Flow & Transport Equations

Multi dimensional flow and Transport equations all drop to a common form:

Flow: $$S_s \frac{\partial h}{\partial t} = K \frac{\partial^2 h}{\partial x^2}$$

Transport $$R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - v_x \frac{\partial c}{\partial x}$$

$$R \frac{\partial \psi}{\partial t} = D \frac{\partial^2 \psi}{\partial x^2} - V \frac{\partial \psi}{\partial x}$$

System	Dependent variable, ψ	R $\frac{\partial \psi}{\partial t}$	D $\partial^2\psi / \partial x^2$	V $\partial \psi / \partial x$
Flow	h	S_s	K	0
Aqueous transport	C_w	$\left(1 + \frac{\rho_b}{\theta} K_d\right)$	$D_w^* + \alpha_L \lvert V \rvert$	V_w^a (advective)
gas diffusion	C_g	$1 + \frac{\theta_w}{\theta_g}\frac{1}{H} + \frac{\rho_b}{\theta_g}\frac{K_d}{H}$	D_g^*	0
gas diffusion + aqueous transport	C_g	"	"	$\frac{\theta_w}{\theta_g} \cdot \frac{1}{H} V_w^a$

Correspondingly – any model that solves the "master" equation will represent all of these systems.

General Equation form:

$$R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} + D \frac{\partial^2 c}{\partial y^2} - v_x \frac{\partial c}{\partial x} - v_y \frac{\partial c}{\partial y}$$

Transforms to:

$$\underbrace{R \int_V \underline{b}^T \underline{b} \, dV}_{\underline{S}} \, \dot{\underline{c}} + \underbrace{\int_V \underline{a}^T \underline{D} \, \underline{a} \, dV}_{\underline{K}_1} \, \underline{c} + \underbrace{\int_V \underline{b}^T \underline{v} \, \underline{a} \, dV}_{\underline{K}_2} \, \underline{c} = \underline{q}$$

Where: $\underline{b}$ = a matrix of shape functions $b = [b_1, b_2 \ldots b_n]$

$\underline{a} = \begin{Bmatrix} \partial/\partial x \\ \partial/\partial y \end{Bmatrix} \underline{b}$ derivative of shape functions

$\underline{D}$ = Dispersion tensor or hydraulic conductivity

$\underline{c}$ = Concentration at nodal points

$\dot{\underline{c}} = \frac{\partial}{\partial t} \underline{c}$

$\underline{v}$ = vector of advective velocities, $\underline{v} = [v_x, v_y]$

q = Fluxes at prescribed boundaries (mass fluxes).

FINAL FORM

$$\underline{S} \, \dot{\underline{c}} + [\underline{K}_1 + \underline{K}_2] \, \underline{c} = q$$

Steady: $[K_1 + K_2] \, \underline{c} = \underline{q}$ Solve for $\underline{c}$ as linear system of equations.

MATHEMATICAL MODELS

- Flow
- Transport.

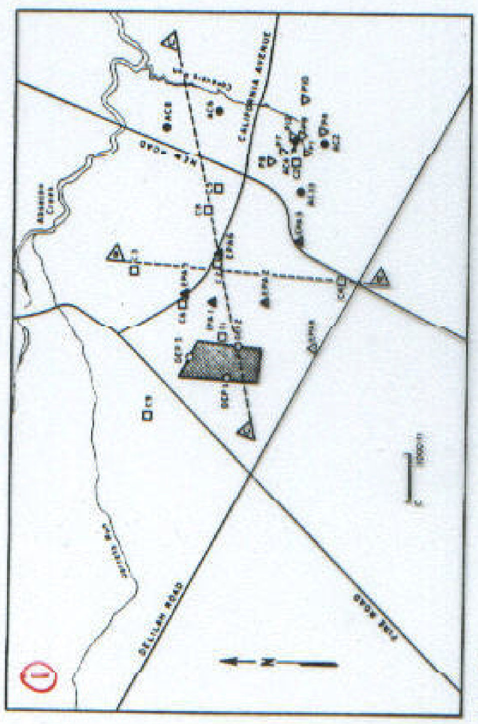


Figure 5.1.
Vicinity of Price Landfill (shaded region) with wells screened in upper zone of the Cohansey indicated (after Gray and Hoffman, 1983).

