

MULTIPHASE FLOW

GeoEE 500

1. Uncontained Flows

- 1.1. Bubble Mechanics
- 1.2. Rayleigh-Taylor Instability

2. Porous-medium Flows

- 2.1. Immiscible Displacement – Definition of terms
- 2.2. Significance to Environmental Problems
- 2.3. Static Behavior
 - 2.3.1. Interfacial Relationships and Capillary Pressures
 - 2.3.1.1. Single Pore Models
 - 2.3.1.2. P_c - S_w Diagrams – Porous and Fractured Media
 - 2.3.1.3. Drainage and Imbibition
 - 2.3.1.4. Fingering
- 2.4. Mobile Behavior
 - 2.4.1. Equation of Motion
 - 2.4.2. Darcy's Law – Relative Permeabilities
 - 2.4.3. Multiphase Transport Equations
- 2.5. Behavior of LNAPLs
- 2.6. Behavior of DNAPLs
- 2.7. Unsaturated Flow – Vadose Zone
 - 2.7.1. Brooks-Corey (1966) Relations
 - 2.7.2. Van Genuchten (1980) Relations

MULTIPHASE FLUIDS

- Two or multiple fluid phases

• Reactors - Bubbles } large surface area $\therefore$ RXN - 2 fluids
 Spraying }

- Porous media - Oil/Water/Gas + Solid
 Water/Gas + Solid

Refractive/G/w content.
Vadose zone

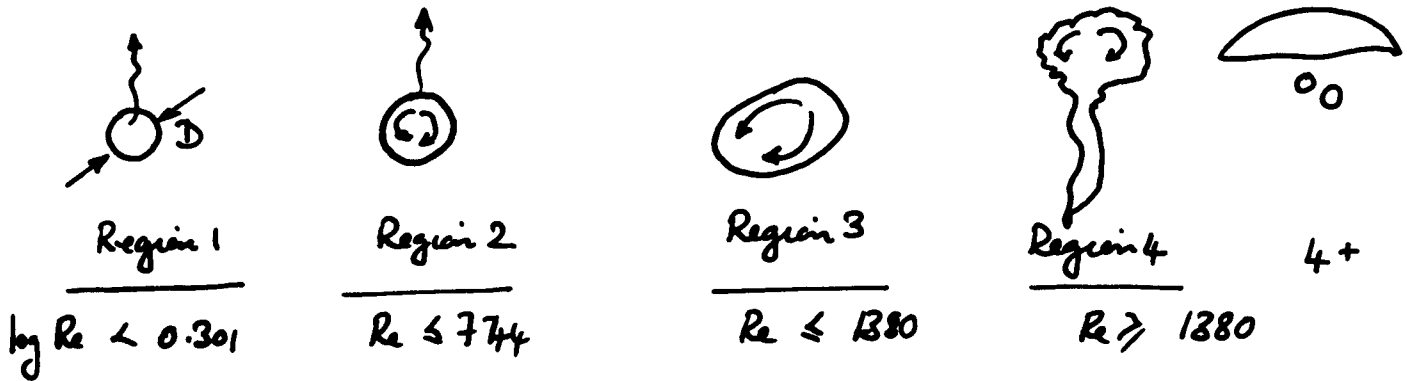
- Similar processes apply in each.
- Main additional complication is the presence of interfacial tension is contributing to response, σ .

- Extra non-dimensional parameter.

- e.g. Stokes settling: $Re \rightarrow$ Friction factor
 $Fr \rightarrow$ Buoyant motion

$$\text{Weber No.} = \frac{\text{Inertial force}}{\text{Surface tension}}$$

IN THE ABSENCE OF POROUS MEDIUM - Bubble Mechanics



Controlled by 3 parameters:

Primarily by parameters of the fluid (not gas): $\rho, \mu, \sigma + g + D + v$

$$Re = \frac{Dv\rho}{\mu} = \frac{\text{Inertial force}}{\text{Viscous force}}$$

$$Fr \Rightarrow Fr \left(\frac{\rho_L}{\rho_L - \rho_g} \right) = \frac{v^2}{Dg} \frac{\rho_L}{(\rho_L - \rho_g)} \approx \frac{v^2}{Dg} = \frac{\text{Inertia}}{\text{Gravity}} = Fr$$

$$We = \frac{Dv^2\rho}{\sigma} = \frac{\text{Inertial}}{\text{Surface tension}}$$

Note: Also the "Bond No" in porous media flow

$$\text{Bond No} = Bo = \frac{\text{Gravity}}{\text{Surface tension}} = \frac{We}{Fr} = \frac{\rho D^2 g}{\sigma}$$

If liquid defined a priori, then steady rise of bubble is constrained in the plane defined by:

$$Re^4 Fr We^{-3} \equiv \rho \sigma^2 g^{-1} \mu^{-4} \quad \text{i.e. } f(\text{fluid properties only, } +g)$$

Bubbles must propagate in this plane, since the properties define a single fluid.

$$Re^4 Fr We^{-3} = \text{Constant} = C$$

$$4 \log Re + \log Fr + 3 \log (1/We) = \log C = \text{constant}$$

May define behaviour of 2 fluids in this plane.

1. Define C for any fluid at any Temp Fig 10.11.3

2. Behaviour confined to this plane

e.g. Bubble shape = $f(Re)$ only Fig. 10.11.4

3. Bubble trajectories shown Fig 10.11.5

4 Evaluate Drag Coeffs.

		Drag Coeff, C_D
Regime	1	$24 Re^{-1}$
	2	$18.2 Re^{-0.682}$
	3	$0.366 We Fr^{-1}$
	4	2.61

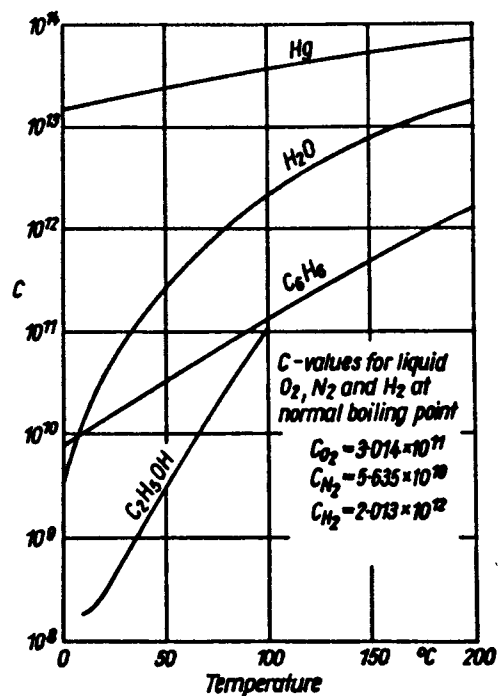


FIG. 10.11.3. Liquid constant C for various liquids as a function of temperature.

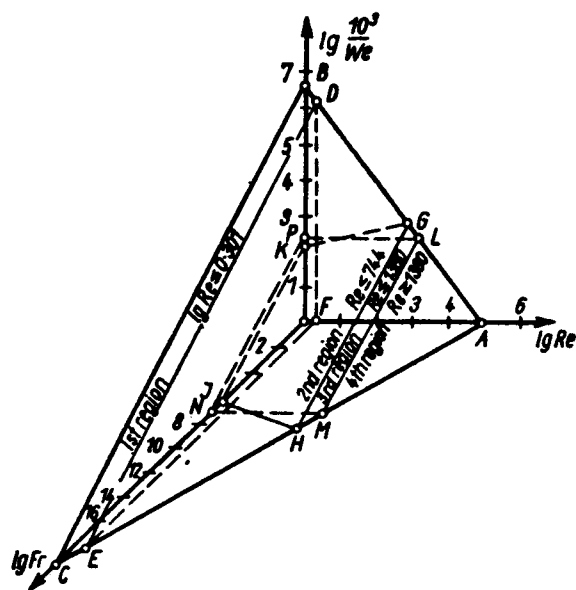


FIG. 10.11.4. Spatial representation of the liquid plane with the existence ranges of the different forms of bubbles.

