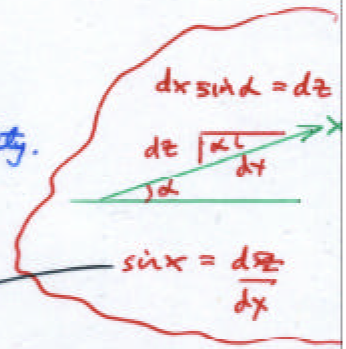
3.5 BUCKLEY – LEVERETT (1942) EQUATIONS



Assume: Neglect gravity, capillarity, liquid compressibility.

Homogeneous reservoir of thickness, b, inclined @ α°.

$$q_w = -(k_w/\mu_w)(\partial p_w/\partial x + \rho_w g \sin\alpha)$$

$$q_{nw} = -(k_{nw}/\mu_{nw})(\partial p_{nw}/\partial x + \rho_{nw} g \sin\alpha)$$

Relative permeability and Darcy's Law.

$$n\,\partial S_w/\partial t + \partial q_w/\partial x = 0 \qquad (1)$$

$$n\,\partial S_{nw}/\partial t + \partial q_{nw}/\partial x = 0 \qquad (2)$$

$$S_w + S_{nw} = 1 \qquad (3)$$

$$p_c = p_{nw} - p_w \qquad (4)$$

4 equns for 4 unknowns.

Initial and boundary conditions

Irreducible wetting saturation

$t \leq 0 \qquad 0 \leq x \leq L \qquad S_{nw} = 1 - S_{w_0}$

$t > 0 \qquad x = 0 \qquad q_w = \text{constant}; \quad q_{nw} = 0$

Wetting fluid injected to displace non-wetting fluid (oil) at irreducible wetting saturation $S_{nw} = 1 - S_{w_0}$

Injected @ constant rate at $x = 0$.

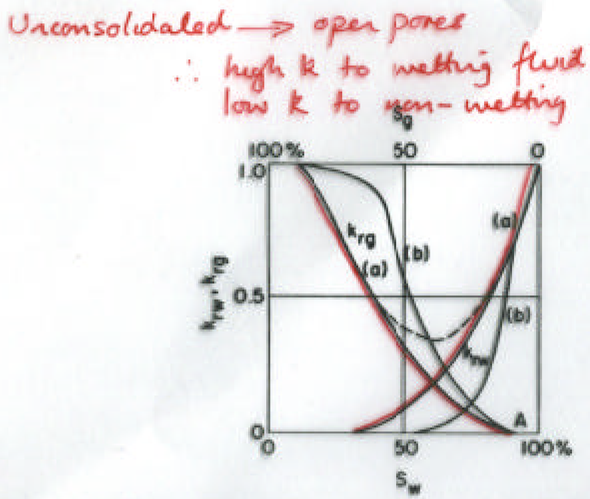


FIG. 9.3.2. Typical relative permeability to gas and water. (a) Unconsolidated sand. (b) Consolidated sand (Botset, 1940).

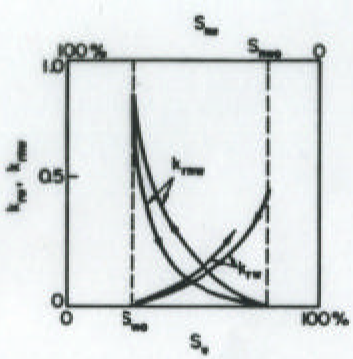


FIG. 9.3.3. Effect of hysteresis on relative permeability.