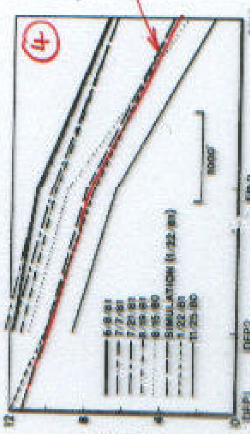


Figure 5.6.
Measured and simulated head gradient along section AA' of Figure 5.5 (after Gray and Hoffman, 1983).

Simulation fitted to data — ∴ K distribution.

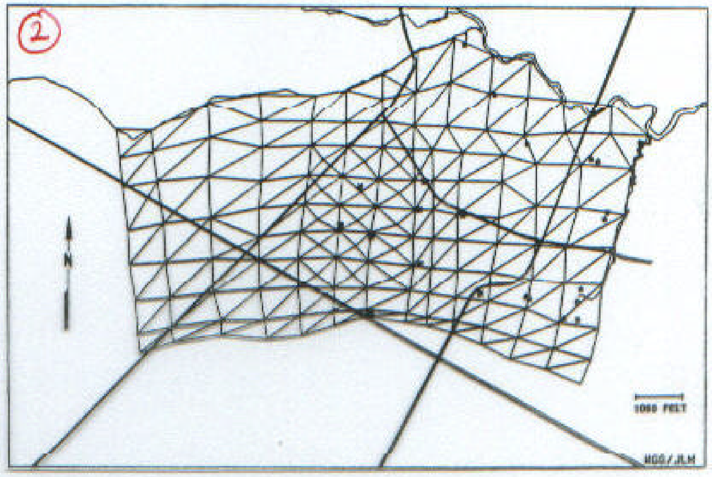


Figure 5.7.
Triangular finite element grid used in numerical simulation study (after Gray and Hoffman, 1983).

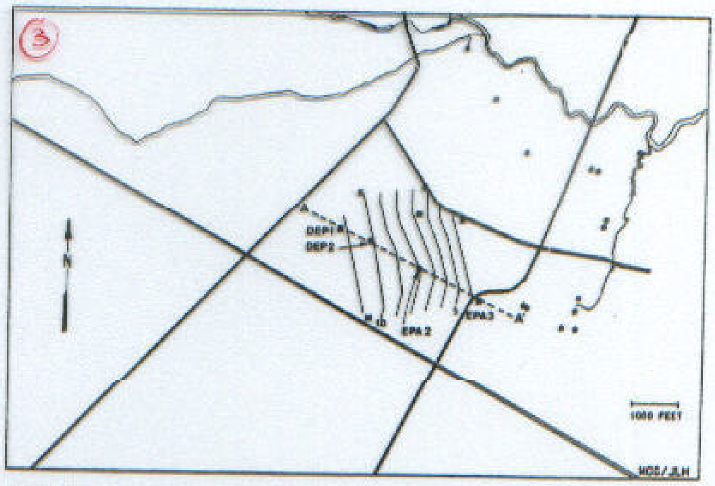


Figure 5.5.
Linearly interpolated water table contours for January 22, 1981 (elevations in feet above MSL) (after Gray and Hoffman, 1983).

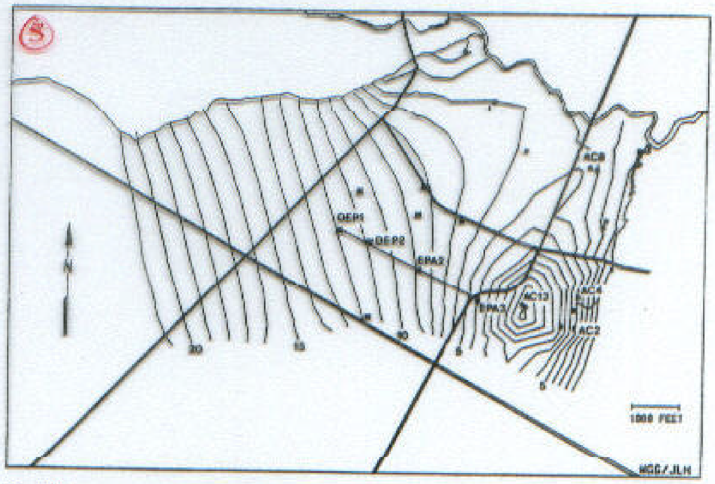


Figure 5.9.
Simulation of water table in the Upper Cohansey using average pumping rates of last ten years (after Gray and Hoffman, 1983).