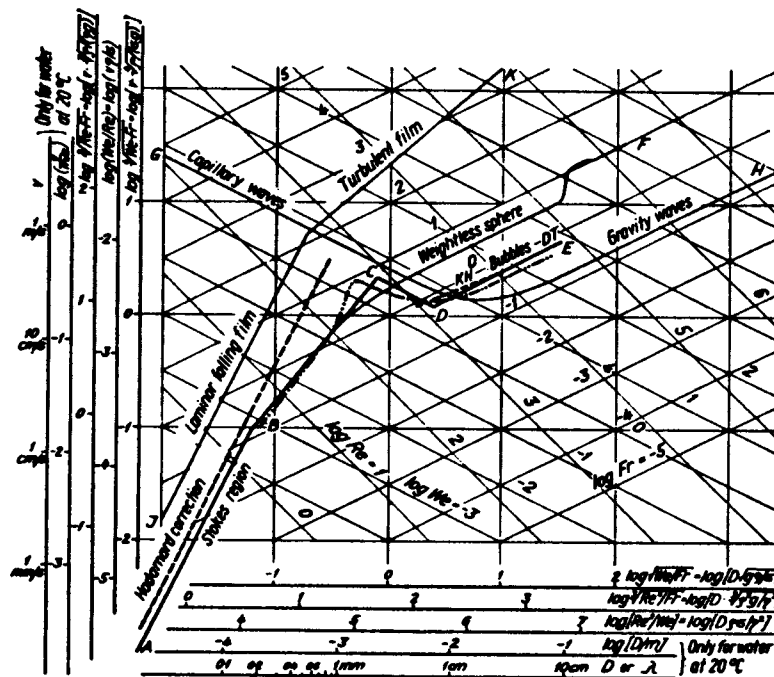


FIG. 10.11.5. Representation of various flow processes in the liquid plane for $C = 3.9 \times 10^{10}$ (e.g. water at 20 °C). — Rate of rise of bubbles; curve ABCD after Peebles and Garber (1953); curve DE after Davies and Taylor (see p. 705, footnote †).

KH Results of Van Krevelen and Hofstijzer (1950). — **ABF** Rate of rise of weightless sphere with ζ according to Fig. 10.3.1. — Stokes law taking Hadamard correction into account. — Curve according to W. M. Haberman and R. K. Morton for air bubbles in filtered and distilled water at 19 °C. **JK** Falling film [eqns. (7.24.6) and (7.24.9)]. **GH** Surface waves [eqn. (10.11.6)]. For the literature references see p. 702, footnote †.

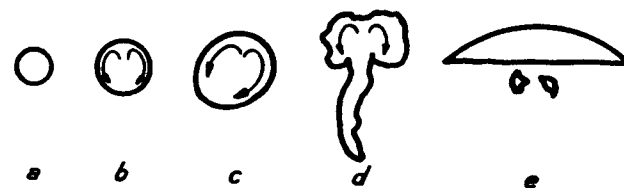


FIG. 10.12.1. The main shapes assumed by bubbles on rising in a liquid: (a) small spherical bubble with no internal circulation; (b) small spherical bubble with internal circulation; (c) elliptical bubble rising along a spiral path; (d) irregular-shaped mushroom bubble; (e) spherical-cap bubble, the upper boundary of which, according to Davies and Taylor, has exactly the shape of a spherical cap.

Gravitational Instability of the Bottom Boundary Layer

The initiation of plumes must clearly begin with the formation of upwelling regions. One approach is to investigate the gravitational instability of a horizontal layer of fluid lying under a denser layer of fluid. The lower layer will then develop Rayleigh-Taylor instability. The wavelength of maximum growth rate and the exponential time constant of growth have been theoretically and numerically predicted for a number of geometries and boundary conditions (Rayleigh 1900, Dobrin 1941, Chandrasekhar 1955, Dančs 1964, Selig 1965, Ramberg 1963, 1967a, 1968a,b,c, 1970, Biot 1966, Biot & Ode 1965, Berner et al. 1972, Whitehead & Luther 1975, Marsh 1979, Whitehead et al. 1984). Demonstration experiments with putty and other non-Newtonian fluids have been extensively photographed and compared with geological formations by Nettleton (1934, 1943), Parker & McDowell (1955), and Ramberg (1963, 1967b, 1970). There was no intercomparison between the laboratory experiments and theory due to the unknown rheology of the laboratory materials.

The limit in which one layer is thin and of lower viscosity than the other is particularly relevant in the geophysical context. Take the configuration shown in Figure 6, in which a thin layer of density $\rho - \Delta\rho$, depth d , and viscosity μ_1 lies under a semi-infinite fluid of density ρ and viscosity μ_2 . Let the wavelength l of the disturbance $\eta(x, t)$ to the horizontal interface be larger than the depth d of the thin layer, so that the lateral velocity u in the thin layer is larger than the vertical velocity. The scaling argument used here can be found in a number of the cited studies. The force balance is between the lateral pressure gradient p/l and viscosity, and thus we have

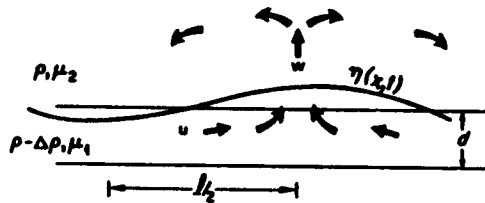


Figure 6 Definition parameters for the analysis of Rayleigh-Taylor instability of a thin layer lying under a deep fluid.

$$\frac{p}{l} = \mu_1 \frac{u}{d^2}. \quad (1)$$

In the deep viscous fluid that lies overhead, the force balance is between stress due to the vertical velocity w of the interface, pressure, and buoyancy, which gives us

$$\frac{p}{l} = \mu_2 \frac{w}{l^2} + \frac{g\Delta\rho\eta}{l}. \quad (2)$$

Combining (1) and (2), we have

$$\mu_1 \frac{u}{d^2} - \mu_2 \frac{w}{l^2} = \frac{g\Delta\rho\eta}{l}. \quad (3)$$

Using continuity

$$\frac{u}{l} + \frac{w}{d} = 0, \quad (4)$$

kinematics of the interface

$$\frac{\partial\eta}{\partial t} = w, \quad (5)$$

and the fact that the growth will be exponential ($w = w_0 e^{\sigma t}$), we find that

$$\sigma = \frac{g\Delta\rho}{\mu_2} \frac{l}{\frac{\mu_1}{\mu_2} \frac{l^3}{d^3} + 1}. \quad (6)$$

The growth rate σ is shown as a function of the normalized wavelength l/d in Figure 7. Note that the maximum growth occurs at

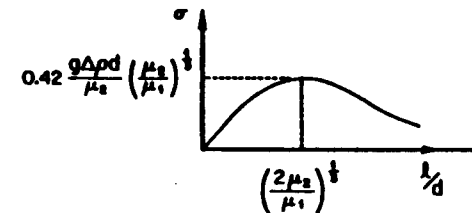


Figure 7 Growth rate as a function of wavelength for one layer undergoing Rayleigh-Taylor instability.

$$\frac{l}{d} = \left(\frac{2\mu_2}{\mu_1} \right)^{1/3} = 1.26 \left(\frac{\mu_2}{\mu_1} \right)^{1/3}, \quad (7)$$

where the growth rate is

$$\sigma = 0.42 \left(\frac{g\Delta\rho d}{\mu_2} \right) \left(\frac{\mu_2}{\mu_1} \right)^{1/3}. \quad (8)$$

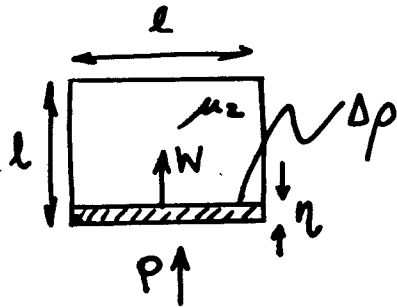
The central finding here is that the fastest growth length scale is proportional to $(\mu_2/\mu_1)^{1/3}$ (which may be a large number). Two superior (but



Figure 8 An experimental demonstration of Rayleigh-Taylor instability.

