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Modeling of Naturally Fractured Reservoirs Using Deformation Dependent Flow Mechanism

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As special cases of a multi-porosity/multi-permeability formulation based on the mixture theory, an array of deformation-dependent flow models of various porosities and permeabilities relevant to the characterization of naturally fractured reservoirs is presented. Some key relationships are identified in the parametric investigation. A finite element discretization is developed for the multi-porosity/multi-permeability media. A case study is focused on a reservoir simulation using a dual-porosity/dual-permeability model. The preliminary study identifies the strong coupling between fluid flow and solid deformations. The effect of this coupling is also controlled by the constitutive properties of the formation.

INTRODUCTION

For the purpose of accurately characterizing the pressure buildup or depletion history of reservoirs, considerable interest has been focused on developing realistic mechanisms depicting the interporosity flow in naturally fractured reservoirs [1,2,3,4,5]. However, most cited models do not explicitly cover the effect of solid deformation on the change of fluid pressure. It is the admissibility of changes in total stress within the system that describes the essence of coupled deformation-dependent flow behavior within porous media and sets it apart from decoupled diffusive (flow) systems. Comprehensive coupling between stresses and pore pressures was first rationalized by Biot [6] and later adopted in many applications to specific deformation flow systems [7,8,9,10,11].

It is important to correctly characterize the behavior of naturally fractured reservoirs. For example, the exceptionally high oil rate recovery in the initial stages of reservoir production may lead to overestimate well production by assuming a higher storage to exist than reality. It was assumed that the high matrix block storage would continuously render the supply to the well through highly permeable fracture channels. In fact, many reservoirs that produce at high initial rates decline drastically after a short period of time because the oil has been stored in the fracture system (Fig. 1). As Aguilera [12] pointed out, it is important to visualize that the storage capacity of naturally fractured reservoirs varies extensively, depending on the degree of fracturing in the formation and the value of the primary porosity. Different from this scenario, Fig. 2 illustrates a case where only a small percentage of the total porosity is resident in the fractures. Averaging from the two extremes, an ideal situation for oil production is depicted in Fig. 3 where about equal storage capacity exists in the fractures and matrix blocks. Therefore, it is not safe to say that the storage capacity of a fractured system is negligible compared to the storage of the matrix. In more general cases, however, it is a fair assumption that in typical fractured reservoirs, fractures provide high-conductivity conduits amenable to rapid hydraulic flows; whereas, the high-porosity matrix blocks contain the majority of the storage. In any case, the behavior of naturally fractured reservoirs is radically different from that of a conventional reservoir comprised solely of intergranular porosity and permeability.

In naturally fractured reservoirs, where the medium consists of discrete fractions of varying solid compressibilities and permeabilities, a multi-porosity/multi-permeability approach appears more appropriate. It is well-known in continuum theory of mixtures that the mixture may be viewed as a superposition of a number of single continua, each following its own motion. In addition, at any time, each position in the mixture is occupied simultaneously by several different particles, each possessing particular constituents. The theory of mixture was originally developed as a thermodynamic framework to describe thermomechanical behaviors of materials consisting of more than one constituent [13]. The theory was extended to fluid flow in porous media which was viewed as a bi-substance [14]. To rationalize the behaviors of a multiple substance such as a fractured porous medium, Aifantis [15,16] proposed a multi-porosity theory based on the theory of mixture, declaring that any media that exhibit finite discontinuities in the porosity field are considered to possess a multi-porosity property.

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reservoir with uniform porosity and permeability.

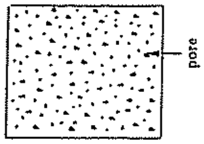

Fig. 4. Single-porosity/single-permeability system

Dual-porosity/single-permeability

For a fractured medium, it is generally recognized that the fractures add secondary porosity to the original porosity by breaking the porous medium into blocks. The dual-porosity conceptualization of a fractured medium considers the fluid in fractures and the fluid in matrix blocks as separate and overlapping continua. However, unlike the common assumption for a dual-porosity medium where the fluid flows primarily through highly permeable fractures, the nonpercolating fractured system depicted in Fig. 5 suggests an equivalent single-permeability behavior in a medium with distinctly different porosities. A fractured reservoir with relatively low permeability but high storage (tight reservoir) may be characterized by this dual-porosity/single-permeability model. The governing equations of solid deformation may be expressed as follows

