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Evaluation of Stresses and Displacements Induced in Discretely Layered Media

M. BAI†
D. ELSWORTH†‡
Z. LI§
N. TOMLIN¶

An analytical model based on the stress function approach is presented for the evaluation of stresses and displacements induced in discretely layered elastic media as a result of single seam extraction. The method is presented in generalized form for plane strain conditions with an arbitrary number of layers. Appropriate boundary conditions are those for both partial and complete seam extraction. Stresses and displacements may be evaluated throughout the section with the elastic strain distribution being used to predict the extents of potential strata failure. A parametric example is presented to illustrate the utility of the method and to identify generic response to changing extractive geometries. Comparison of the proposed model with results from a well-documented case study for which well-defined material and geometric parameters are available shows favourable agreement.

INTRODUCTION

Displacements induced by seam extraction below water-bearing strata may have serious implications for water flows into mined excavations. A prerequisite to determining the magnitude of anticipated inflows and the potential stability of the structure is knowledge of the extraction induced displacement field throughout the mass. A common idealization of the geological conditions may be as a series of horizontally layered elastic strata between which compatibility of displacements and equilibrium of stresses are enforced at layer interfaces. Idealization as a sequence of elastic layers, extending to infinity in the horizontal direction, and subject to the requirements of plane strain allows stresses and displacements in the mass to be determined semi-analytically.

ANALYTICAL MODEL

The model accommodates isotropic linear elastic behaviour within a stratified 2-D section. The basic solution of stress, displacement and strain fields in a semi-infinite plane is obtained by means of the Fourier complex variation method. The overlying strata may be divided into as many rock layers as practical. The solutions for stresses and displacements within the mass are presented as recurrence relation from which surface displacement profiles may also be recovered.

†Department of Mineral Engineering, The Pennsylvania State University, University Park, PA 16802, U.S.A.
‡Formerly at Waterloo Centre for Groundwater Research, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.
§Central Coal Mining Research Institute, Beijing, China.
¶Department of Mining Engineering, University of Newcastle-upon-Tyne, Newcastle-upon-Tyne, NE1 7RU, U.K.

Basic solutions

Stresses for the 2-D case ($\sigma_x, \sigma_y, \tau_{xy}$) may be recovered from the Maxwell stress function (ϕ) as:

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\phi}{\partial x^2}, \quad \tau_{xy} = \frac{-\partial^2\phi}{\partial x \partial y}, \tag{1}$$

where the y axis is defined positive in the downwards vertical direction.

The harmonic equation is:

$$\frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2 \partial y^2} + \frac{\partial^4\phi}{\partial y^4} = 0. \tag{2}$$

The Fourier complex variation [$F(Z)$] may be defined as:

$$F(Z) = \int_0^\infty f(x)\cos(xZ)\,dZ, \tag{3}$$

where Z is an integral parameter and the inverse function is given by $f(x)$ as:

$$f(x) = \frac{2}{\pi}\int_0^\infty F(Z)\cos(xZ)\,dZ. \tag{4}$$

Assuming that, at infinity, the excavation induced stresses vanish, then it follows that:

$$\frac{\partial\phi}{\partial x} = 0 \quad \text{when} \quad x = 0.$$

Incorporating the stress function of equation (1) with the Fourier variation of equation (3) yields:

$$\bar{\sigma}_x = \int_0^\infty \sigma_x \cos(xZ)\,dx$$
$$= \int_0^\infty \frac{\partial^2\phi}{\partial y^2}\cos(xZ)\,dx$$

or

$$\bar{\sigma}_x = \frac{\partial}{\partial y^2}\int_0^\infty \phi\cos(xZ)\,\mathrm{d}x = \frac{\partial^2\bar{\phi}}{\partial y^2}, \tag{5}$$

where

$$\bar{\phi} = \int_0^\infty \phi\cos(xZ)\,\mathrm{d}x \tag{6}$$

and similarly

$$\bar{\sigma}_y = \int_0^\infty \sigma_y\cos(xZ)\,\mathrm{d}x = -Z^2\bar{\phi}, \tag{7}$$

$$\bar{\tau}_{xy} = \int_0^\infty \tau_{xy}\sin(xZ)\,\mathrm{d}x = Z\frac{\partial\bar{\phi}}{\partial y}. \tag{8}$$

Applying the Fourier variation to the harmonic equation [equation (2)], and substituting equations (5–8), yields:

$$\int_0^\infty \nabla^4\phi\cos(xZ)\,\mathrm{d}x = \left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} - Z^2\right)\bar{\phi} = 0, \tag{9}$$

where $\bar{\phi}$ may be derived from this ordinary differential equation as:

$$\bar{\phi} = A\cosh(yZ) + BZy\cosh(yZ) + C\sinh(yZ) + DZy\sinh(yZ). \tag{10}$$

The linear constitutive relation in the form of Hooke's law [1] may be substituted with the stresses defined in terms of the Maxwell function of equation (1) as:

$$\frac{\partial U}{\partial x} = \frac{1+\mu}{E}\left[(1-\mu)\frac{\partial^2\phi}{\partial y^2} - \mu\frac{\partial^2\phi}{\partial x^2}\right], \tag{11}$$

with μ = Poisson's ratio and U = horizontal displacement.