FORCES AND MASS BALANCES

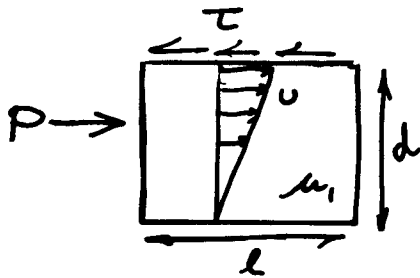
OVERLAYING LAYER



$$P \cdot l = \tau \cdot l + (g \Delta \rho) \eta \cdot l$$

$$\text{or } \frac{P}{l} = \mu_2 \frac{W}{l^2} + \frac{(g \Delta \rho) \eta}{l} \quad (2)$$

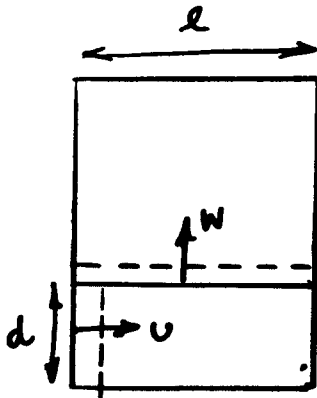
UNDERLAYING LAYER



$$P \cdot d = \tau \cdot l = \mu_1 \frac{u}{d} \cdot l$$

$$\text{or } \frac{P}{l} = \mu_1 \frac{u}{d^2} \quad (2)$$

CONTINUITY



$$\frac{u \cdot d}{l \cdot d} + \frac{W \cdot l}{l \cdot d} = 0$$

$$\frac{u}{l} + \frac{W}{d} = 0 \quad (4)$$

IMMISCIBLE DISPLACEMENT - POROUS MEDIA

Controlled by properties of:

Fluids - ρ, μ, σ

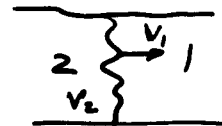
Medium - pore diameter, D .

spacing between pores, l

Appropriate Dim. dimensional parameters:

Ca Capillary No. $\equiv \frac{\text{Viscous forces}}{\text{Capillary forces}} \equiv \left\{ \begin{array}{l} Re = \text{Inertial/Viscous} \\ We = \text{Inertial/Interfacial} \end{array} \right\} \frac{We}{Re} = Ca$

M Mobility Ratio $\equiv \frac{v_1}{v_2} \equiv \frac{k_{r1} \mu_2}{\mu_1 k_{r2}} \approx \frac{\mu_2}{\mu_1}$



$M \geq 1$ No viscous fingering

$M < 1$ Viscous fingering

Bo Bond No $\equiv \frac{\text{Gravity}}{\text{Interfacial}} \equiv \frac{We}{Fr}$

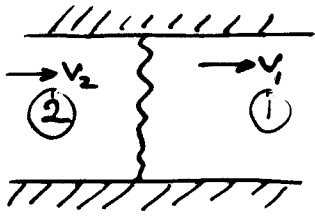
Note: Ca & Bo includes:

$\left. \begin{array}{l} Re \\ We \\ Fr \end{array} \right\}$ but some info lost since 2 parameters (groups) for 3 individual.

ok. since always laminar - no transition

M gives information on instability - a new length scale.

IMMISCIBLE DISPLACEMENT



VISCIOUS
FINGERING

$$\log(Ca) \quad Ca = \frac{v_2 \mu_2}{\sigma}$$

$$Ca = \frac{\text{Viscous}}{\text{Capillary}}$$

2 is the displacing fluid

Absence of buoyancy effects:

- Viscous fingering when low viscosity fluid displaces a high viscosity fluid.
- Front stabilizes if high viscosity displaces low viscosity.
- Capillary fingering results at low flow rates.

Presence of buoyancy effects:

$$Bo = \frac{g(\rho_2 - \rho_1)(D/2)l}{\sigma} \equiv \frac{\text{Gravity}}{\text{Interfacial}}$$

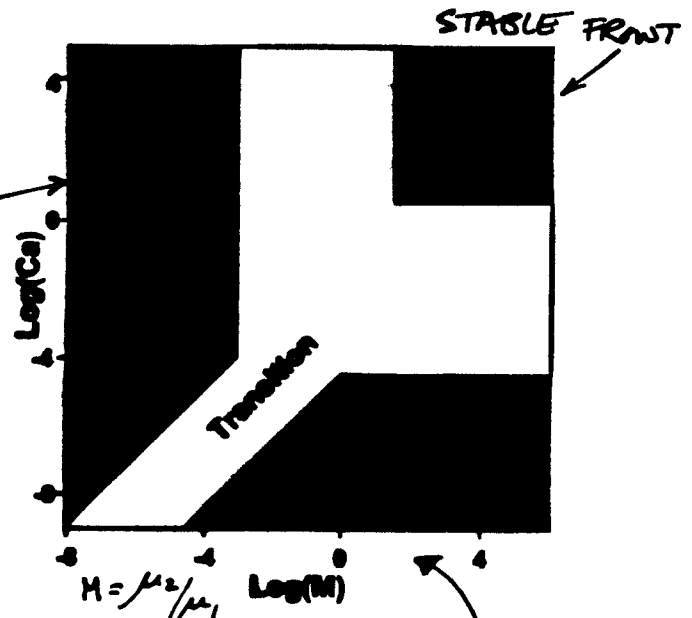
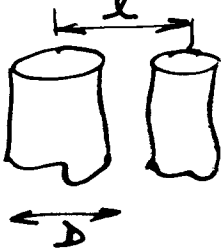


Figure 1. Two-dimensional phase diagram proposed by Lenormand et al. [1988], showing immiscible displacement regimes as a function of their position in M, Ca space. (Copyright Cambridge University Press; reprinted with permission.)

CAPILLARY FINGERING

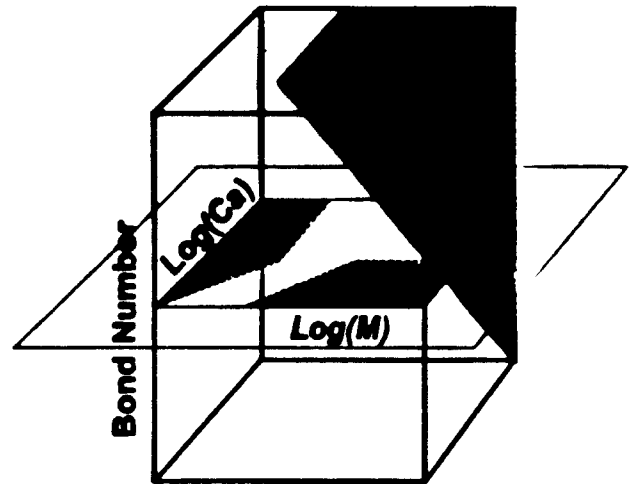


Figure 2. Extension of Lenormand et al.'s [1988] phase diagram to three dimensions, showing the hypothesized configuration of the stable flow regime. Lenormand et al.'s [1988] diagram is the $Bo = 0$ plane. Because no physical experiments were performed, values other than zero are deliberately omitted from the vertical axis. (After Berkowitz and Ewing [1998]. Copyright Kluwer Academic Publishers; reprinted with permission.)

FLOW - REFRESHER

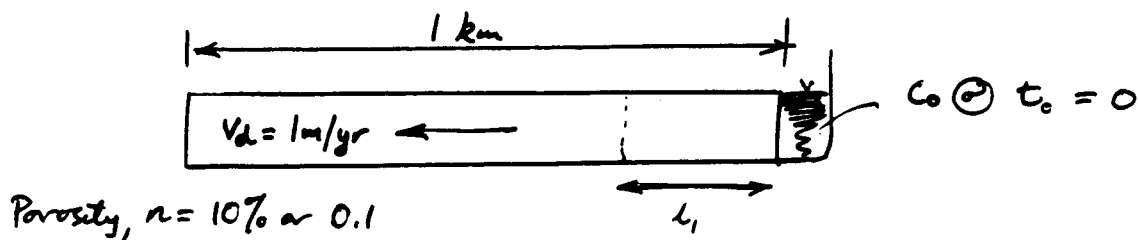
$$\frac{k}{\mu} = \frac{K}{\rho g}$$

$\swarrow$ Porosity (L^2) $\swarrow$ Hydraulic conductivity (LT^{-1})

$$Q = -AK \frac{dh}{dx} \quad ; \quad v_d = -K \frac{dh}{dx}$$

$$v_a = \frac{v_d}{n}$$

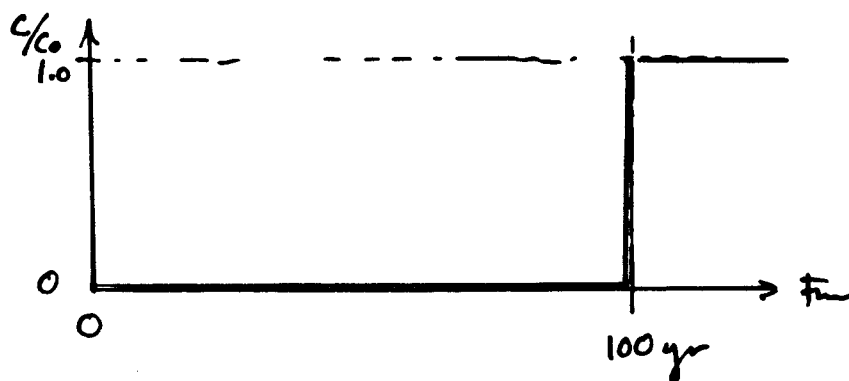
RTD (Residence Time Distribution) or "Breakthrough" Curves



