:  
   20% Participation  
   80% Final project
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<td>1. Introduction &amp; 6. References</td>
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<td>2.1 Governing Equations</td>
<td>Define conservation equations, constitutive relations, and boundary and initial conditions</td>
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<td>2.3 Solution</td>
<td>Solve using model</td>
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<td>Complete paper and present results</td>
<td>10 minute presentation with one slide for each of 1; 2.1; 2.2; 2.3; 3; 4; 5 (seven slides)</td>
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