$$Gu_{i,jj} + (\lambda + G)u_{k,ki} + \sum_{m=1}^{2} \phi_m p_{m,i} = 0, \tag{8}$$

where m=1 and 2 represent fractures and matrix blocks, respectively. The governing equation for the fluid phase is

$$-\frac{1}{\mu} k p_{m,kk} = \phi_m \dot{\epsilon}_{kk} - \phi_m^* \dot{p}_m \pm \xi(\Delta p), \tag{9}$$

where k is the equivalent single permeability, or a permeability averaged over the total system, and ξ corresponds to a fluid transfer rate representing the intensity of flow between the fractures and matrix driven by the pressure gradient, Δp. A positive sign indicates outflow from the matrix and a negative sign indicates inflow into the matrix.

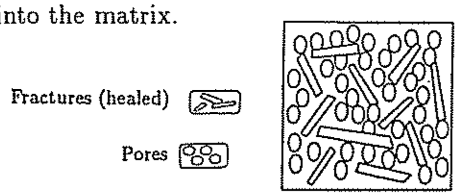

Fig. 5. Dual-porosity/single-permeability system

The major difference between the dual-porosity single-permeability system and the previous single-porosity/single-permeability system is that both inter-porosity flows are permitted in the former. No distinction between fracture and matrix permeabilities may be identified, which distinguishes it from the conventional dual-porosity/dual-permeability system.

Dual-porosity/dual-permeability

This is a typical naturally-fractured reservoir model in which the fracture and matrix phases are distinctly different in both porosity and permeability. Specifically, high porosity/ low permeability matrix and low porosity/high permeability fractures are main characteristics of the medium. In an idealized model of dual-porosity/dual-permeability the pattern shown in Fig. 6 is frequently used.

In view of the governing equations, the model carries an identical form in the solid phase as the dual-porosity/single-permeability model. However, the equation for the fluid phase is different, i.e.,

$$-\frac{1}{\mu} k_m p_{m,kk} = \phi_m \dot{\epsilon}_{kk} - \phi_m^* \dot{p}_m \pm \xi(\Delta p), \tag{10}$$

where k_m is the permeability of phase m.

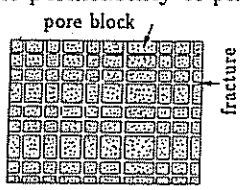

Fig. 6. Dual-porosity fractured medium

Under the assumption of low matrix permeability, a fracture flow mechanism may be incorporated in the formulation. The dual-porosity/dual-permeability model is suitable for the simulation of the fractured reservoir with low-permeability matrix blocks.

Triple-porosity/dual-permeability

For severely fractured reservoirs, however, a dual-porosity model may not be appropriate, even when considering local geometry. An immediate extension of the dual porosity conceptualization is to triple porosity. An example of a triple-porosity model is where a dominant fracture system intercepts a less pervasive and nested fracture system, which in turn is set within a porous matrix.

In this paper, a triple-porosity system or matrix-fissure-crack system is proposed. For a triple-porosity/dual-permeability system, matrix pores are interwoven with nonpercolating fissures, and they interact with open cracks through fluid exchange among different phases (Fig. 7). The governing equations for the solid phase are given by

$$Gu_{i,jj} + (\lambda + G)u_{k,ki} + \sum_{m=1}^{3} \phi_m p_{m,i} = 0, \tag{11}$$

where m=1, 2, and 3 are the subscripts for cracks, fissures and matrix, respectively.