Darcy Velocity: $v_d = 1 \text{ m/yr} = \frac{1}{1000} \text{ km/yr}$

Advective velocity: $v_a = \frac{v_d}{n} = \frac{10^{-3}}{0.1} = 10^{-2} \text{ km/yr}$

Breakthrough time: $v_a = \frac{l}{t} \therefore t = l/v_a = \frac{1 \text{ km}}{10^{-2} \text{ km/yr}} = 100 \text{ yr}$



LNAPL

Idealized behavior

eg. Gasoline, kerosene.

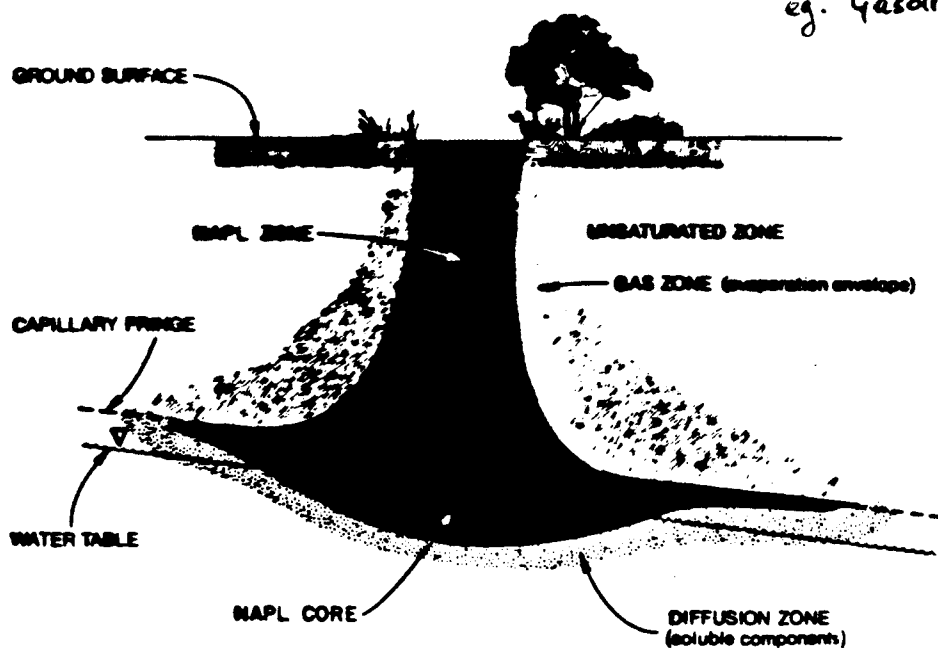


Fig. 1. Schematic representation of lighter than water NAPL movement through the unsaturated and into the saturated zone [after Abrisola and Pinder, 1985a].

LNAPL

eg. Dry cleaning solvents, TCE, PERC.

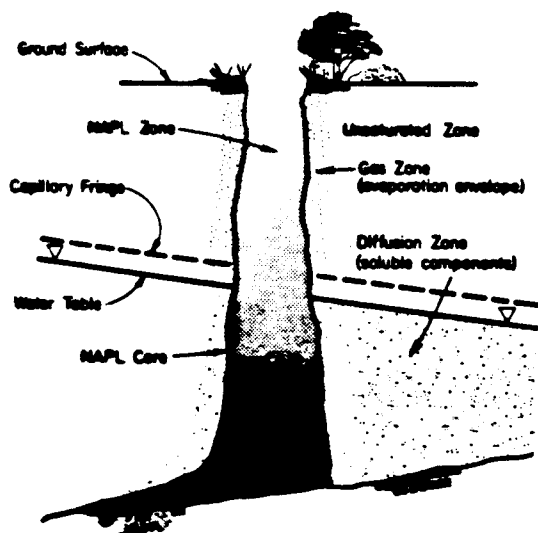


Fig. 2. Schematic representation of heavier than water NAPL movement through the saturated and unsaturated zones.

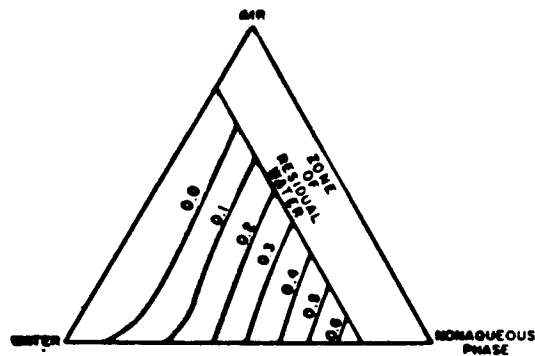
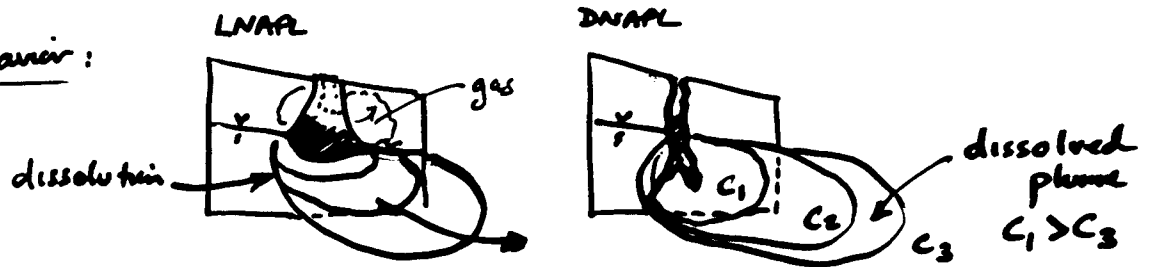


Fig. 3. Ternary diagram showing the relative permeability of the nonaqueous phase as a function of phase saturations [after Faust, 1985].

Immiscible Transport

Idealized behavior:



If contaminants are immiscible then these processes are important

True behavior:

1. Fingering results
2. Erratic and unpredictable distribution
3. DNAPLs "very" penetrative $\left\{ \begin{array}{l} \text{low viscosity} \\ \text{high density} \\ \text{low interfacial tension} \end{array} \right.$

Questions:

- How far will they migrate
- What are the controls on penetration
 - clay - vs - sand aquitards
 - fractured - vs - unfractured
- How do they dissolve / bind / retard
- How may they be:
 - Remobilized
 - Chemically immobilized
 - Physically isolated.

FLOW OF IMMISCIBLE FLUIDS

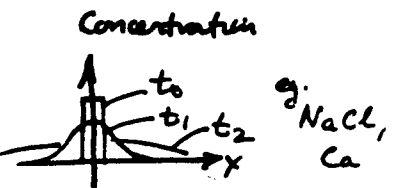
Bear, J., Dynamics of Fluids in Porous Media,
Chapter 9, 1988.

1. Types of Fluid Flows:

Miscible displacement: Two fluids completely soluble

No interface $\therefore$ no interfacial tension

Hydrodynamic dispersion/diffusion



Immiscible displacement: Simultaneous flow of two fluids

Capillary pressure difference. eg. Air-water

Oil-water (gas)



Some mixing at interface is possible
but likely small.

