

### 7.1.1 Diffusive Transport

Fick's first law  $q_D = \Theta_g D_g^* \frac{\partial c_g}{\partial x}$

$q_D$  = diffusive mass flux

$c_g$  = concentration of VOC  
in gas phase

$\Theta_g$  = volumetric gas content

$D_g^*$  = "Effective" diffusion coefficient of gas.

### Fick's second law

Mass continuity/Conservation of mass equation.

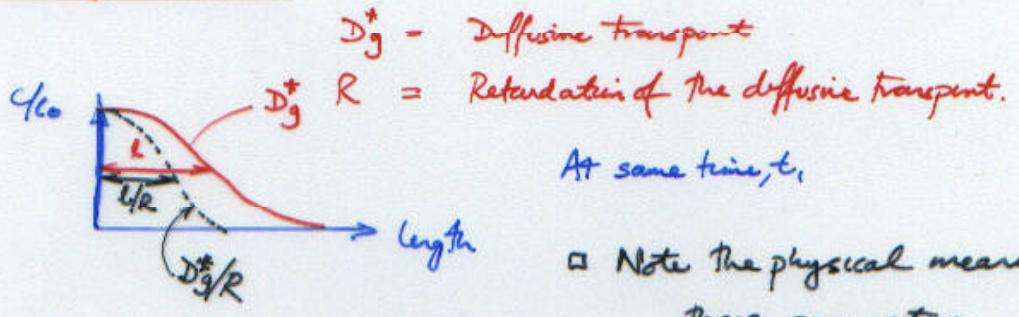
$$R \frac{\partial c_g}{\partial t} = D_g^* \frac{\partial^2 c_g}{\partial x^2}$$

$R$  = Retardation factor  
- accounts for sorption  
between phases

This ignores:

1. Advection of vapors due to pumping (vacuum stripping)
2. Advection due to density/temperature effects
3. Infiltration due to rain/etc.

## Controlling Parameters



## "Effective" Diffusion Coefficient, $D_g^*$

$$D_g^* = \tau D_g$$

$\tau$  = tortuosity

$D_g$  = "Free" diffusion coefficient ( $L^2/T$ )

### Tortuosity

#### 1. Experimental Measurement



$$\tau = \left(\frac{L}{h}\right)^2$$

#### 2. Empirical estimates

$$\text{eg. } \tau = \frac{\Theta_g^{2/3}}{\Theta_t^{1/3}}$$

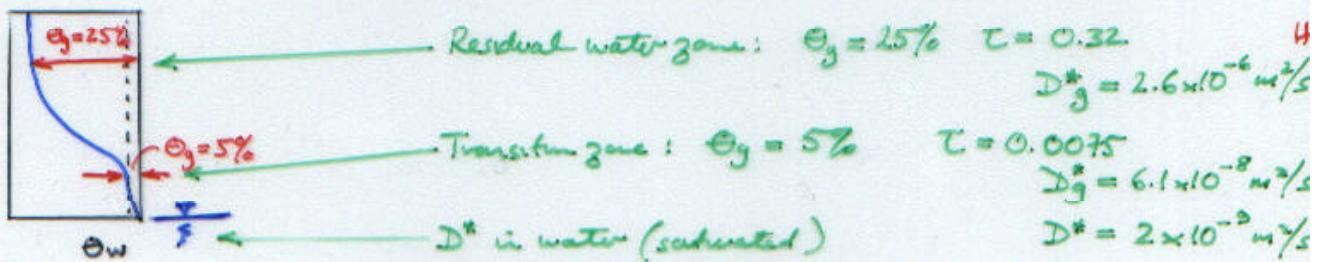
(Millington and Quirk, 1961)

Total porosity (i.e. Saturated)

## Estimates of $D_g^*$

TCE @ 20°C       $D_g = 8.1 \times 10^{-6} \text{ m}^2/\text{s}$  (typical for chlorinated solvent).

Assume  $\Theta_g = 35\%$



## PARTITIONING BEHAVIOR

Retention results from two mechanisms

$$\left\{ \begin{array}{l} \text{Gas} \rightarrow \text{Water partitioning} \quad ① \\ \text{Water} \rightarrow \text{Solid partitioning} \quad ② \end{array} \right.$$

### ① Gaseous/Aqueous Partitioning

Described by Henry's Law.

$$H = \frac{C_g}{C_w}$$

$C_w$  = water concentration (in same units as  $C_g$ )

$H$  = "Dimensionless" Henry law coeff.

