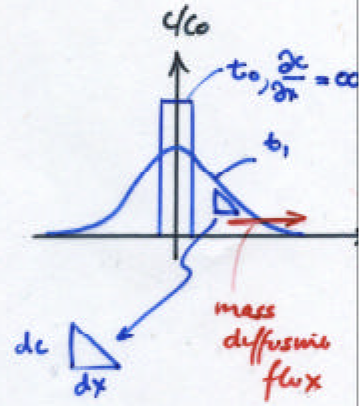
5.1 DIFFUSION



- Driven by concentration gradient
- Process of mollecular diffusion (Brownian motion)
- Characteristic for entropy (disorder) to increase
- Stagnant fluid

Fick's first law:

$$F = -D_d \frac{dc}{dx}$$

F = mass of solute per unit area per unit time (M/L^2T)

D_d = diffusion coefficient (L^2/T)

$\frac{dc}{dx}$ = concentration gradient $((M/L^3)/L)$

$(D_d \approx 10^{-9}\ m^2/s$ (range)$)$

Time dependent concentration

=1 since not porous medium

$$n \frac{\partial c}{\partial t} = -\left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right)$$

Substituting Fick's first law:

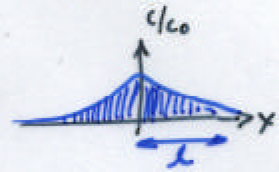
$$\frac{\partial c}{\partial t} = D_d \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right)$$

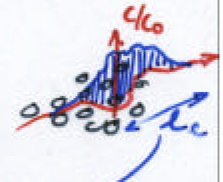
One dimensional equation

$$\frac{\partial c}{\partial t} = D_d \frac{\partial^2 c}{\partial x^2}$$

D_d is the "free" diffusion coefficient (i.e. in a beaker)

In porous medium the "effective" diffusion coefficient is used $D_d \rightarrow D^*$ due to the tortuous flow path



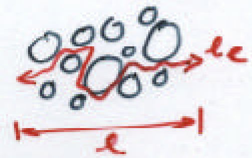


Effective length due to tortuous flow path

$$D^* = \omega D_d$$

ω is related to tortuosity, $T = l_e / l$

$T \geq 1$



Laboratory studies $\quad 0.01 < \omega \leq 0.5$

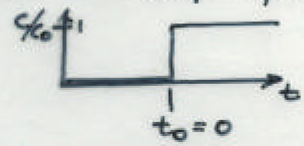
but lab studies not very useful.

SOLUTION OF DIFFUSION EQUATION

Solve
$$\frac{\partial c}{\partial t} = D^* \frac{\partial^2 c}{\partial x^2}$$

$c = 0 \qquad 0 > t \qquad$ initial condition (no solute)

$c = c_0 \qquad x = 0 \quad t \geq 0 \qquad$ step input



Solution:
$$\frac{c}{c_0}(x,t) = \mathrm{erfc}\left(\frac{x}{2\sqrt{D^* t}}\right) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\,du$$

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$$