2. Interfacial Tension and Capillary Pressure:

2.1 Saturation and Fluid Content:

$$S_\alpha = \frac{\text{volume of fluid } \alpha \text{ within REV}}{\text{volume of voids within REV}}; \sum_\alpha S_\alpha = 1$$

$$S = V_f / V_v$$

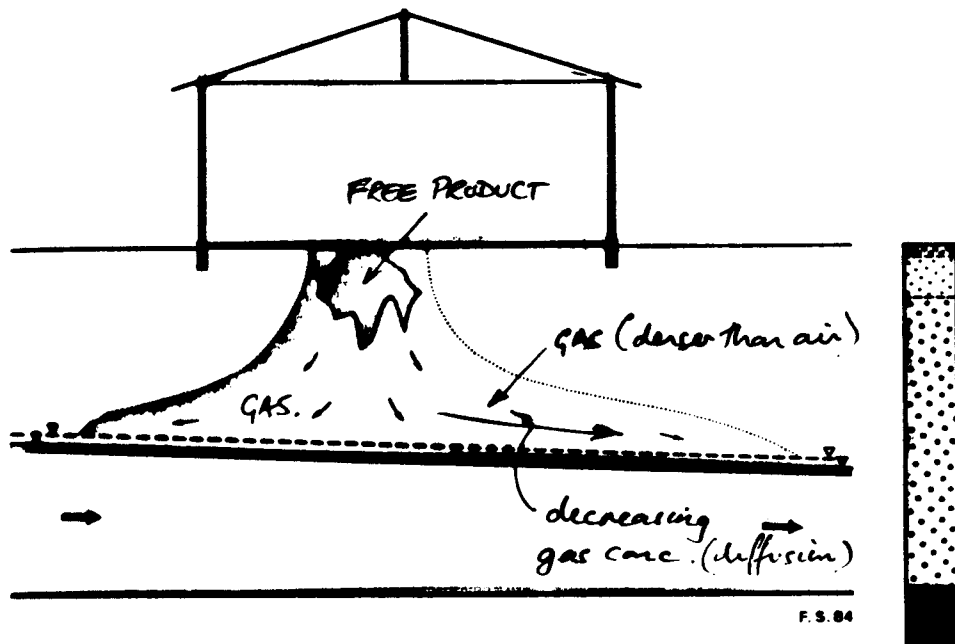
Unsaturated flow; volumetric moisture content, Θ : $\Theta = \frac{\text{vol. water in REV}}{\text{bulk vol. of REV}} \neq 1$

$$\Theta = V_w / V_T; V_T = V_v + V_s$$

Note that moisture content in soil mechanics is by
weight, not volume. i.e.

$$m/c = \frac{W_w}{W_s}$$

III.a.



III.b.

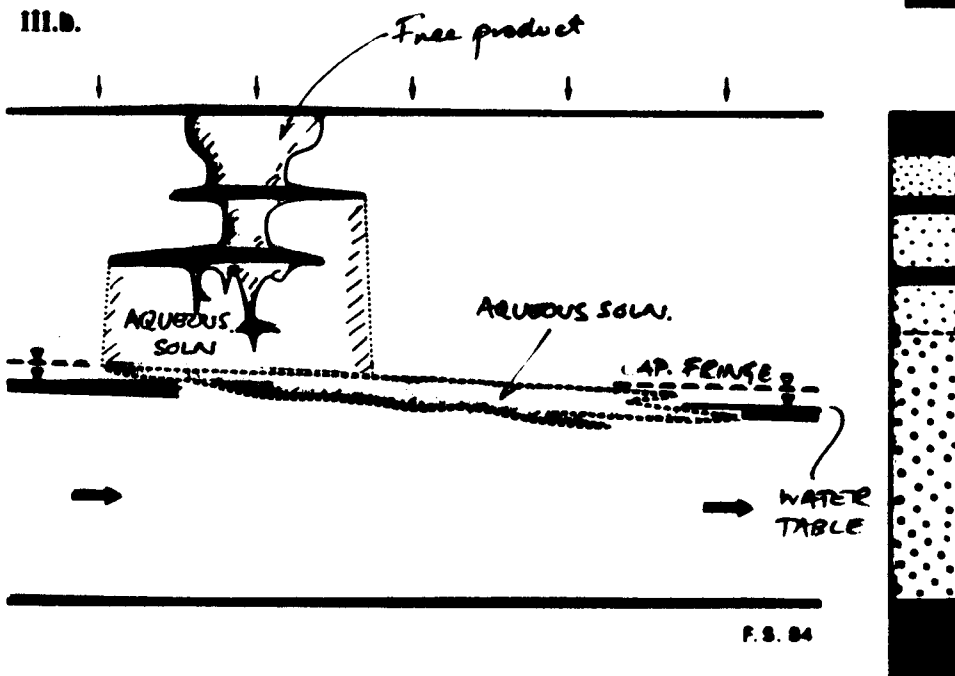


Figure III.a. Schematic of a small spill in a permeable unsaturated zone with a resulting mound of CHC gas. Concentration decreases with distance from spill. Grainsize shown at right.

Figure III.b. Larger spill than in III.a., but still not large enough to exceed the retention capacity of the unsaturated zone; no liquid CHC reaches the

XVI.a.

INITIALLY UNSAT (DIST)



XVI.b.

SATURATED



Figure XVI.a. Beads initially moist; diameter range = 0.85 - 1.23 mm. PER then dripped in from above. The PER accumulated as a sheath around a zone of high water content.

Figure XVI.b. Beads initially saturated with water; diameter range = 0.49 - 0.70 mm. PER then applied from above. When the flow of PER was discontinued, the front portion of the PER stream broke off and beaded

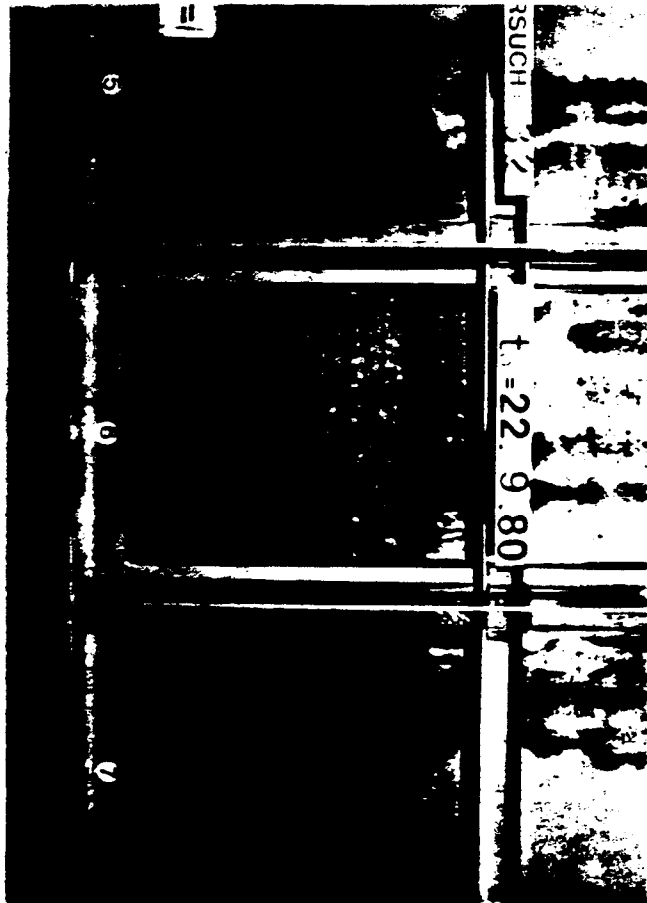
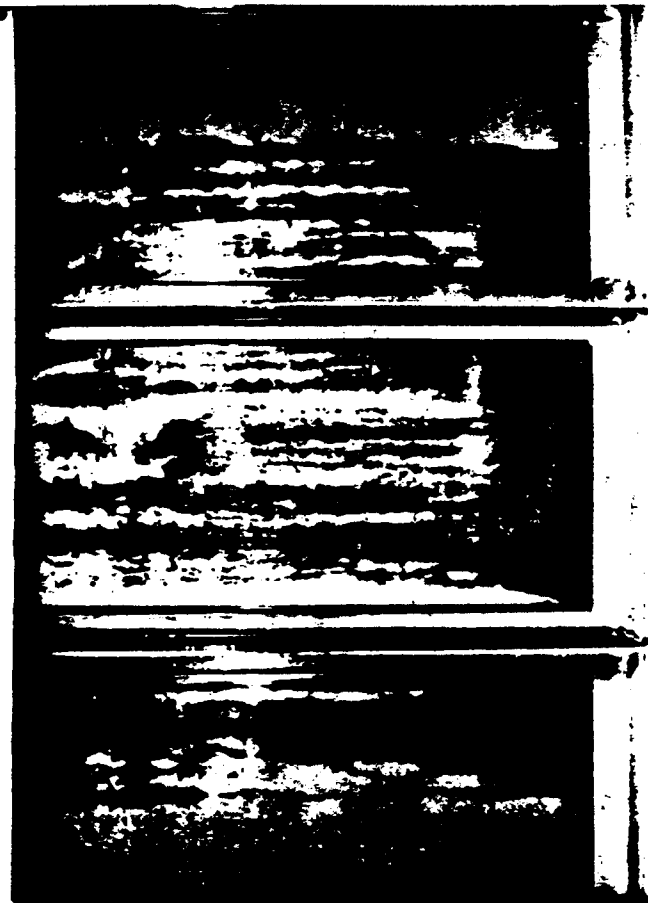
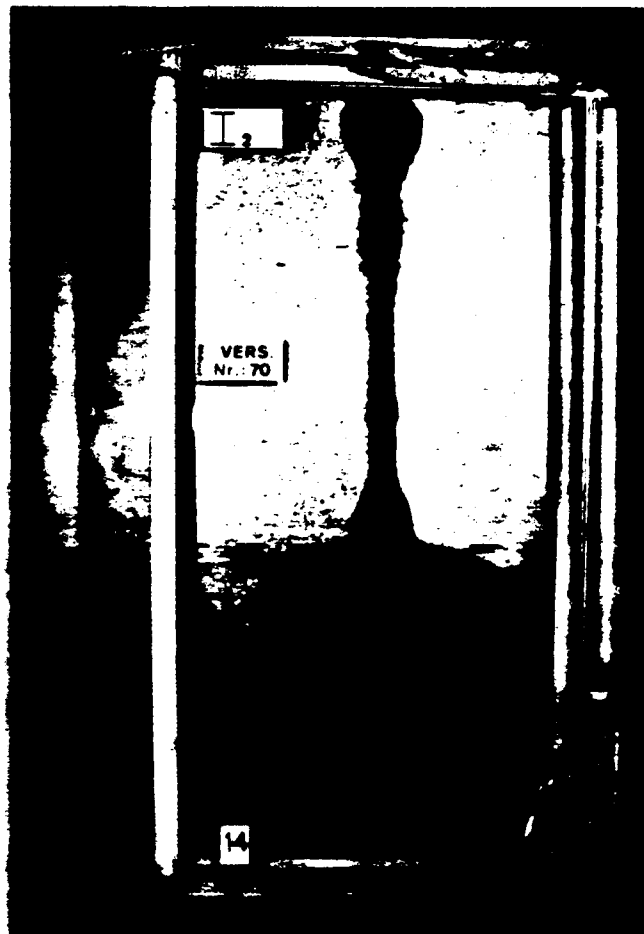


