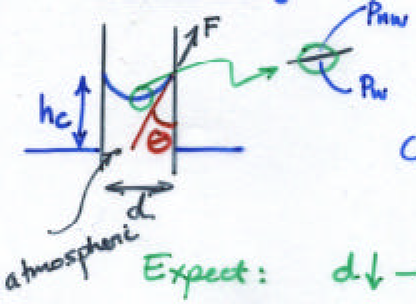
2.3 Capillary Pressure

Important: Controls penetration of immiscible fluids

Water-wet system



Note (a) $p_x = p_y = p_z$ within any phase for static conditions

(b) p_w depends on surface curvature

Capillary rise, h_c: $\gamma h_c \pi \frac{d^2}{4} = \sigma_{12} \pi d \cancelto{1}{\cos\theta}$ $\therefore h_c = \frac{4\sigma_{12}}{d\gamma_w}$

Expect: $d\downarrow \rightarrow h_c\uparrow$ $\qquad$ d = pore throat or fracture aperture

---

Capillary pressure, p_c: $\qquad p_c = p_{nw} - p_w \quad \Rightarrow \quad \frac{p_c}{\gamma_w} = \frac{1}{\gamma_w}(p_{nw} - p_w) = h$

In the capillary pressure relationship, $h_c = \frac{4\sigma_{12}}{d\gamma_w}$ the assumption that $\theta \rightarrow 0$ is made for clean glass. Not good for soils/rocks

Most important deduction: $\qquad p_c = h_c \gamma_w \propto \frac{\sigma_{12}}{d}$

Capillary pressure inversely proportional to pore dia, d.
Smaller d, requires larger p_c to penetrate.

Since many potential pore throat diameters exist, a capillary model may be replaced by a grain-grain contact model.

CAPILLARY PRESSURES - POROUS MEDIA

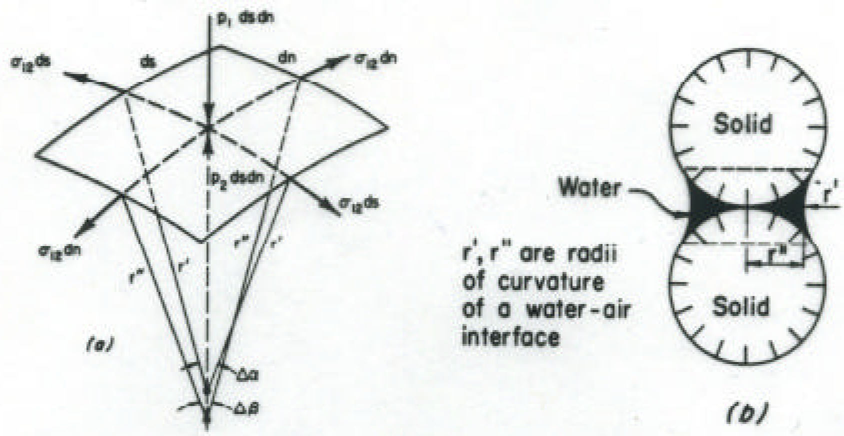


FIG. 9.2.4. Equilibrium at a curved interface between two immiscible fluids.

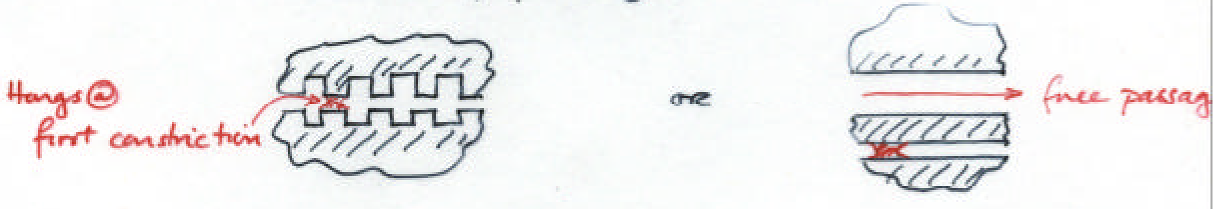
Soil suction or tensi

$$\Delta p = p_c = p_2 - p_1 = \sigma_{12}\left(\frac{1}{r'} + \frac{1}{r''}\right) = \boxed{\frac{2\sigma_{12}}{r^*} = p_c}$$

r^* = mean radius => $\frac{2}{r^*} = \left(\frac{1}{r'} + \frac{1}{r''}\right)$ (Laplace eqn.).

PROBLEM: r^* is difficult to determine

- Multiple grain sizes (and pore throat sizes)
- Distribution of pore sizes



- Pore geometry
- Fluids (σ_{12}) and contact angles ($\cos\Theta$).