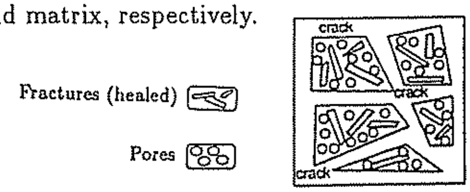

Fig. 7. Triple-porosity/dual-permeability system

For the fluid phase, it is convenient to write out each equation and the corresponding subscript separately

$$\nu_u = \frac{\nu + \psi}{1 - \psi}, \quad \nu = \nu_u - \psi(1 + \nu_u). \tag{22}$$

It is understood from equation (21) that $\psi \geq 0$; therefore, $\nu_u \geq \nu$. If $\psi = 0$ and $\nu_u = \nu$, then $\phi = 0$ and the fluid flow is fully decoupled from the solid deformations. Since [21] $\frac{1}{2} \geq \nu_u \geq \nu$, the following relation exists $0 \leq \psi \leq \frac{1-2\nu}{3}$.

By back-substituting equation (22), Skempton's constant B may be represented by a simple expression, extended for a multi-porosity medium,

$$B_m = \frac{1}{\phi_m^*}\left(\frac{1}{K} - \frac{1}{K_s}\right). \tag{23}$$

Depending on the specific circumstances, ϕ_m^* may be selected from equations (16) or (17).

FINITE ELEMENT MULTI-POROSITY MODEL

For the general representation of the multi-porosity/multi-permeability model proposed previously, the finite element approximation technique is chosen. Applying the effective stress law enables the stress-strain relationship to be written for a multi-porosity medium as

$$\partial\underline{\sigma} = \underline{D}(\partial\underline{\epsilon} + \sum_{m=1}^{I} \underline{C}\,\underline{m}\phi_m \partial p_m), \tag{24}$$

where I is the number of distinct porosity dimensions, $\underline{\sigma}$ and $\underline{\epsilon}$ are the vectors of stress and strain, respectively; p_m is the fluid pressure for phase m, $\underline{C}$ is a compliance matrix, $\underline{D}$ is an elasticity matrix and $\underline{m}$ is a one-dimensional vector. For 3-D problems, $\underline{m}^T = \{\,1\,1\,1\,0\,0\,0\,\}$.

Invoking the principle of virtual work and applying the incremental equilibrium to the total stress state, and substituting equation (24) into the general equilibrium equations, enables the governing finite element discretization for the solid phase to be given as

$$\underline{K}_T \frac{d\underline{u}}{dt} + \underline{R}\frac{d\underline{p}}{dt} = \frac{d\underline{F}}{dt}, \tag{25}$$

where

$$\begin{cases} \underline{K}_T = \int_V \underline{B}^T \underline{D}\,\underline{B} dV, \quad \underline{F} = \int_S \underline{N}\underline{f} dS, \\ \underline{R} = \sum_{m=1}^{I} \int_V \underline{B}^T \underline{D}\,\underline{C}\,\underline{m}\phi_m \underline{N} dV, \end{cases} \tag{26}$$

where $\underline{B}$ is the strain-displacement matrix, $\underline{f}$ is a vector of applied boundary tractions, V is the integration domain, $\underline{N}$ is a vector of shape functions and S is the surface domain.

In the fluid phase, the rate of fluid accumulation per unit volume of space may be defined as

$$\nabla^T \underline{v} = \underline{m}^T \phi_m \frac{\partial\underline{\epsilon}}{\partial t} - \phi_m^* \frac{\partial\underline{p}}{\partial t} + (-1)^m \sum_{m=1}^{I} \xi_m(\Delta p_m), \tag{27}$$

where Δp is the fluid pressure difference between two phases. Substituting Darcy's velocity into equation (27) and invoking the Galerkin finite element procedure yield

$$\underline{E}\underline{p} + \underline{L}\frac{\underline{u}}{dt} + \underline{M}\frac{d\underline{p}}{dt} = \underline{Q}^* \Delta\underline{p} + \underline{G}\,\underline{Z}, \tag{28}$$

where

$$\begin{cases} \underline{E} = -\frac{1}{\mu} \int_V \nabla \underline{N}^T \underline{k} \nabla \underline{N} dV, \\ \underline{L} = \phi_m \int_V \underline{N}^T \underline{m}^T \underline{C}\,\underline{D}\,\underline{B} dV = \underline{R}^T \\ \underline{M} = -\phi_m^* \int_V \underline{N}^T \underline{N} dV, \quad \underline{G} = \frac{1}{\mu} \int_V \nabla \underline{N}^T \underline{k} \nabla \underline{N} dV, \\ \underline{Q}^* = \sum_{m=1}^{I} \xi_m \int_V \underline{N}^T \underline{N} dV \end{cases} \tag{29}$$

where $\underline{k}$ is the permeability matrix, and $\underline{Z}$ is the vector of elevation control volume. Equations (25) and (28) completely define the finite element formulation of a multi-porosity problem.