Figure X.a. Sheet-like spill of 36.3 L of PER. View of spill above the capillary fringe.

Figure X.b. Sheet-like spill of 36.3 L of PER. View of spill below the capillary fringe. Time = ~ 10 min.

XI.a. AFTER 1 HOUR



re XI.a. of PER. Kinematic viscosity = 0.54 mm²/s. Time = ~1 h.

re XI.b. of PER. Later view of spill depicted in re XI.a. Kinematic viscosity = 0.54 mm²/s.

XI.b.



2.2 Interfacial Tension and Wettability

Interfacial tension due to molecular attraction - different in two fluids

Interfacial tension, σ_{ik} , between two fluids i and k .

σ_{ik} is temperature dependent $\therefore$ Capillary pressure = $f(T)$.

Dupré's formula

$$W_{ik} = \sigma_i + \sigma_k - \sigma_{ik} \Rightarrow \text{work to separate into substrate } i \text{ and } k \text{ with vapor interface}$$

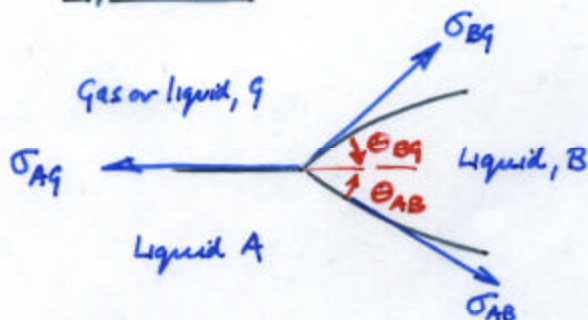
σ_i = surface tension

σ_{ik} = interfacial tension

$$\frac{i}{k} \Rightarrow \frac{\text{vapor } i}{k}$$

σ_i = surface tension of fluid with its own vapor.

Equilibrium



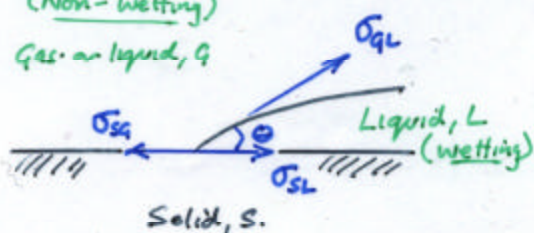
$$\text{Equilibrium: } \sigma_{AG} = \sigma_{AB} \cos \theta_{AB} + \sigma_{BG} \cos \theta_{BG}$$

Only satisfied if $\sigma_{AG} < (\sigma_{AB} + \sigma_{BG})$
 $\therefore$ lens of B formed.

If $\sigma_{AG} > (\sigma_{AB} + \sigma_{BG})$
 then B spreads between A and G.

(Non-wetting)

Gas or liquid, G



By convention, θ measured in denser fluid

$$\text{Equilibrium: } \sigma_{GL} \cos \theta = \sigma_{SG} - \sigma_{SL}$$

$$\cos \theta = (\sigma_{SG} - \sigma_{SL}) / \sigma_{GL}$$

$\therefore$ May determine interface angle for known σ (contact angle, θ).
 - influenced by surface roughness.

If $(\sigma_{sq} - \sigma_{sl})/\sigma_{ql} > 1 \Rightarrow$ No equilibrium
Liquid flows over solid $\rightarrow$ wettability

Wettability

Defined by θ

$\theta < 90^\circ$ eg Fluid, L, wets the solid; wetting fluid

$\theta > 90^\circ$ eg Fluid G, is non wetting fluid

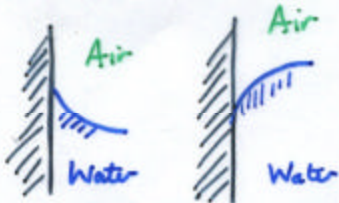
$\theta = 90^\circ$ Zero adhesion tension since equal affinity.

Wettability is controlled by: a) Chemical composition of fluids
eg. adding dopant

b) Sequence of wetting

eg. Advancing or receding.

$\therefore$ hysteresis - wetting or draining



Water wets
solid

Water non-wets
solid

Most geologic systems are water wet.

petroleum/water

air/water

NAPL/water

IMPORTANT ASPECTS OF WETTING

Water-Wet

- a) Pendular rings @ grain contacts
- no continuous water phase
 - except mono-molecular coating of H_2O
- b) $\uparrow$ wetting phase saturation $\rightarrow$
- continuous water (wetting) phase
 - equilibrium saturation = when phase is continuous
 $\downarrow$ (wetting)

Implication: wetting phase may be removed.

- c) Above critical saturation (equilibrium) funicular saturation
- Now non-wet phase non-continuous.
 - Can only remove non-wetting phase if large pressure gradient to squeeze through pore throat.

Funicular saturation critical in remediation!

Remediation @ residual saturations:

Important since difficult
to remove last
few % of saturation.

Grains



Pools



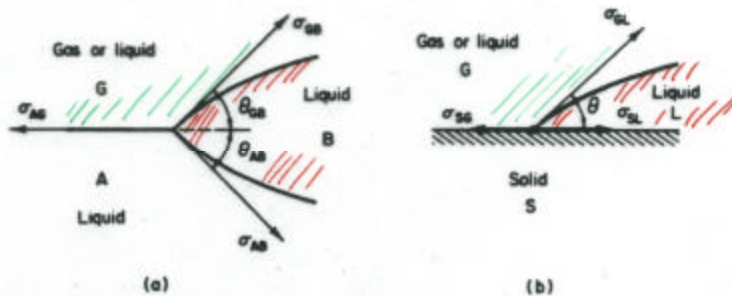


FIG. 9.2.1. Interfacial tensions.