Alternate (usual) form of Henry's Law

$$H' = \frac{C_g'}{C_w'} = \frac{P_v}{S} \quad \text{i.e. } P = H'C_w$$

$P_v$  = vapor pressure

$S$  = saturation of "solute" in water. Max value = solubility

$H'$  units of atm.  $\cdot$   $m^3/mol$

### ② Aqueous/Solid Partitioning

- Identical to saturated zone.

i.e.

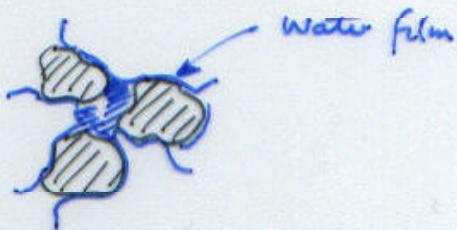
$$K_D = K_{ow} \cdot f_{oc}$$

## RETARDATION FACTOR

Assume equilibrium partitioning between phases { water  
solid }

$$R = 1 + \underbrace{\frac{\Theta_w}{\Theta_g} \cdot \frac{1}{H}}_{\text{Partitioning to Water}} + \underbrace{\frac{\rho_b}{\Theta_g} \cdot \frac{k_2}{H}}_{\text{Partitioning to solid via water}}$$

Partitioning to Water



Note:  $H$  adjusts the proportion present in the water as a function of concentration in the gas phase i.e.  $H = \frac{C_g}{C_w}$

This is the non-dimensional Henry's constant.

## RETARDATION FACTOR MAGNITUDES

Magnitudes are dependent on the two retardation mechanisms  $\begin{cases} \text{gas} \rightarrow \text{water} \\ \text{gas} \rightarrow \text{solid} \\ (\text{via water}) \end{cases}$

$$\text{TCE @ } 20^\circ\text{C} \quad H = 0.3$$

$$K_{oc} = 126 \text{ ml/g.}$$

Assume:  $\Theta_w = 10\%$  and  $\rho_b = 1.65 \text{ g/cm}^3$   
 $\Theta_g = 25\%$

Contrast low and high organic content soils.

$$f_{oc} = 0.01\% \quad f_{oc} = 1\%$$

$f_{oc}$	$\frac{\Theta_w}{\Theta_g} \cdot \frac{1}{H}$	$\frac{\rho_b \cdot K_a}{\Theta_g H}$	$R$
0.01%	1.33	.27	2.61
1%	1.33	27.7	30.1

- 3 As expected, higher  $f_{oc}$  has proportionately greater sorption of vapor
  - a Note vapor sorbed to  $f_{oc}$  is sorbed via water.  
i.e.  $\text{gas} \rightarrow \text{water} \rightarrow \text{solid}$

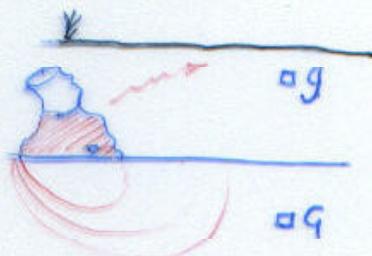
DIFFERENTIAL RETARDATION - o Gas movement  
 o Aqueous dissolution and movement

Aqueous. Effective velocity, =  $\frac{V}{R_a}$

$$R_a = [1 + \frac{\rho_b K_d}{\theta_w \text{sat.}}]$$

Gaseous. Retardation,  $R_g = [1 + \frac{\theta_w}{\theta_g} \cdot \frac{1}{H} + \frac{\rho_b}{\rho_g} \cdot \frac{K_d}{H}]$

Implication of these two different retardation coeffs is  
 that the order of arrival @ a "vadose" monitoring location  
 and a "groundwater" monitoring location may be different.



Dimensionless Henry's coeff.

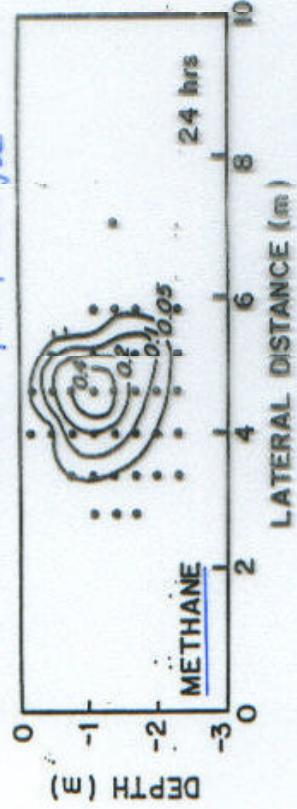
Table 5-2. Parameters and values used to calculate the relative order of transport velocity for TCA, TCE, and MC and Chapter 5.3.15 (reprinted from ACS, 1992)