Concentrations after pumping – No remedial measures

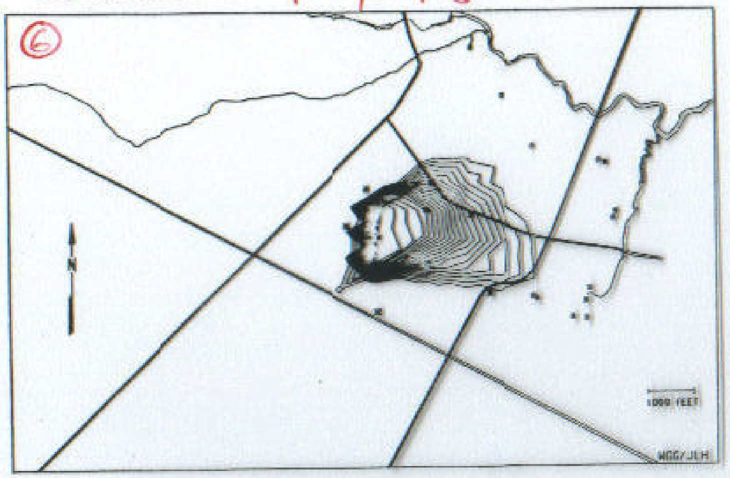


Figure 5.11.
Contaminant distribution after ten years using the head distribution of Figure 5.9 (after Gray and Hoffman, 1983).

Concentrations after 10 yr

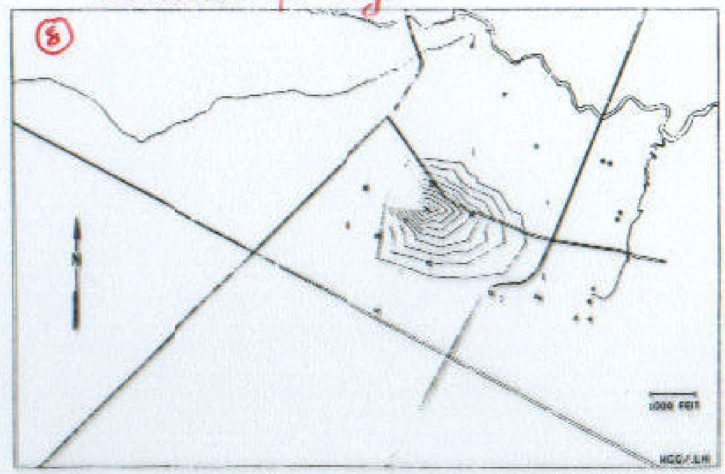


Figure 5.16.
Concentration distribution for RS2 after ten years (after Gray and Hoffman, 1983).

Revised water table contours – interceptor wells

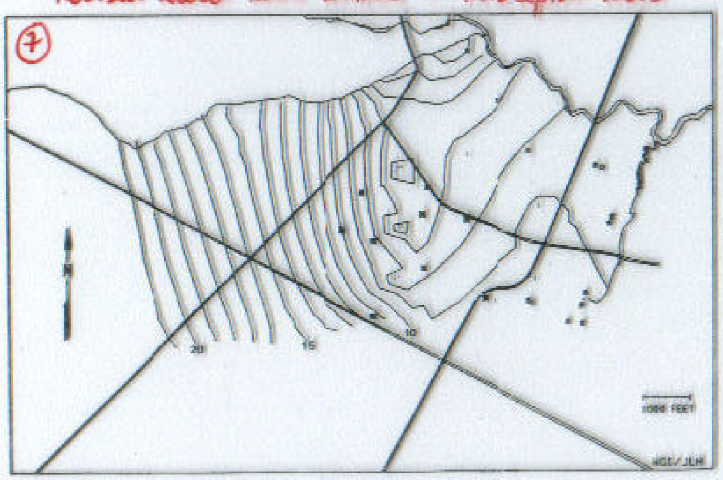


Figure 5.14.
Water-table contours for RS2 pumping 10 mgm from each well at "P" and injecting 2 mgm into each well at "I" (after Gray and Hoffman, 1983).