Applying the Fourier cosine variation to $\partial U/\partial x$ gives:

$$\int_0^\infty \frac{\partial U}{\partial x}\cos(xZ)\,\mathrm{d}x = \frac{1+\mu}{E}\int_0^\infty\left[(1-\mu)\frac{\partial^2\phi}{\partial y^2} - \mu\frac{\partial^2\phi}{\partial x^2}\right]\cos(xZ)\,\mathrm{d}x, \tag{12}$$

which, according to equations (5) and (7) may be equated to:

$$\int_0^\infty \frac{\partial U}{\partial x}\cos(xZ)\,\mathrm{d}x = Z\bar{U}, \tag{13}$$

where

$$\bar{U} = \frac{1+\mu}{EZ}\left[(1-\mu)\frac{\partial^2\bar{\phi}}{\partial y^2} + \mu Z^2\bar{\phi}\right]. \tag{14}$$

For shear strains (γ_{xy}) the constitutive law may be given as:

$$\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} = \frac{2(1+\mu)}{E}\tau_{xy}, \tag{15}$$

or

$$\frac{\partial V}{\partial x} = \frac{2(1+\mu)}{E}\tau_{xy} - \frac{\partial U}{\partial y}, \tag{16}$$

where V = vertical displacement.

Applying the Fourier sine variation to $\partial V/\partial x$ and using equations (8) and (14), in a similar manner to that used in deriving $\bar{U}$, $\bar{V}$ is obtained. This is given as:

$$\bar{V} = \frac{-1}{Z}\int_0^\infty \frac{\partial V}{\partial x}\sin(xZ)\,\mathrm{d}x$$

or

$$\bar{V} = \frac{-(1+\mu)}{EZ^2}\left[(2-\mu)Z^2\frac{\partial\bar{\phi}}{\partial y} - (1-\mu)\frac{\partial^3\bar{\phi}}{\partial y^3}\right]. \tag{17}$$

Finally, substituting $\bar{\phi}$ of equation (10) into equations (5), (7), (8), (14) and (17), the basic solution is obtained as follows:

$$\bar{\sigma}_y = -Z^2[A\cosh(yZ) + BZy\cosh(yZ) + C\sinh(yZ) + DZy\sinh(yZ)], \tag{18}$$

$$\bar{\sigma}_x = Z^2\{A\cosh(yZ) + B[2\sinh(yZ) + Zy\cosh(yZ)] + C\sinh(yZ) + D[2\cosh(yZ) + Zy\sinh(yZ)]\}, \tag{19}$$

$$\bar{\tau}_{xy} = Z^2\{A\sinh(yZ) + B[\cosh(yZ) + Zy\sinh(yZ)] + C\cosh(yZ) + D[\sinh(yZ) + Zy\cosh(yZ)]\}, \tag{20}$$

$$\bar{U} = \frac{1+\mu}{E}\{A\cosh(yZ) + B[2(1-\mu)\sinh(yZ) + Zy\cosh(yZ)] + C\sinh(yZ) + D[2(1-\mu)\cosh(yZ) + Zy\sinh(yZ)]\}, \tag{21}$$

$$\bar{V} = \frac{1+\mu}{E}\{-A\sinh(yZ) + B[(1-2\mu)\cosh(yZ) - Zy\sinh(yZ)] - C\cosh(yZ) + D[(1-2\mu)\sinh(yZ) - Zy\cosh(yZ)]\}. \tag{22}$$

The inverse functions of strains may be obtained by differentiating $\bar{V}$ and $\bar{U}$ with respect to y and x, respectively, such that:

$$\bar{\epsilon}_y = \frac{\partial\bar{V}}{\partial y} = -Z\frac{1+\mu}{E}\{A\cosh(yZ) + B[2\mu\sinh(yZ) + Zy\cosh(yZ)] + C\sinh(yZ) + D[2\mu\cosh(yZ) + Zy\sinh(yZ)]\}, \tag{23}$$

and similarly for

$$\bar{\epsilon}_x = \frac{\partial\bar{U}}{\partial x}. \tag{24}$$

The desired stresses, displacements and strains can be derived from the inverse functions of the basic solutions as:

$$\sigma_y = \frac{2}{\pi}\int_0^\infty \bar{\sigma}_y\cos(xZ)\,\mathrm{d}Z, \tag{25}$$

$$\sigma_x = \frac{2}{\pi}\int_0^\infty \bar{\sigma}_x\cos(xZ)\,\mathrm{d}Z, \tag{26}$$

