3.3 Mass Conservation in Multiphase Flow

Continuity equation: $$\frac{\partial}{\partial t}(n S_\alpha \rho_\alpha) + \frac{\partial}{\partial x_i}(\rho_\alpha q_\alpha) = 0 \qquad \alpha = 1, 2 \text{ or } \alpha = 1, 2, 3.$$

For an incompressible fluid and medium $\frac{\partial}{\partial t}(n \text{ and } \rho) = 0$

Substitute q_α from relative permeability relation:
Results in 4 equations:

$$n \frac{\partial S_1}{\partial t} - \frac{\partial}{\partial x_i}\left[k \frac{k_{r1}}{\mu_1}\left(\frac{\partial p_1}{\partial x_j} + \rho_1 g \frac{\partial z}{\partial x_j}\right)\right] = 0$$

$$n \frac{\partial S_2}{\partial t} - \frac{\partial}{\partial x_i}\left[k \frac{k_{r2}}{\mu_2}\left(\frac{\partial p_2}{\partial x_j} + \rho_2 g \frac{\partial z}{\partial x_j}\right)\right] = 0$$

$$S_1 + S_2 = 1$$

$$p_2 - p_1 = p_c(S_1)$$

Solve for 4 unknowns: S_1, S_2, p_1, p_2

with $h_1 = z + \frac{p_1}{\rho_1 g}$; $h_2 = z + \frac{p_2}{\rho_2 g}$

Solve using numerical techniques.