CASE STUDY

An attempt is made in the following to apply the dual-porosity/dual-permeability model, described previously, to a simple, hypothetical case study. The objective of this preliminary investigation is to delineate the influence of the changes in formation properties and deformation magnitudes on the transient change of fluid pressure distribution. For generality, dimensionless analysis prevails. For simplicity, a quasi-steady flow interaction between fracture and matrix block proposed by Warren and Root (1963) is used.

A flat-lying reservoir is located at a depth of 457 m (1500 feet) and contains three layers distinguished by material properties listed in Table 1 for the case with stiffer rock of layer 3 (hard layer 3). The elastic modulus for the case of softer rock of layer 3 (soft layer 3) is 10% of the value shown in Table 1. The specific weight of overburden is 2.5. A gravitational cover load is assumed to act on the top of the reservoir. The oil-saturated reservoir is being produced at a constant rate of $1.64 \times 10^{-4} m^3/sec$ (500 ft^3 per day) through a horizontal well situated in the cross-sectional center of the reservoir. It is assumed that no gas mixture is found in the oil, and no water-drive mechanism needs to be considered. No flow conditions are assumed, both at the lateral boundary as well as at the bottom layer of the reservoir. As a result of symmetry, only one-half of the reservoir in the lateral direction needs to be simulated in the finite element model (Fig. 9).

The normalized fluid pressure (with respect to the maximum pressure at t=0.01 day) depletion curves at the well are shown in Fig. 10. In general, the pressure declines faster between 0.01 and 0.1 day, and slower afterwards. The pressure magnitude is smaller and declines faster for the case of soft layer 3. The cross-sectional fluid pressure distributions for both hard and soft layer 3 cases at t=0.01 day are illustrated in Figs. 11 and 12, respectively. It is of interest to note that the pressure variation is more localized with smaller magnitudes for the case of soft layer 3. This scenario represents a weaker modification of fluid pressure due to larger compliance of the reservoir formation. The fluid pressure profiles at various distances from the well and at different times are given in Figs. 13 and 14. For both hard and soft layer 3 cases, the pressure declines

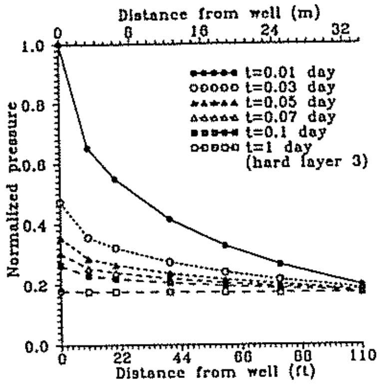

Fig. 13. Pressure depletion from well in hard layer 3 case

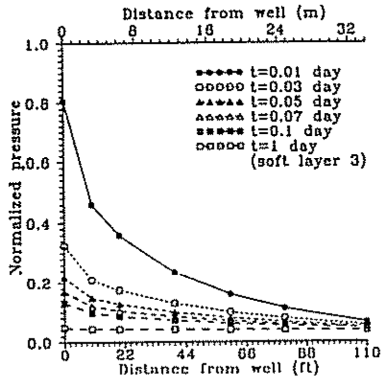

Fig. 14. Pressure depletion from well in soft layer 3 case

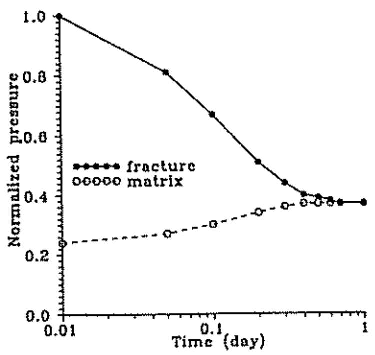

Fig. 15. Early time fluid exchange

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