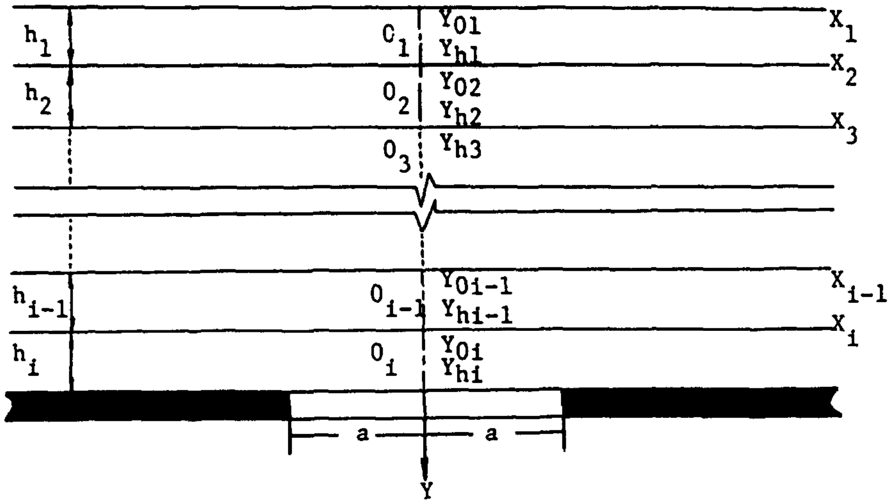

Fig. 1. Floating coordinate system.

$$\tau_{xy} = \frac{2}{\pi}\int_0^\infty \bar{\tau}_{xy}\sin(xZ)\,\mathrm{d}Z, \tag{27}$$

$$U = \frac{2}{\pi}\int_0^\infty \bar{U}\sin(xZ)\,\mathrm{d}Z, \tag{28}$$

$$V = \frac{2}{\pi}\int_0^\infty \bar{V}\cos(xZ)\,\mathrm{d}Z, \tag{29}$$

$$\epsilon_x = \frac{2}{\pi}\int_0^\infty \bar{U}Z\cos(xZ)\,\mathrm{d}Z, \tag{30}$$

$$\epsilon_y = \frac{2}{\pi}\int_0^\infty \bar{\epsilon}_y\cos(xZ)\,\mathrm{d}Z, \tag{31}$$

where ϵ_y = vertical strain and ϵ_x = horizontal strain.

Boundary conditions

As a first approximation, the shear stresses between adjacent rock layers are assumed to be zero. The vertical stresses and displacements, together with shear stresses at the lower boundary of each layer, are presumed to be the same values as those at the upper boundary of the contiguous layer. The boundary conditions of this simplified model can be described relative to Fig. 1 as:

At the upper boundary of the first layer at $y = 0_1$:

$$\bar{\sigma}_{y01} = 0,$$
$$\bar{\tau}_{xy01} = 0. \tag{32}$$

At successive interfaces between layers i and $i+1$ at $y = h_i$:

$$\bar{\sigma}_{yhi} = \bar{\sigma}_{y0i+1},$$
$$\bar{\tau}_{xyhi} = \tau_{xy0i+1},$$
$$\bar{\tau}_{xyhi} = 0,$$
$$\bar{V}_{yhi} = \bar{V}_{y0i+1}. \tag{33}$$

Where layer $i+1$ becomes the extracted or excavated seam the boundary conditions are:

$$\bar{\tau}_{xyhi} = 0,$$

$$\bar{V}_{yhi} = \int_0^a M\cos(xZ)\,\mathrm{d}x = \frac{M}{Z}\sin(xZ), \tag{34}$$

where M = thickness of the extracted seam and a = semi-extracted width.

In the above it is assumed that the gob is filled with caved material to a width $2a$. Alternatively, under room and pillar extraction, V_{yhi} can be defined as follows:

$$\bar{V}_{yhi} = \int_{b/2}^{b/2+a} M\cos(xZ)\,\mathrm{d}x + \int_{3b/2+a}^{3b/2+2a} M\cos(xZ)\,\mathrm{d}x + \cdots + \int_{(2n+1)b/2+na}^{(2n+1)b/2+(n+1)a} M\cos(xZ)\,\mathrm{d}x, \tag{35}$$

which may be evaluated as

$$\bar{V}_{yhi} = \frac{2M}{Z}\sin\frac{aZ}{2}\left[\cos\frac{(a+b)}{2}Z + \cos\frac{3(a+b)}{2}Z + \cdots + \cos\frac{(2m+1)(a+b)}{2}Z\right], \tag{36}$$

where a = room width, b = pillar width and $\bar{M}$ = one half the number of rooms ($m = 1, 2, \ldots, \bar{M}$) as illustrated in Fig. 2.

It is apparent that $4N$ constants must be determined from equations (18–22) by applying appropriate boundary conditions. The coefficients to be determined are A_i, B_i, C_i and D_i for $i = 1, 2, \ldots, N$ where N is the number of layers present within the system.