$\therefore$ Use a statistical average => Determine $p_c = p(S_w)$ Lab ok

Field be

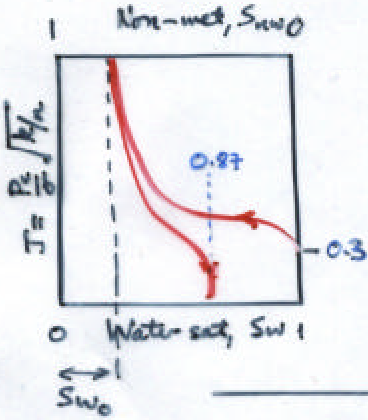
HOW TO DETERMINE $P_c = P_c(S_w)$

For capillary tube of radius, r.

$$p_c = \frac{2\sigma_{12}}{r} \cos\theta$$

Semi-empirical approach, Leverett (1941). Dimensional analysis gives:

$$J = J(S_w) = \left(\frac{P_c}{\sigma}\right)\sqrt{k/n} \quad ; \qquad P_c = P_c(S_w)$$

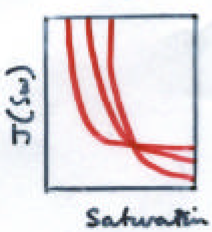


J = Leverett function reduces to a common curve for different materials

k = permeability (L^2)
n = porosity

$\sqrt{k/n}$ $\propto$ to mean pore diameter

May also be influenced by $\cos\theta$ (factors influencing contact angle).



$$J = J(S_w) = \left(\frac{P_c}{\sigma\cos\theta}\right)\sqrt{k/n}$$

∴ Dependent on formation type

Brooks & Corey (1964)

$$P_c = P_c(S_e) \qquad S_e = \frac{(S_w - S_{wo})}{(1 - S_{wo})}$$

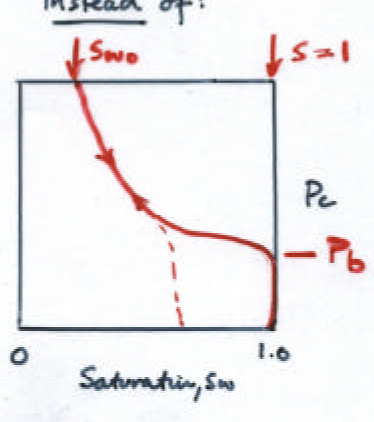
S_e = effective saturation

S_{wo} = irreducible wetting fluid saturation

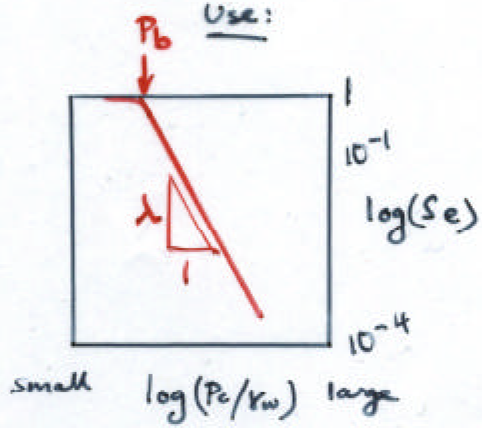
Gives a straight line relationship in log-log S_e versus P/γ_w space except close to $S_e = 100\%$

BROOKS – COREY CURVES

Instead of:



Use:



$$S_e = \frac{(S_w - S_{wo})}{(1 - S_{wo})}$$

$S=1$

S_e = effective saturation

S_{w_o} = irreducible wetting fluid saturation

Curve defined by two parameters:

λ = –ve slope of curve (pore size distribution)

P_b = intercept of line and $S_e = 100\%$

also termed "bubbling pressure"

Pressure needed to force a "bubble" of fluid through the pore throat

REPRESENTED AS:

$$S = (1-S_{wo})\left(\frac{P_c}{P_b}\right)^{-\lambda} + S_{wo}$$

or

$$S_e = \frac{(S-S_{wo})}{(1-S_{wo})} = \left(\frac{P_c}{P_b}\right)^{-\lambda}$$