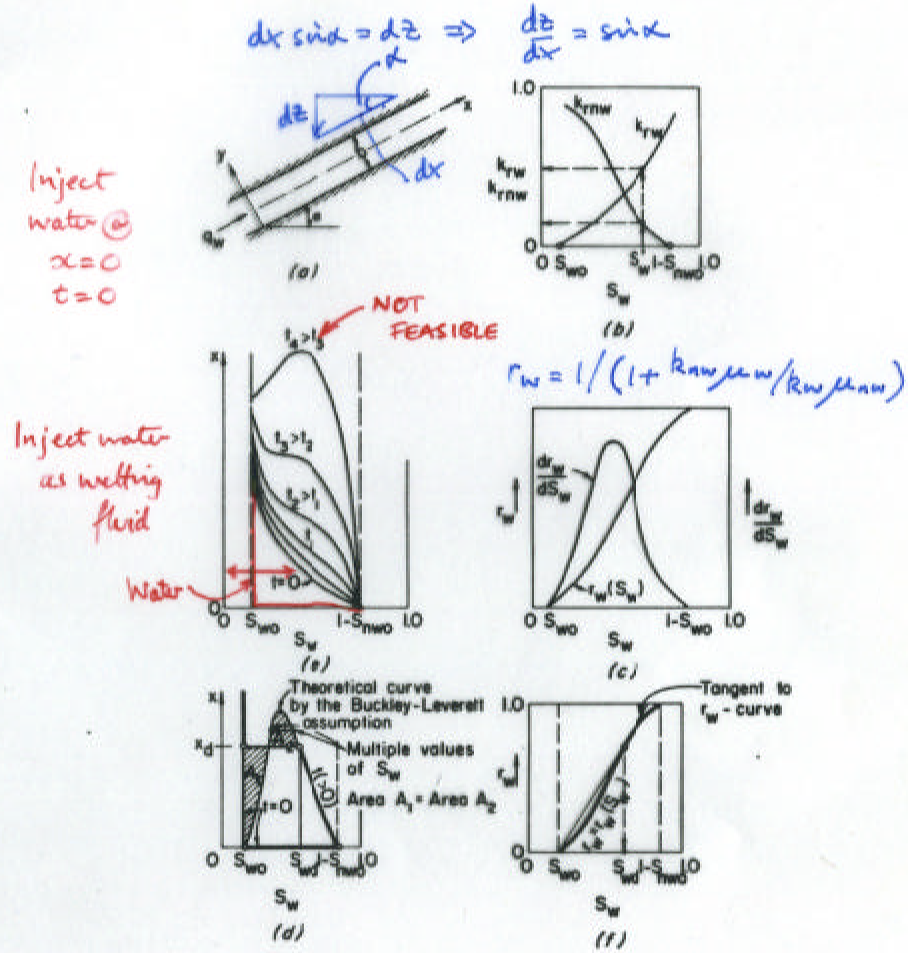


FIG. 9.3.4. The Buckley-Leverett solution for a linear immiscible displacement.

B-L Equations (cont'd)

Enables evaluation of front movement, see figure 9.3.4.(e) $M = \frac{V_w}{V_{nw}}$

$$r_w = 1/(1 + k_{nw}\mu_w / k_w \mu_{nw}) = [1/(1 + 1/\text{Mobility ratio})]$$

Note since capillarity neglected @ t_4, two saturations exist.

∴ only applicable to high flow rates

– where capillary effects are masked

$$M = \frac{k_w \mu_{nw}}{k_{nw} \mu_w} = \frac{V_w}{V_{nw}}$$

General Comments

① Techniques developed for petroleum industry. Difficult interpretation for saturations less than irreducible saturation. This is perhaps most important region for groundwater contamination.

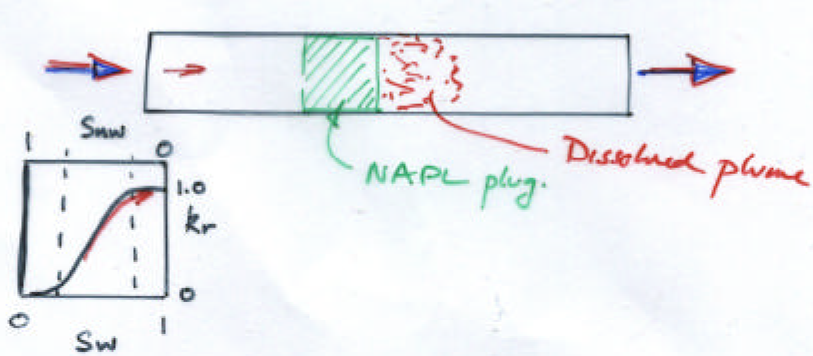
□ Need to incorporate:

Dissolution effects.

② For pump-and-treat need to understand two effects

a) Role of k_r in controlling water flow rates through NAPL plug. Plug does not move since funicular water saturation