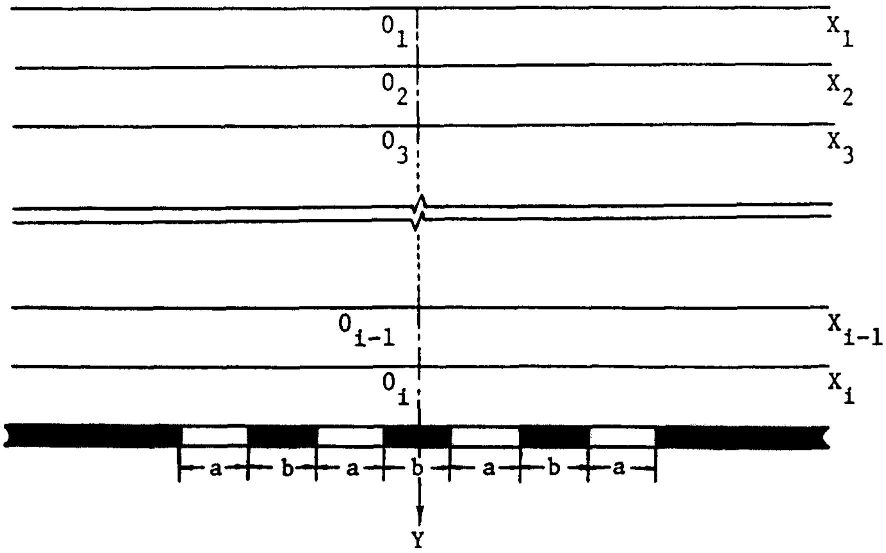

Fig. 2. Model for partial extraction.

Integration procedure

The integrals of equations (25–31) must be evaluated within the infinite region. This is most conveniently facilitated if the infinite region is mapped onto a finite region by utilizing the substitution:

$$Z_1 = \exp(-Z), \tag{37}$$

to yield transformation of the integral and associated limits to the form:

$$\frac{2}{\pi}\int_0^\infty f(xZ)\,dZ = \frac{2}{\pi}\int_0^1 \frac{f[-\ln(Z_1)x]}{Z_1} = dZ_1, \tag{38}$$

where the integral of equation (38) is evaluated numerically using Simpson's rule.

Recursion relations

In the general case, $4N$ constants must be determined for N layers. $4N$ equations will be established from the $4N$ boundary conditions to obtain the $4N$ constants and, as such, a unique solution to the problem is determined.

The general problem is solved beginning from the surface of the first layer in the model. In addition, it is assumed that there are no vertical stresses or shear stresses acting on the surface. Consequently, the constant A_1 is readily available from these boundary conditions. From consistency at layer interfaces, there exists the condition that: $B_i = -C_i (i = 1, 2, \ldots, N)$. As a result, the $4N$ constants are reduced to only $3N$ if the substitution is made to either replace B_i or replace C_i in representing A_i or D_i.

With respect to boundary conditions, the constants are obtained from the recursive rules:

$$\begin{aligned} A_i = {} & A_{i-1}\cosh(h_{i-1}Z) \\ & + B_{i-1}[h_{i-1}Z\cosh(h_{i-1}Z) - \sinh(h_{i-1}Z)] \\ & + D_{i-1}h_{i-1}Z\sinh(h_{i-1}Z), \end{aligned} \tag{39}$$

$$\begin{aligned} B_i = {} & \{-A_{i-1}\sinh(h_{i-1}Z) + B_{i-1}[2(1-\mu_{i-1}) \\ & \times \cosh(h_{i-1}Z) - h_{i-1}Z\sinh(h_{i-1}Z)] \\ & + D_{i-1}[(1-2\mu_{i-1})\sinh(h_{i-1}Z) - h_{i-1}Z \\ & \times \cosh(h_{i-1}Z)]\} \\ & \times [(1+\mu_{i-1})E_i/[E_{i-1}2\cdot(1-\mu_i^2)]. \end{aligned} \tag{40}$$

The recursion rule for D_i is obtained by meeting boundary conditions in the final layer, which is associated with the mining configuration at the working area. Thus,

$$D_i = \frac{P}{[2(1-\mu_i)\sinh(h_iZ)]} - \frac{B_i\cosh(h_iZ)}{\sinh(h_iZ)}, \tag{41}$$

where

$$P = \frac{E_iM}{Z(1+\mu_i)}\sin(aZ). \tag{42}$$

For numerical solution, the recursive rules may also be written as a system of equations as:

$$[G]\{E\} = \{P\}, \tag{43}$$

where $[G]$ is the matrix:

$$
\begin{bmatrix}
-1 & 0 & 0 & g_{1(i-1)} & g_{2(i-1)} & g_{3(i-1)} & & & & & & \\
 & -1 & 0 & g_{4(i-1)} & g_{5(i-1)} & g_{6(i-1)} & & & 0 & & & \\
 & & -1 & g_{7(i-1)} & g_{8(i-1)} & g_{9(i-1)} & & & & & & \\
 & & & \ddots & & \ddots & & & & & & \\
 & & & & & -1 & 0 & 0 & g_1 & g_2 & g_3 & \\
 & 0 & & & & & -1 & 0 & g_4 & g_5 & g_6 & \\
 & & & & & & & -1 & g_7 & g_8 & g_9 & \\
 & & & & & & & & 1 & 0 & 0 & \\
 & & & & & & & & & 1 & P_1 & \\
g_{10i} & g_{11i} & g_{12i} & 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 &
\end{bmatrix},
$$

where g_k, $k = 1, 2, \ldots [9(i-1)+3]$ are given implicitly in the recursive equations (39–41). Also:

$$P_1 = \frac{\sinh(h_1 Z) + h_1 Z \cosh(h_1 Z)}{Z h_1 \sinh(h_1 Z)}. \qquad (44)$$

The vector $\{E\}$ in equation (43) can be expressed as:

$$\{E\} = \{A_i B_i D_i \ldots A_1 B_1 D_1\}^{\mathrm{T}}. \qquad (45)$$

Similarly, vector $\{P\}$ is written as:

$$\{P\} = \{0\,0\,P_2\,0\,0\,0 \ldots 0\,0\,0\}^{\mathrm{T}}, \qquad (46)$$

where

$$P_2 = \frac{P}{2(1-\mu_i)\sinh(h_i Z)}, \qquad (47)$$

where P is given in equation (42).