Chemical	$K_{ow}$ (ml/g)	$K_d$ (ml/g)	$H$	$K_H$	$R_a$	$R_g$	Relative Order of Movement in Groundwater	Relative Order of Movement in Vapor
1,1,1-Trichloroethane (TCA)	152	2.58	16.9	0.599	55.8	3		2
Trichloroethylene (TCE)	126	2.14	14.2	0.379	73.3	2		3
Methylene Chloride (MC)	8.8	0.15	1.93	0.084	34.9	1		1

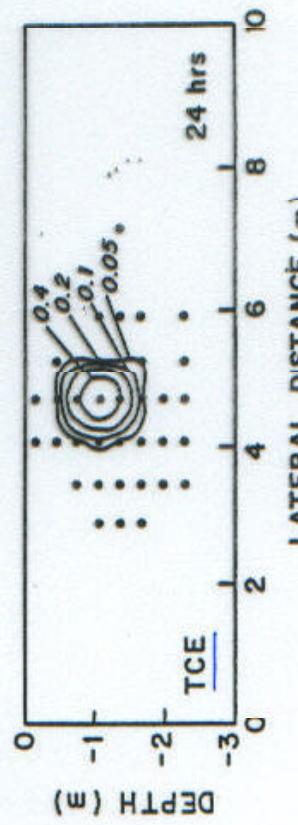
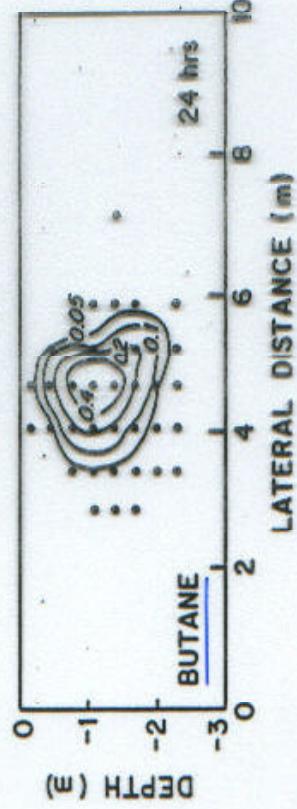
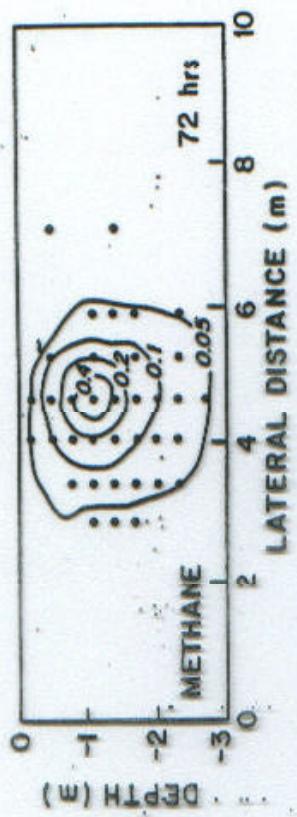
### Variation Experiment (04-)

Hydrogen  
Butane  
Trichloroethane(TCE) } + Nitrogen carriers

Monitoring @ 60 cm x 25 cm cadres.



Residual m/c. of Low for



[RELATIVE CONCENTRATIONS ]

[RELATIVE CONCENTRATIONS ]

FOR EXPERIMENTAL RESULTS

Henry's Law Coeffs.

Methane	~ 8
Butane	~ 12
TCG	~ .4

$$\therefore H_{\text{meth}} \approx H_{\text{butane}} > H_{\text{TCG}}$$

$$\therefore \text{since } R = 1 + \frac{\Theta_w}{\Theta_g} \frac{1}{H} + \frac{P_b}{\Theta_g} \frac{k_a}{H}$$

$$R_{\text{meth}} ; R_{\text{butane}} < R_{\text{TCG}}$$

and TCG vapors migrate more slowly.