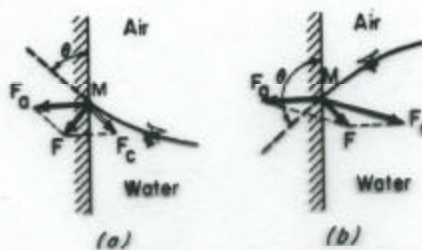


FIG. 9.2.2. Contact angle between a water-air interface and a solid. (a) Water wetting the solid. (b) Water nonwetting the solid.

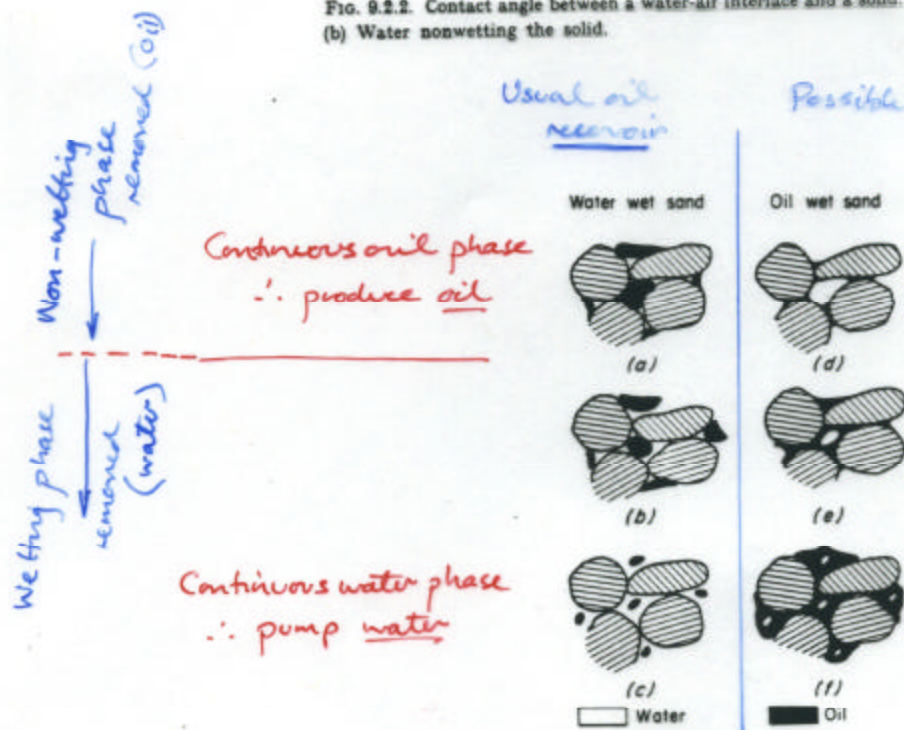
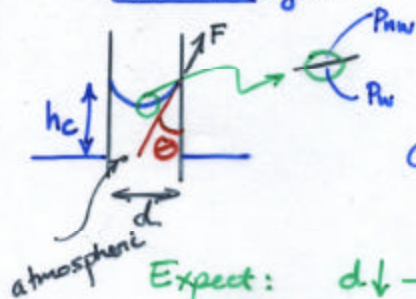


FIG. 9.2.3. Possible fluid saturation states.

2.3 Capillary Pressure

Water-wet system



Important: Controls penetration of immiscible fluids

Note (a) $p_x = p_y = p_z$ within any phase for static conditions

(b) p_w depends on surface curvature

Capillary rise, h_c : $\gamma h_c \pi \frac{d^2}{4} = \sigma_{12} \pi d \cos \theta \quad \therefore h_c = \frac{4 \sigma_{12}}{d \gamma_w}$

Expect: $d \downarrow \rightarrow h_c \uparrow$

d = pore throat or fracture aperture

Capillary pressure, p_c :

$$p_c = p_{nw} - p_w \Rightarrow \frac{p_c}{\gamma_w} = \frac{1}{\gamma_w} (p_{nw} - p_w) = h$$

In the capillary pressure relationship, $h_c = \frac{4 \sigma_{12}}{d \gamma_w}$

the assumption

that $\theta \rightarrow 0$ is

made for clean glass.

Not good for soils/rocks

Most important deduction:

$$p_c = h_c \gamma_w \propto \frac{\sigma_{12}}{d}$$

Capillary pressure inversely proportional to pore dia, d .

Smaller d , requires larger p_c to penetrate.

Since many potential pore throat diameters exist,
a capillary model may be replaced by
a grain-grain contact model.

CAPILLARY PRESSURES - POROUS MEDIA

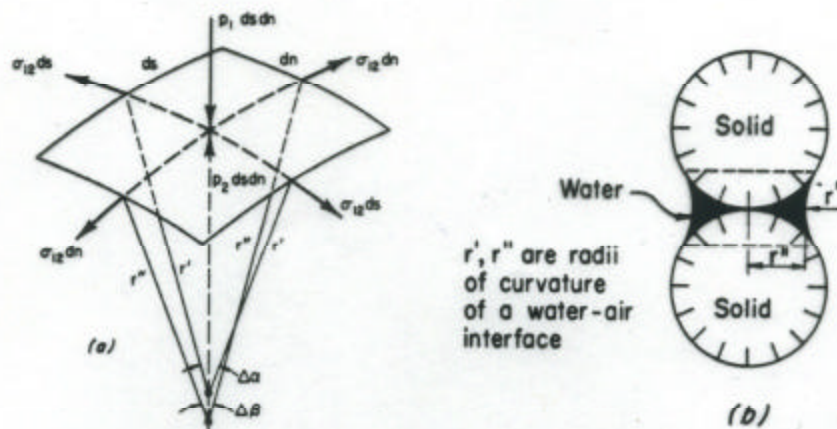


FIG. 9.2.4. Equilibrium at a curved interface between two immiscible fluids.

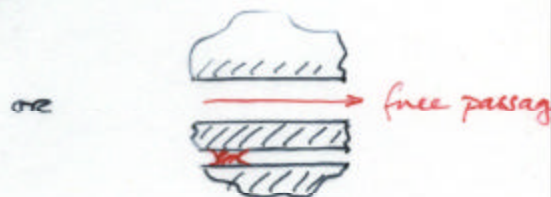
Soil suction or tension

$$\Delta p = p_c = p_2 - p_1 = \sigma_{12} \left(\frac{1}{r'} + \frac{1}{r''} \right) = \boxed{\frac{2\sigma_{12}}{r^*} = p_c}$$

$$r^* = \text{mean radius} \Rightarrow \frac{2}{r^*} = \left(\frac{1}{r'} + \frac{1}{r''} \right) \quad (\text{Laplace eqn}).$$

PROBLEM: r^* is difficult to determine

- Multiple grain sizes (and pore throat sizes)
- Distribution of pore sizes



- Pore geometry
- Fluids (σ_{12}) and contact angles ($\cos \theta$).