All constants are determined by solving equation (43). Consequently, the stress and strain fields may be found for the model with an arbitrary number of layers.

Surface deformation

Since we have obtained the required constants for all general cases in the previous section, the surface movement and deformation within the body may be determined from the following formulae:

(1) surface subsidence:

$$V_{y01} = \frac{2}{\pi} \int_0^\infty \bar{V}_{y01} \cos(xZ)\, \mathrm{d}x, \qquad (48)$$

where $\bar{V}_{y01}$ refers to equation (22);

(2) surface horizontal displacement:

$$U_{y01} = \frac{2}{\pi} \int_0^\infty \bar{U}_{y01} \sin(xZ)\, \mathrm{d}x, \qquad (49)$$

where $\bar{U}_{y01}$ refers to equation (21);

(3) surface slope:

$$T_{y01} = \frac{\partial V}{\partial x} = -\frac{2}{\pi} \int_0^\infty Z \bar{V}_{y01} \sin(xZ)\, \mathrm{d}x, \qquad (50)$$

where $\bar{V}_{y01}$ refers to equation (48);

(4) surface curvature:

$$K_{y01} = \frac{\partial^2 V}{\partial x^2} = -\frac{2}{\pi} \int_0^\infty Z^2 \bar{V}_{y01} \cos(xZ)\, \mathrm{d}x; \qquad (51)$$

(5) surface strain:

$$E_{y01} = \frac{\partial U}{\partial x} = \frac{2}{\pi} \int_0^\infty Z \bar{U}_{y01} \cos(xZ)\, \mathrm{d}x, \qquad (52)$$

where $\bar{U}_{y01}$ refers to equation (49).

Inter-strata strain field

The strain field in the overlying strata can be generally expressed as follows where ϵ_x, ϵ_y refer to equations (30) and (31); $\bar{\epsilon}_{yi}$ refers to equation (23); $\bar{U}_{xi}$ refers to equation (21); $i = 1, 2, \ldots, N$:

(a) vertical strain:

$$\epsilon_{yi} = \frac{2}{\pi} \int_0^\infty \bar{\epsilon}_{yi} \cos(xZ)\, \mathrm{d}x; \qquad (53)$$

(b) horizontal strain:

$$\epsilon_{xi} = \frac{2}{\pi} \int_0^\infty Z \bar{U}_{xi} \cos(xZ)\, \mathrm{d}x. \qquad (54)$$

STRATA FAILURE CRITERION

For mining under bodies of water, the criteria for determining strata failure are based either on a correlation between the thickness of the extracted seam and the depth of mining, or the intensity of mining induced ground strain.

In the late 1960s, after a series of extensive investigations in Britain, the National Coal Board [2] postulated a rule for sub-sea mining stating that the tensile strain at the seabed should not exceed 0.01 (10 mm/m) for longwall mining. The strain was to be calculated by the same method as used for surface strain in the SEH [3]. In accordance with regional experience, the United States [4] suggested that the tensile strain adjacent to bodies of water should not exceed 8.75 mm/m. Similarly,

Table 1. Permissible strain for buildings

Country	Compressive strain (mm/m)	Tensile strain (mm/m)
China		2.0
France	1–2	0.5
West Germany	0.6	0.6
Poland	1.5	1.5
Japan	0.5	0.5
Soviet Union	2–4	2–4
United States	0.8	0.4

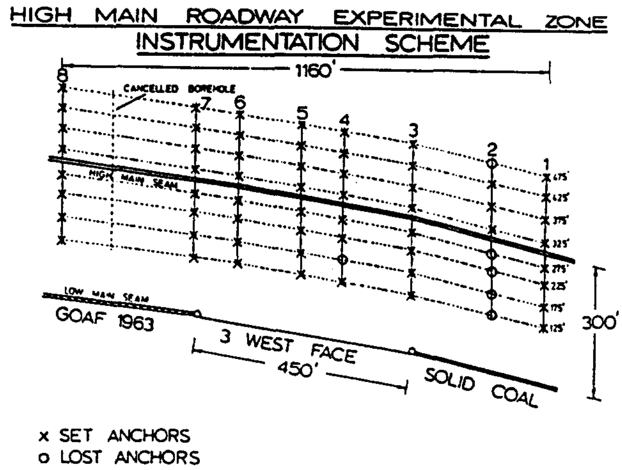

Fig. 3. *In* situ instrumentation scheme (after Dowdell [7]).