b) Role of dissolution as S_{nw} ↓



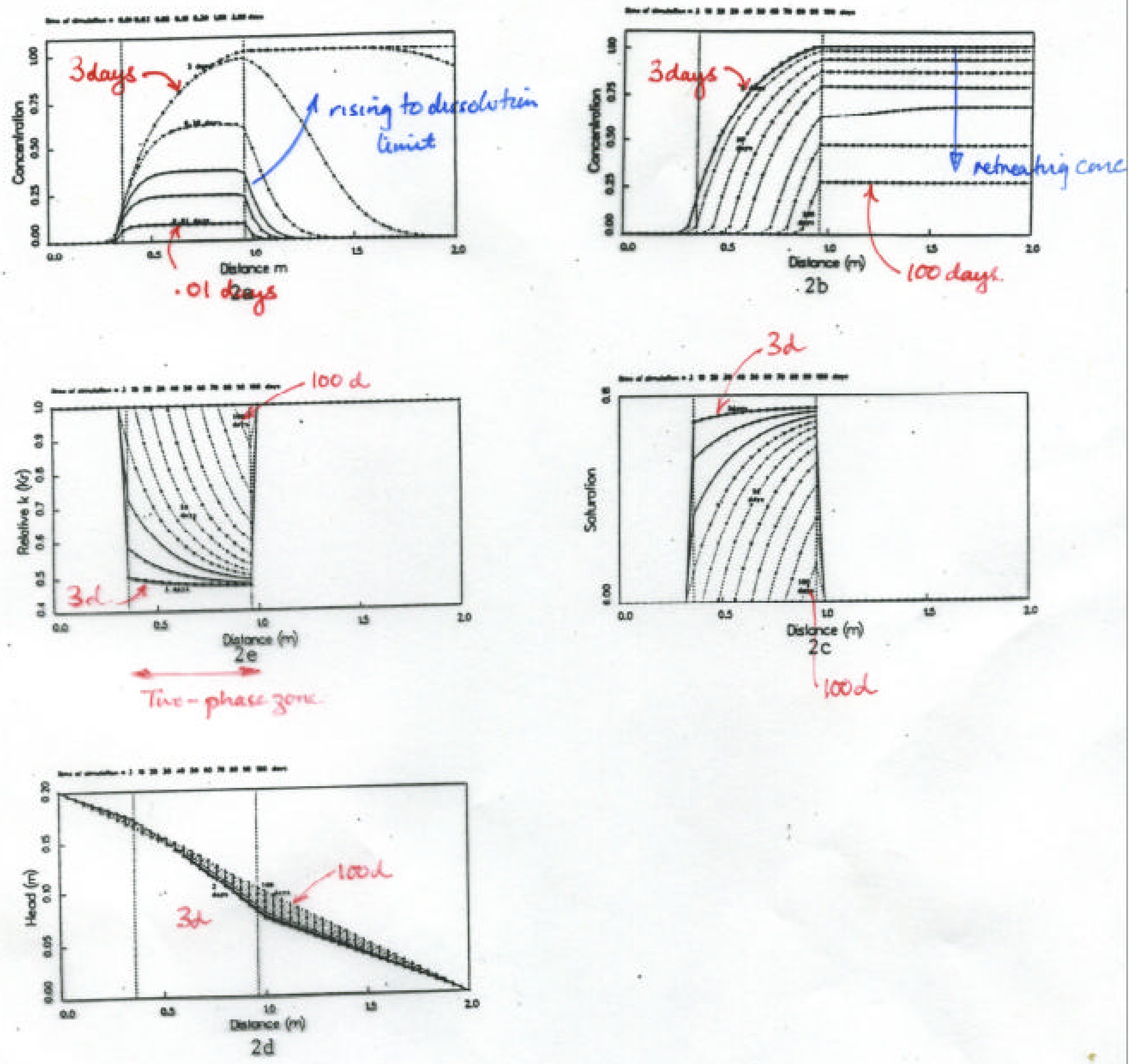
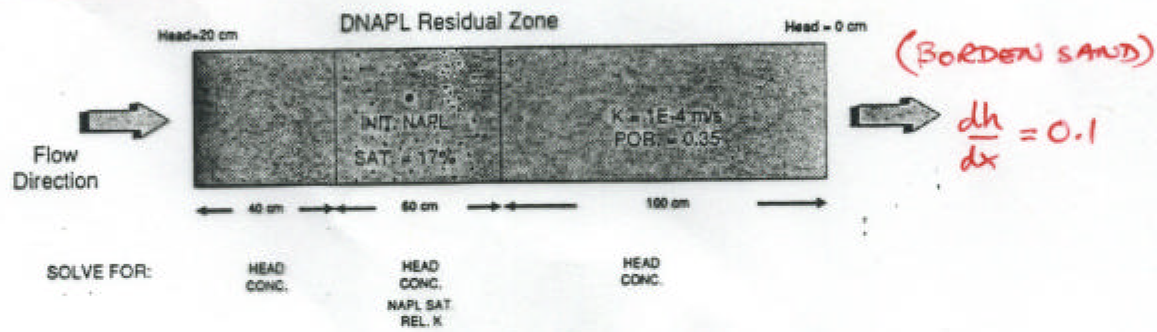
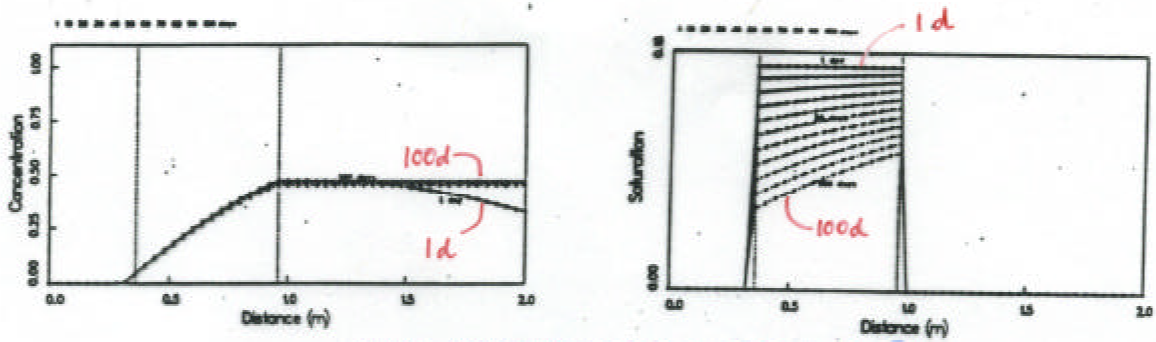