## HENRY'S LAW COMPARISON

Two forms:

$$H = \frac{C_g}{C_w}$$

$$H' = \frac{P_v}{C_w}$$

~~$C_w \text{ in } (\text{mol}/\text{m}^3)$~~ 

Ideal gas law:  $Pv = \rho \frac{\bar{R}}{M} T$

$\bar{R}$  = universal gas const

$$= 8.20575 \times 10^{-5} [(\text{atm} \cdot \text{m}^3)/(\text{mol} \cdot \text{K})]$$

$$T = ^\circ\text{K} = ^\circ\text{C} + 273.16 \text{ }^\circ\text{K}$$

Substituting:

$$H' = \frac{P_v}{C_w} = \rho \frac{\bar{R} T}{M C_w}$$

$M$  = mass of 1 mole  
i.e. Atomic wt.  
(mass/mole)

$$H' = \left( \frac{\rho}{M C_w} \right) (\bar{R} T) = H (\bar{R} T)$$

Correspondingly:

$$H = \frac{H'}{(\bar{R} T)}$$

Check form of  $H$ :

$$H = \frac{\rho}{M C_w} \quad \text{and} \quad \frac{\rho}{M} = C_g$$

Example for TCE @  $20^\circ\text{C} = 293 \text{ }^\circ\text{K}$

$$H'_{\text{TCE}} (K_4) = 9.1 \times 10^{-3} \text{ atm} \cdot \text{m}^3/\text{mol}$$

$$H = \frac{9.1 \times 10^{-3}}{(8.2 \times 10^{-5}) 293} \left[ \frac{\text{atm} \cdot \text{m}^3}{\text{mol}} \cdot \frac{\text{mol} \cdot \text{K}}{\text{atm} \cdot \text{m}^3} \cdot \frac{1}{\text{K}} \right]$$

$$H \approx 0.38$$

$$\text{since} \quad \frac{\text{Grams}}{\text{m}^3} \cdot \frac{1}{\text{Molec. Wt.}} = \frac{\text{moles}}{\text{m}^3}$$

$$\therefore \frac{\rho}{M} = C_g !! \quad \therefore \text{ok.}$$

Table 4-6. Vapor concentration and total gas density data for selected DNAPLs at 25°C (from Paine et al., 1989).

Chemical	Molecular Weight, M g/mole	Vapor Pressure atm (@25°C)	Saturated Vapor Concentration (kg/m³)	Total Gas Density kg/m³
Trichloroethene	131.4	9.9	0.52	1.58
Chloroform	119.4	25.6	1.23	2.11
Tetrachloroethene	165.8	2.5	0.17	1.31
1,1,1-Trichloroethane	133.4	16.5	0.89	1.87
Methylene Chloride	84.9	58.4	2.00	2.50
1,2-Dichloroethene	96.9	43.5	1.70	2.37
1,2-Dichloroethane	99.0	10.0	0.44	1.48
Chlorobenzene	112.6	1.6	0.07	1.23
1,1-Dichloroethane	99.0	30.1	1.20	2.03
Tetrachloromethane	153.8	15.1	0.94	1.93
Air at 1 atm, 25°C	28.6	(101.3)		1.17

25°C

### Check Results

$$\text{TLC: } C_{g\max} = .52 \text{ kg/m}^3 = 520 \text{ mg/l}$$

$$C_{w\max} @ 20^\circ\text{C} = 1100 \text{ mg/l}$$

$$H = \frac{520}{1100} \approx 0.47$$

This compares with .38 previously since vapor data at 25°C. ∴ different temperature.

## CONCENTRATIONS

Vapor Concentrations :  $C_g$

Variety of units :

- molar basis  $\rightarrow \text{mol/m}^3$
- weight basis  $\rightarrow \text{mg/l}$
- volume basis  $\rightarrow \text{ppmv or \%}$
- pressure basis  $\rightarrow \text{atm, Pa or mmHg}$ .

All are related through ideal gas law  $P = \rho \frac{RT}{M}$

Aqueous Concentrations:  $C_w$

Related to  $C_g$  by Henry's Law.  $H = \frac{C_g}{C_w}$

Max  $C_w$  will be solubility.

Note that vapor concentration depends on mixture in liquid container

$$\text{Equilibrium concentration: } C_g = X_i \frac{P_v}{RT} = X_i \frac{P_v}{T} \frac{M_i}{R}$$

*gas const*  $\longrightarrow$  *Universal gas const*

$R = \frac{R}{M}$

$C_g$  has units of  $\text{mol/m}^3$

$X_i$  = mole fraction of compound  $i$  in liquid

Non-equilibrium:

- slow vaporization kinetics
  - dispersed or isolated residual DAPC
- } same as water!!