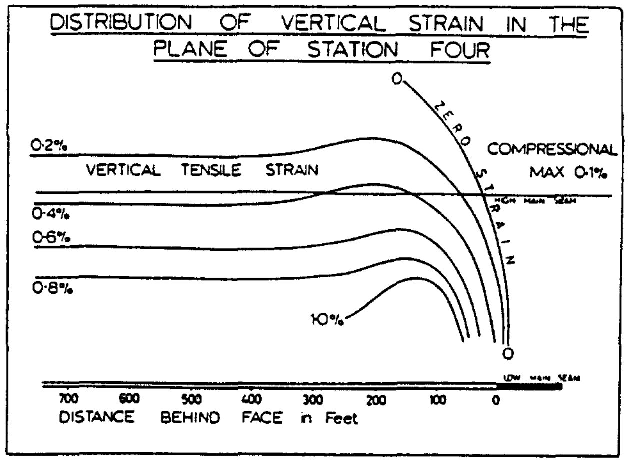

Fig. 4. Distribution of vertical strain (after Dowdell [7]).

Canada [5] proposed a 7.71 mm/m tensile strain limit, Australia [5] 7.5 mm/m and Chile [5] 5.03 mm/m. These criteria all fall within a relatively narrow bound.

Structural limits on deformations in brick and concrete structures are of the order of 0.5–2 mm/m in tension or 0.5–4 mm/m in compression [6] as documented in Table 1. Although it remains uncertain as to whether these criteria are applicable to rocks, the magnitude of these threshold strains occupy the lower limits of those proposed for rock materials above.

Underground strain measurements were carried out as early as the 1960s. An analysis by Dowdell [7] was concerned with the measurement of absolute rock movement in the strata above an advancing longwall face. *In situ* measurements made by Malone and reported in [7] give the distribution of vertical strains derived from the measured displacements at the Vane Tempest colliery as illustrated in Figs 3 and 4.

Dowdell concluded that the upper limit of tensile strain sustainable by the overlying strata was 0.5 mm/m. It was therefore suggested that if the vertical tensile strain was in excess of this value, fracturing or bed separation must be occurring. For strata in vertical compression a threshold of 0.5 mm/m strain was recorded [7] and attributed to the influence of the advancing front abutment.

Farmer *et al.* [8] have also studied the results from *in situ* strain measurements at Wearmouth, Markham and Lynemouth collieries. It was suggested that a compressive strain of 8–10 mm/m would be sufficient to cause shear fracturing and dilation of rock. Moreover, they stated that when the tensile strain exceeded 1 mm/m, tensile fracturing would occur in the rock mass. A distillation of the results reported in the previous allows a strain-based criterion to be proposed for failure modes under both compressive and tensile loading. This concept is key to determination of the extent of failure induced in the overlying strata. The criterion is presented in Table 2.

PARAMETRIC EXAMPLE

A simple five-layer model is examined in the following with constant elastic parameters of $E = 10$ GPa and $\mu = 0.3$ with depth. For a fixed extraction width of 180 m within the 2-m thick seam the influence of four different depths of cover are examined with particular reference to the estimated fracture zone extent. The extents of strata failure induced for cover depths of 250, 350, 500

Table 2. Strata failure criteria

Zone of strata failure	Strains (mm/m) Tensile	Strains (mm/m) Compressive	Note
Fractured zone	>2	>4	Based on horizontal strain
Potential fractured zone	>1	>2	
Bed separation zone	>2		Based on vertical strain
Potential bed separation zone	>1		

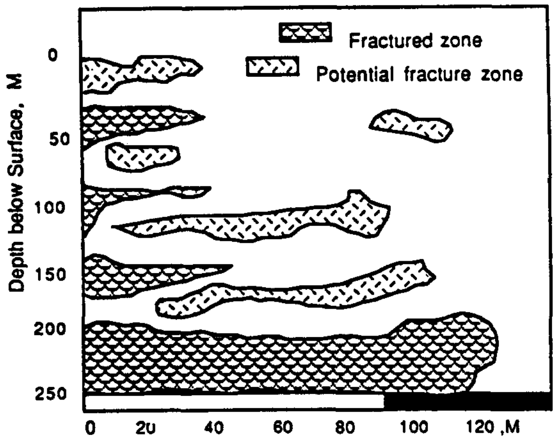

Fig. 5. Strata failure for 250 m mining depth.

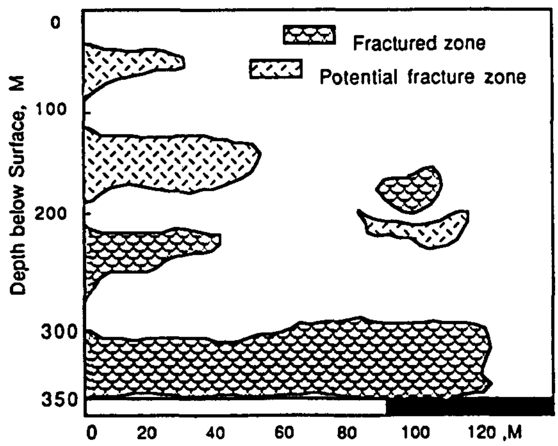

Fig. 6. Strata failure for 350 m mining depth.