Concentrations after 25 yr.

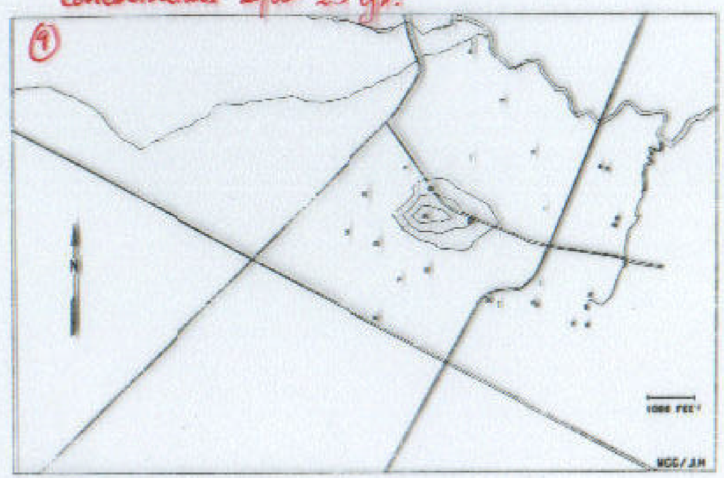
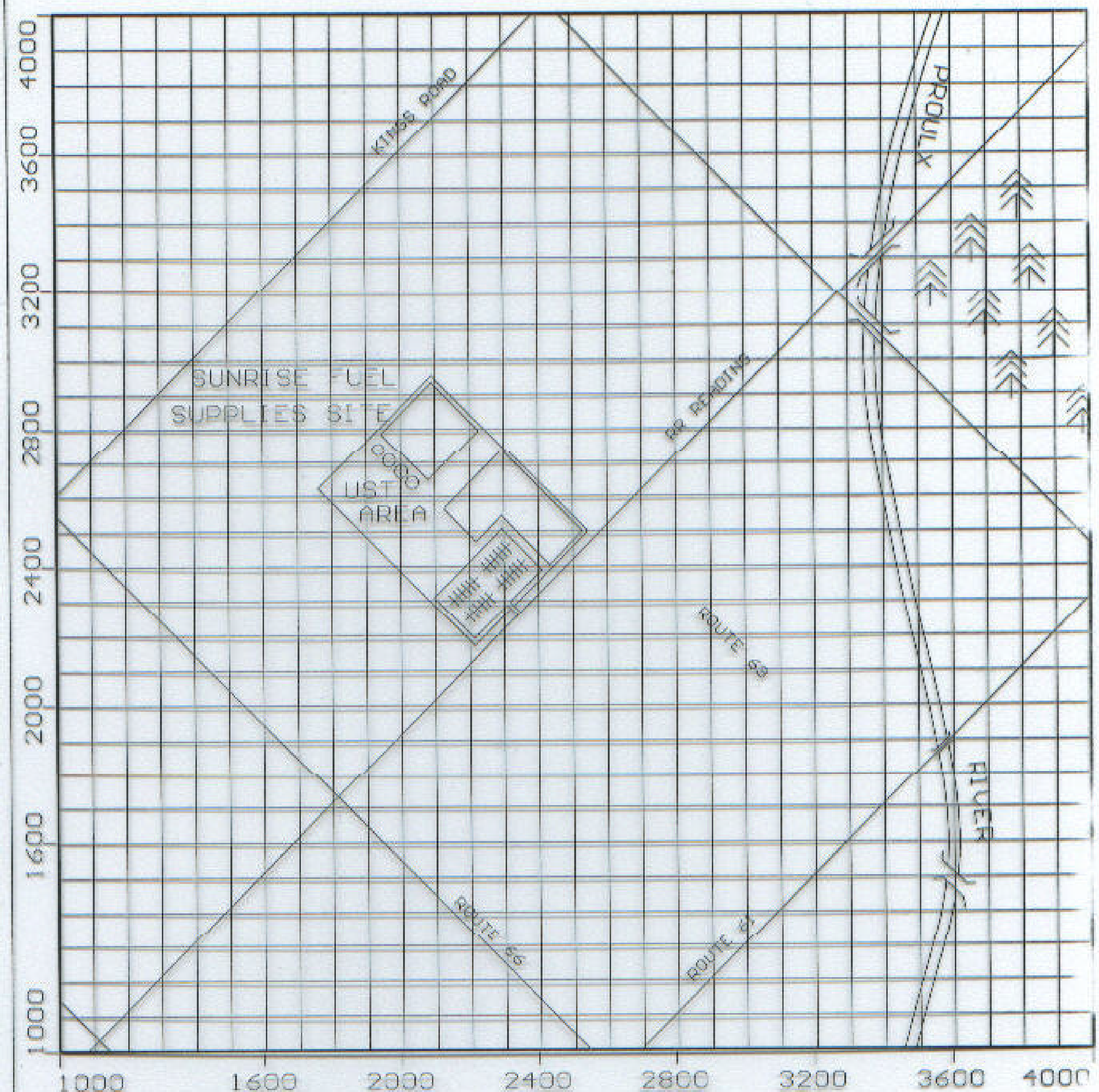