Fig. 2 - Results with Mass Transfer Coefficient = 1x10-4 1/s (a)Conc. at early times (b)Conc. at late times (c)DNAPL Saturation (d)Head distribution (e)Relative permeability.

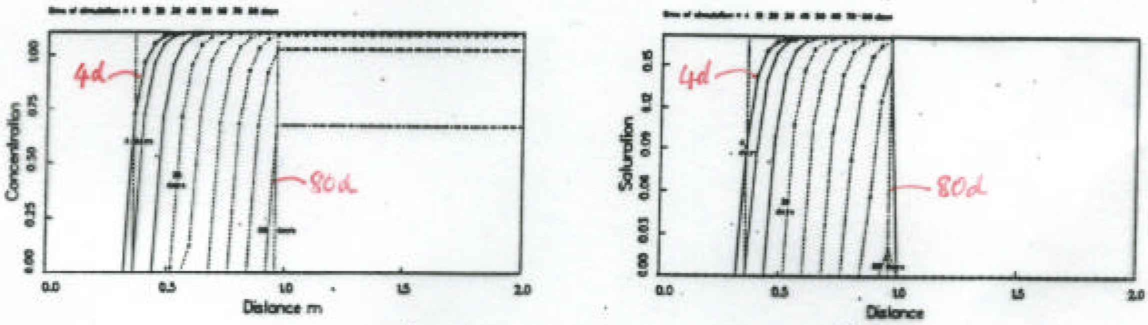
COLUMN EXPERIMENT





SMALL MASS TRANSFER COEFF. 1×10^{-5} 1/s

Fig. 3 - Results with Mass Transfer Coefficient = $1x10^{-5}$ $1/_s$



LARGE ~~SMALL~~ MASS TRANSFER COEFF. 1×10^{-3}

Fig. 4 - Results with Mass Transfer Coefficient = $2x10^{-3}$ $1/_s$

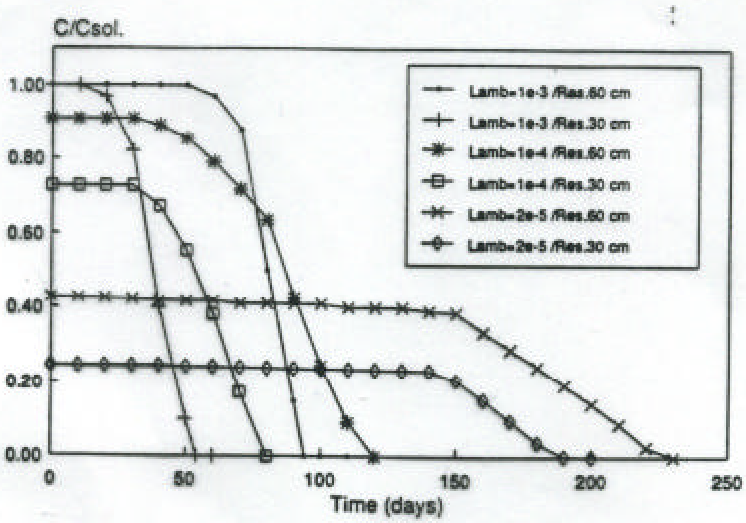


Fig. 5 - Concentration observed at the end of the residual saturation zone for different Mass Transfer Coefficients and lengths of the residual saturation zones