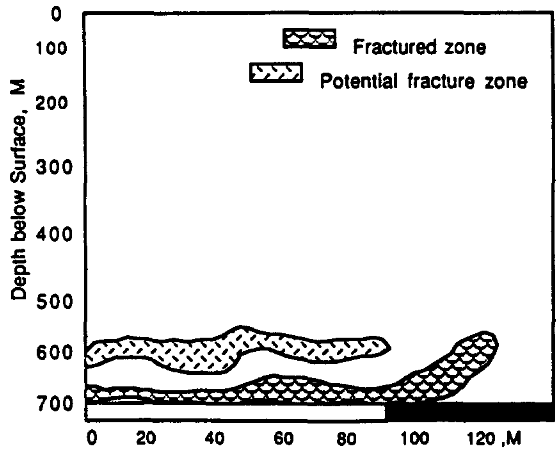

Fig. 8. Strata failure for 700 m mining depth.

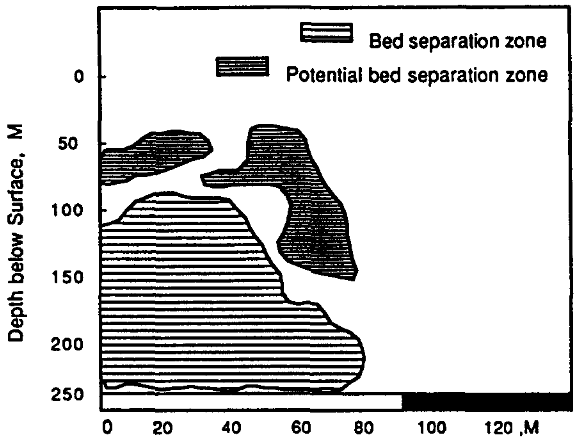

Fig. 9. Bed separation zones for 250 m mining depth.

and 750 m are illustrated in Figs 5–8. It is apparent from the results that the height of the fractured zone increases with a reduction in depth of cover. Where the depth is less than 250 m the fractured zone extends to 210 m above the seam with the zone of potential fracture even closer to the surface, as illustrated in Fig. 5. The height of the fractured zone is significantly reduced relative to depth when the depth is greater than 400 m. When the depth is in the neighbourhood of 700 m or more, both the fractured zone and the potential fractured zone are concentrated within about 150 m above the seam. These calculations also suggest the intuitive result that above a threshold depth of cover, the excavation induced disturbance increases no further. For this particular extraction geometry the threshold depth is of the order of 500 m. Induced strata failure related to vertical deformations are illustrated in Figs 9–12 for the extraction geometries identified previously. At depths greater than 500 m, there is a tendency for the maximum height of the bed separation zone to decrease with an increase in mining depth.

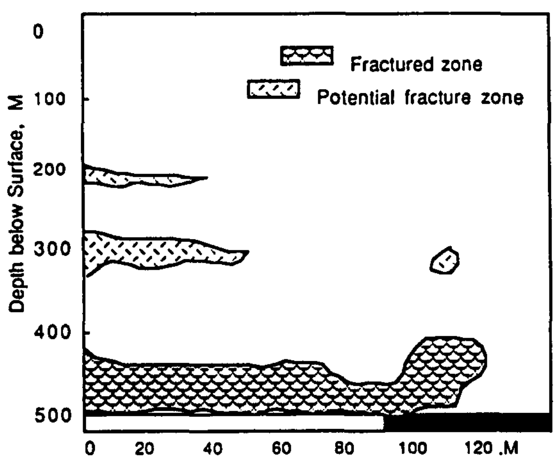

Fig. 7. Strata failure for 500 m mining depth.

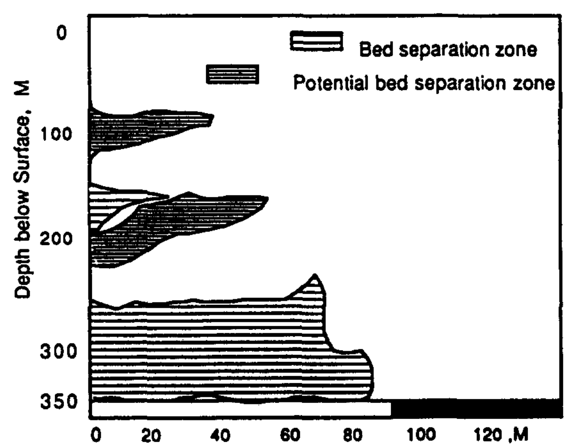

Fig. 10. Bed separation zones for 350 m mining depth.

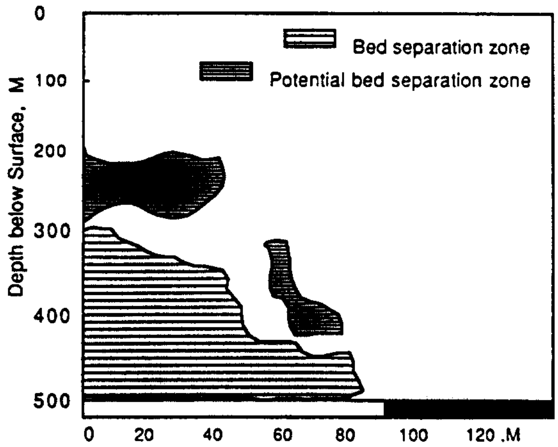

Fig. 11. Bed separation zones for 500 m mining depth.