Figure 5.17.
Concentration distribution for RS2 after 25 years (after Gray and Hoffman, 1983).



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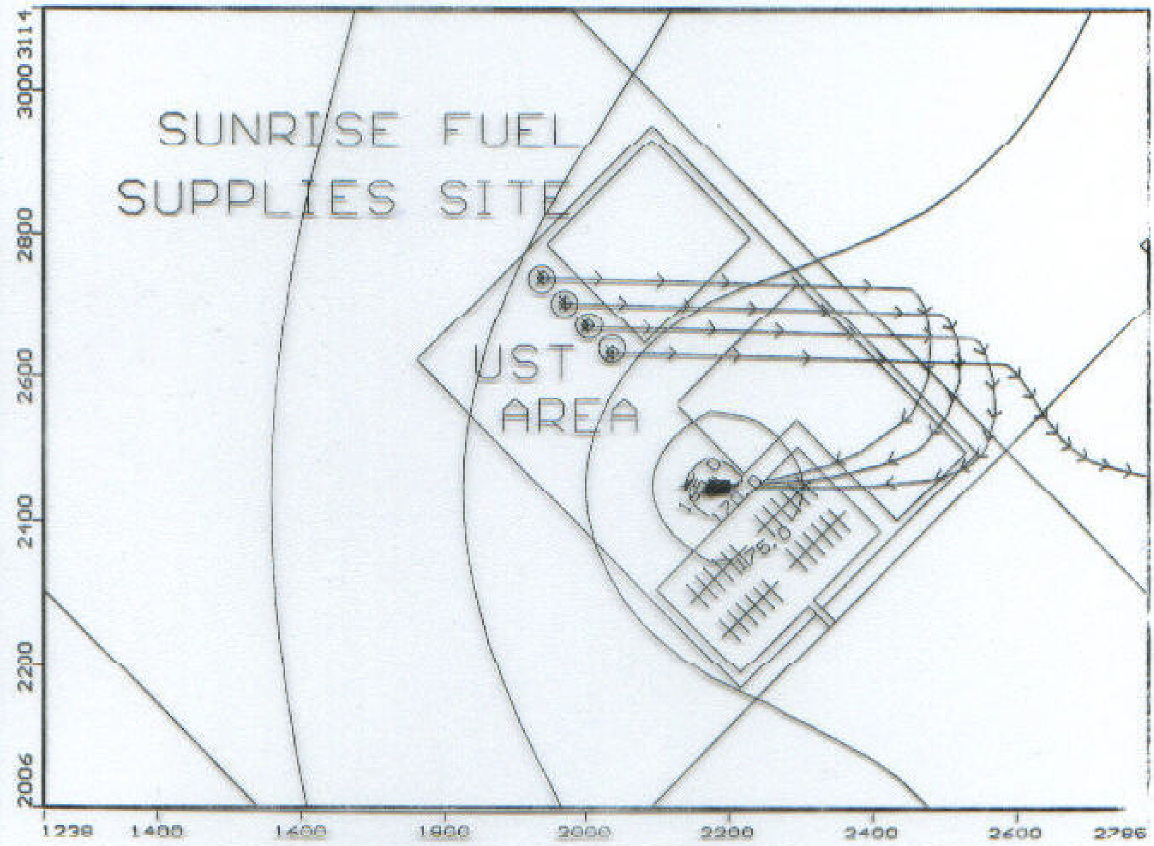

26 Mar 99

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