

## 4.2 FLOW OF WATER IN THE UNSATURATED ZONE

### 4.2.1 Hydraulic Conductivity

$$K(\theta) = k_r(\theta) \frac{k}{\mu_w} \rho_w g$$

$k$  = intrinsic permeability

$k_r$  = relative permeability

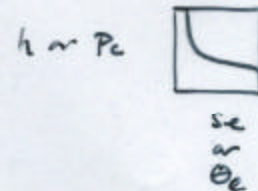
van Genuchten (1980)

Empirically relate  $k_r$  to  $\theta$

$$K(\theta) = K_{sat} Se^{1/2} [1 - (1 - Se^{1/m})^m]^2 \quad Se = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

$$K(h) = K_{sat} \frac{\{1 - (\alpha h)^{n-1} [1 + (\alpha h)^n]^{-m}\}^2}{[1 + (\alpha h)^n]^{m/2}}$$

Note:  $h = \frac{P}{\gamma_w}$  and  $P$  related to  $Se$  or  $\theta$



Useful in complex numerical models for unsaturated flow.

VAN GENUCHTEN CURVES (1980)

Ideally, the relative conductivity, moisture content, and water capacity curves are determined directly by performing a series of tests on the soils involved in the study. However, in many cases they can be approximated using a set of measured or approximated constants and a set of empirical relationships. For example, one option for generating the curves is to use the van Genuchten functions (van Genuchten, 1980). The van Genuchten relationships are:

$$K_r = \theta_e^{0.5} [1 - (1 - \theta_e^{1/\gamma})^2]^2$$

and

$$S_e = \theta_r = [1 + (\alpha h)^\beta]^{-1/\gamma} \quad \text{for } h < 0$$

$$\theta_r = 1 \quad \text{for } h \geq 0$$

where:

$$\theta_w = \theta_r + \theta_e(\theta_r - \theta_e)$$

$$\gamma = 1 - \frac{1}{\beta}$$

and

- $\theta_w$  = moisture content (dimensionless)
- $S_e = \theta_r$  = effective moisture content (dimensionless)
- $\theta_r$  = saturation moisture content (dimensionless)
- $\theta_e$  = residual moisture content (dimensionless)
- $\beta, \gamma$  = soil-specific exponents (dimensionless)
- $\alpha$  = soil-specific coefficient

COMPARISON WITH "FETTER" TERMINOLOGY

$$S_e = \theta_e = \left( \frac{\theta_w - \theta_r}{\theta_s - \theta_r} \right)$$

$$\gamma = m$$

$$\beta = n$$

Free parameters

$\alpha$  represents  $1/(R_e/r_w)$

$\beta$  represents slope of  $P_e$ -vs- $S_e$  }  $S_e = \theta_e$

Table 5.1 lists a set of saturated and residual moisture contents and the van Genuchten  $\alpha$  and  $\beta$  terms for a variety of common soil types. When applying the  $\alpha$  term, care should be taken to convert it to the proper units.

**Table 5.1**  
**Representative Soil Parameters**

Soil Type	Saturated Moisture Content, $\theta_s$	Residual Moisture Content, $\theta_r$	$\alpha$ [cm <sup>-1</sup> ]	$\beta$ ( $\gamma$ )
Clay**	0.38	0.068	0.008	1.09
Clay Loam	0.41	0.095	0.019	1.31
Loam	0.43	0.078	0.036	1.56
Loam Sand	0.41	0.057	0.124	2.28
Silt	0.46	0.034	0.106	1.37
Silt Loam	0.45	0.067	0.020	1.41
Clay	0.36	0.070	0.005	1.09
Silty Clay	0.43	0.089	0.010	1.23
Sand	0.43	0.045	0.145	2.68
Sandy Clay	0.38	0.100	0.027	1.23
Sandy Clay Loam	0.39	0.100	0.059	1.48
Sandy Loam	0.41	0.065	0.075	1.89

\*\* Agricultural soil, less than 60% clay  
Source: Carrel and Parrish (1986)