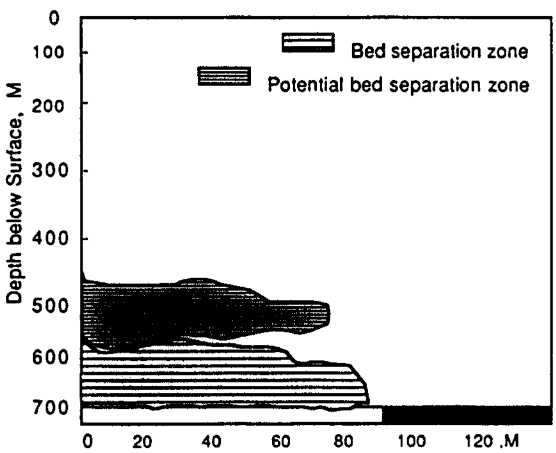

Fig. 12. Bed separation zones for 700 m mining depth.

CASE STUDY

Water inflow during longwall mining was a routine problem encountered in the Barnsley seam at Wistow mine in Britain. The extracted seam height is 2.2 m, average depth of cover is 358 m and width of mining is 180 m. A known aquifer comprising strong waterbearing Permian strata overlies the A1 panel with the distance between the base of the Permian strata and the Barnsley seam being 94 m.

The proposed analytical method is used to analyze this example. The values of elastic parameters were determined from laboratory tests on the rock samples taken directly from the mine [9]. By applying the failure criterion derived from the testing program the zones of strata failure within the section are determined and are illustrated in Figs 13 and 14. The analytical results suggest that the expected height of the fractured and bed separation zones would vary between 70 and 120 m. It is possible that the zones of strata failure had reached the base of Permian strata, resulting in the high recorded water inflow into the mine workings. This postulate is confirmed by geophysical logging in the borehole at the centre of the A1 face as identified in Fig. 15. The logging indicated an exceptional increase in vertical strain occurring 100 m above the Barnsley seam that is quite consistent with the result shown in Fig. 14.

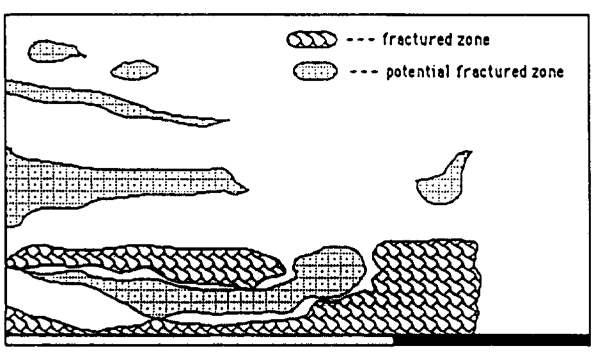

Fig. 13. Strata failure due to longwall mining at Wistow Mine.

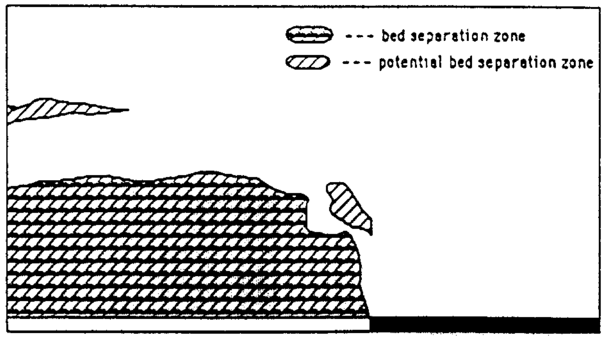

Fig. 14. Bed separation zone in longwall mining at Wistow mine.

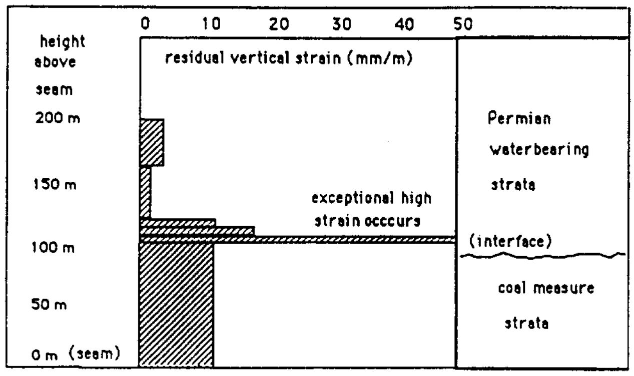

Fig. 15. Residual vertical strain vs depth at Wistow mine.

CONCLUSIONS

An analytical model is presented for the evaluation of stress and displacement distributions in layered media. Coupled with a threshold-strain based failure criterion for the overlying strata, the extent of bed separation and strata failure may be routinely estimated for known or prescribed extraction geometries.

Comparison of the results from the analytical model with displacements measured *in situ* at the Wistow colliery illustrates the utility of the method in predicting the extent of strata failure. Particular reference is made as to the likelihood of potential water incursion as hydraulic connection with an overlying acquifer is completed.

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