i.e. $\log(S_e) = -\lambda \log\left(\frac{P_c}{P_b}\right)$

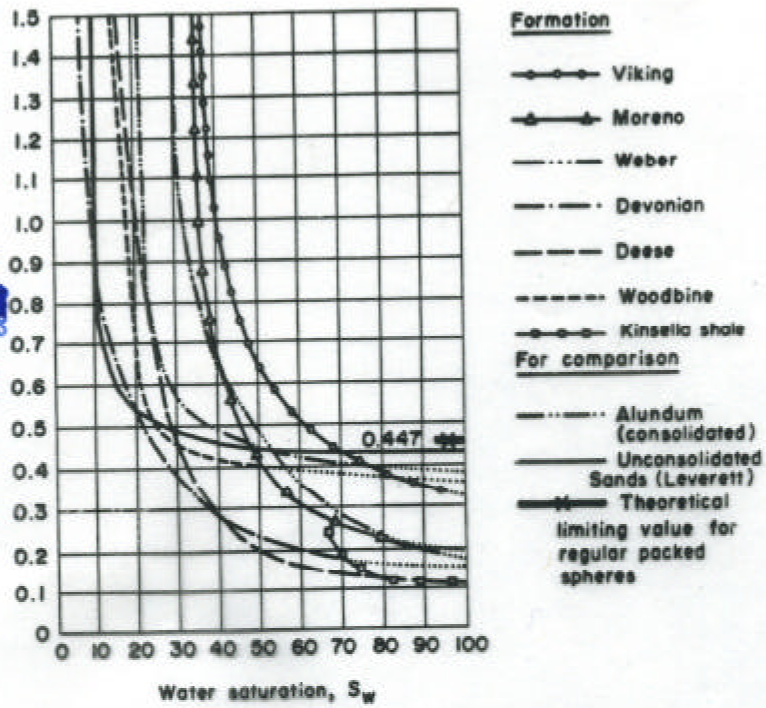


Fig. 9.2.6. Leverett function for various formations (Rose and Bruce, 1949).

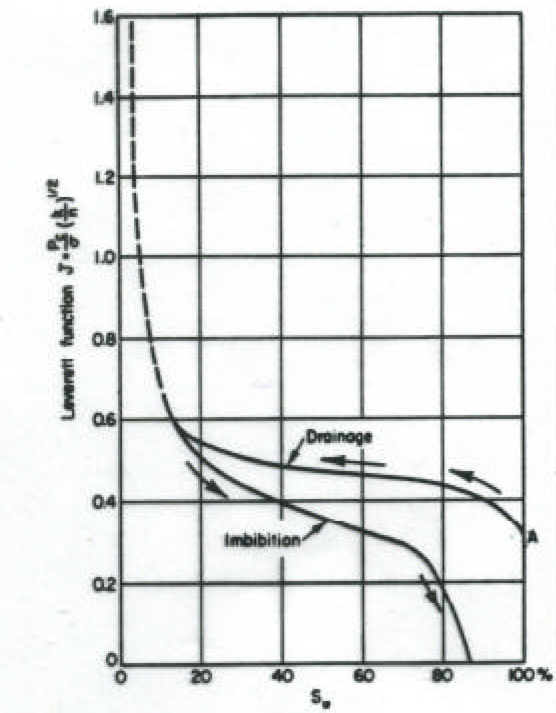


Fig. 9.2.5. Typical Leverett functions for sand (Leverett, 1941

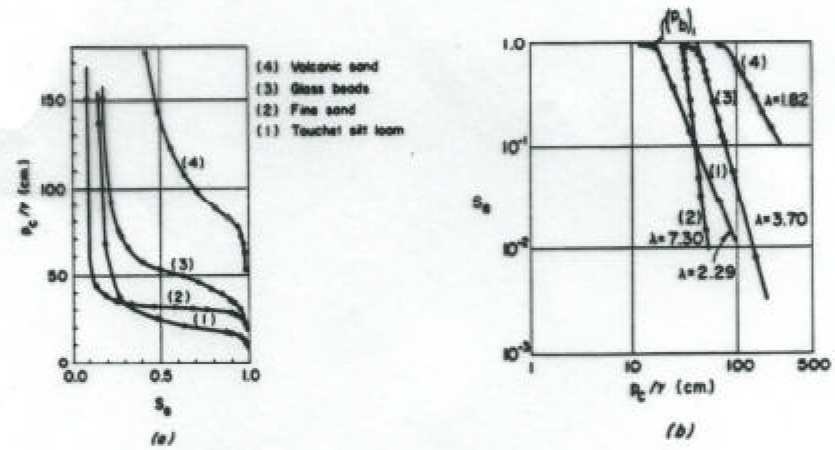
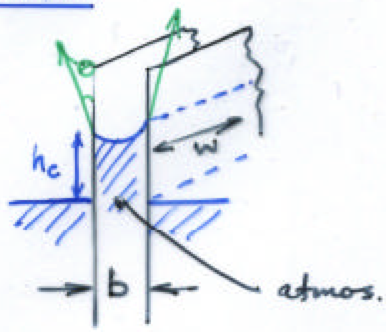


Fig. 9.2.7. Capillary pressure head as a function of effective saturation for porous materials of various pore-size distributions (Brooks and Corey, 1964).

CAPILLARY RISE IN FRACTURES

IDEALIZED



$$\underbrace{w \cdot b \cdot h_c}\,\gamma_w = 2 w \sigma \cancel{\cos\theta}^{\,1}$$

$$h_c = \frac{2\sigma}{\gamma_w b}$$

$$P_{c_0} = \frac{2\sigma}{b} \qquad P_{c_0} \propto \frac{\sigma}{b}$$

REAL $P_c = f(S_w)$

$\sigma \simeq 7 \times 10^{-2}$ N/m.