$\therefore$ Use a statistical average $\Rightarrow$ Determine $p_c = p(S_w)$ Lab ok
Field bad

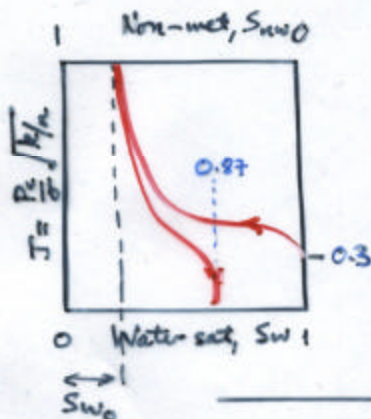
How to DETERMINE $P_c = P_c(S_w)$

For capillary tube of radius, r .

$$P_c = \frac{2\sigma_{12} \cos \theta}{r}$$

Semi-empirical approach, Leverett (1941). Dimensional analysis gives:

$$J = J(S_w) = \left(\frac{P_c}{\sigma} \right) \sqrt{k/n} ; \quad P_c = P_c(S_w)$$

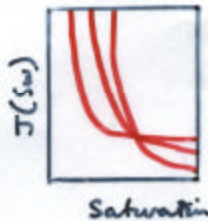


J = Leverett function reduces to a common curve for different materials

k = permeability (L^2)
 n = porosity

$\sqrt{k/n} \propto$ to mean pore diameter

May also be influenced by $\cos \theta$ (factors influencing contact angle).



$$J = J(S_w) = \left(\frac{P_c}{\sigma \cos \theta} \right) \sqrt{k/n}$$

$\therefore$ Dependent on formation type

Brooks & Corey (1964)

$$P_c = P_c(S_e)$$

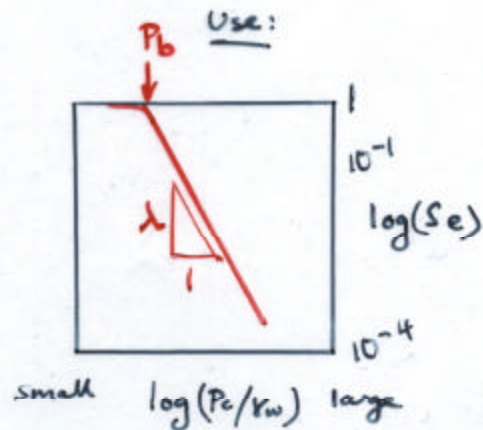
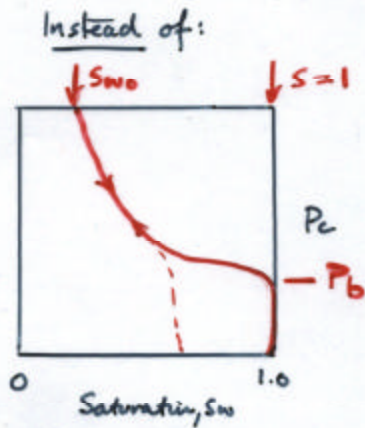
$$S_e = \frac{(S_w - S_{w0})}{(1 - S_{w0})}$$

S_e = effective saturation

S_{w0} = irreducible wetting fluid saturation

Gives a straight line relationship in log-log S_e versus P/S_w space except close to $S_e = 100\%$

BROOKS - COREY CURVES



$$S_e = \frac{(S_w - S_{w0})}{(1 - S_{w0})}$$

$S=1$ (pointing to 1 in denominator)

S_e = effective saturation

S_{w0} = irreducible wetting fluid saturation

Curve defined by two parameters:

λ = -ve slope of curve (pore size distribution)

P_b = intercept of line and $S_e = 100\%$

also termed "bubbling pressure"

Pressure needed to force a "bubble" of fluid through the pore throat

REPRESENTED AS:

$$S = (1 - S_{w0}) \left(\frac{P_c}{P_b} \right)^{-\lambda} + S_{w0}$$



or

$$S_e = \frac{(S - S_{w0})}{(1 - S_{w0})} = \left(\frac{P_c}{P_b} \right)^{-\lambda}$$

i.e. $\log(S_e) = -\lambda \log\left(\frac{P_c}{P_b}\right)$

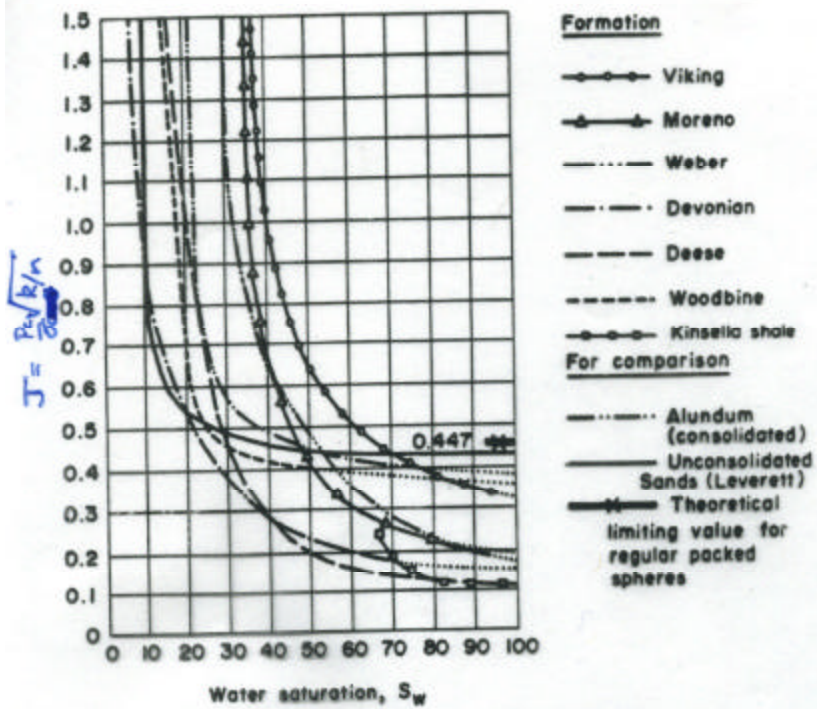


FIG. 9.2.6. Leverett function for various formations (Rose and Bruce, 1949).

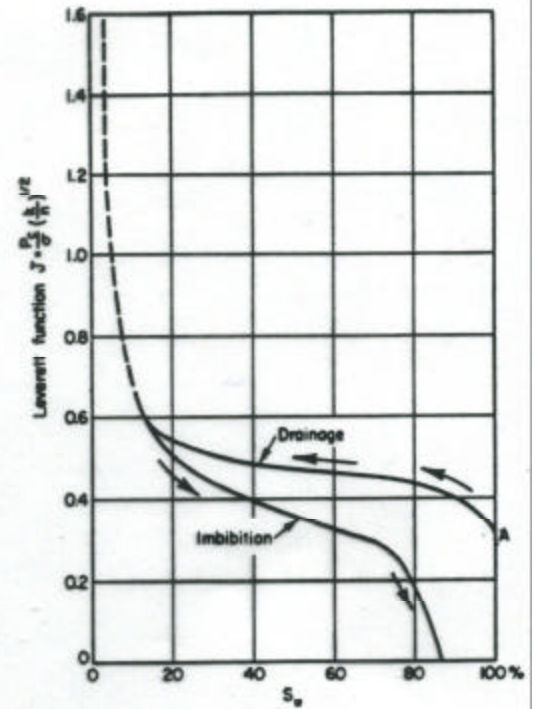


FIG. 9.2.5. Typical Leverett functions for sand (Leverett, 1941).

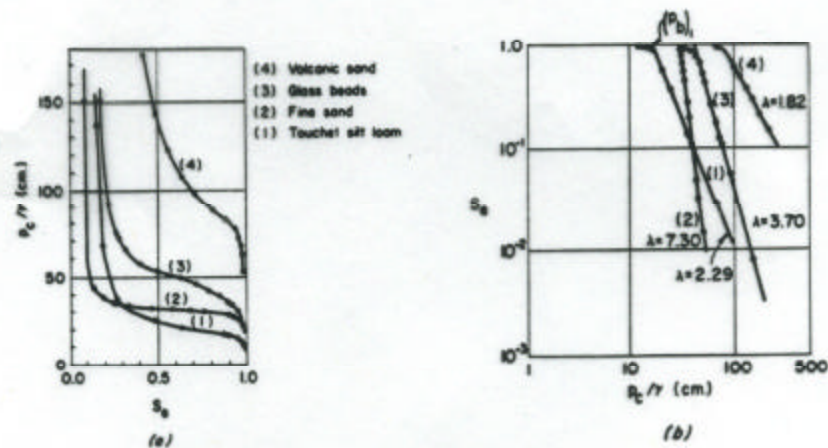
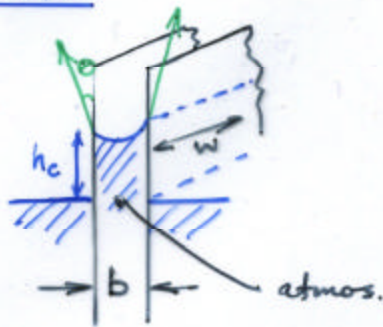


FIG. 9.2.7. Capillary pressure head as a function of effective saturation for porous materials of various pore-size distributions (Brooks and Corey, 1964).

CAPILLARY RISE IN FRACTURES

IDEALIZED



$$w \cdot b \cdot h_c \gamma_w = 2w\sigma \cos \theta$$

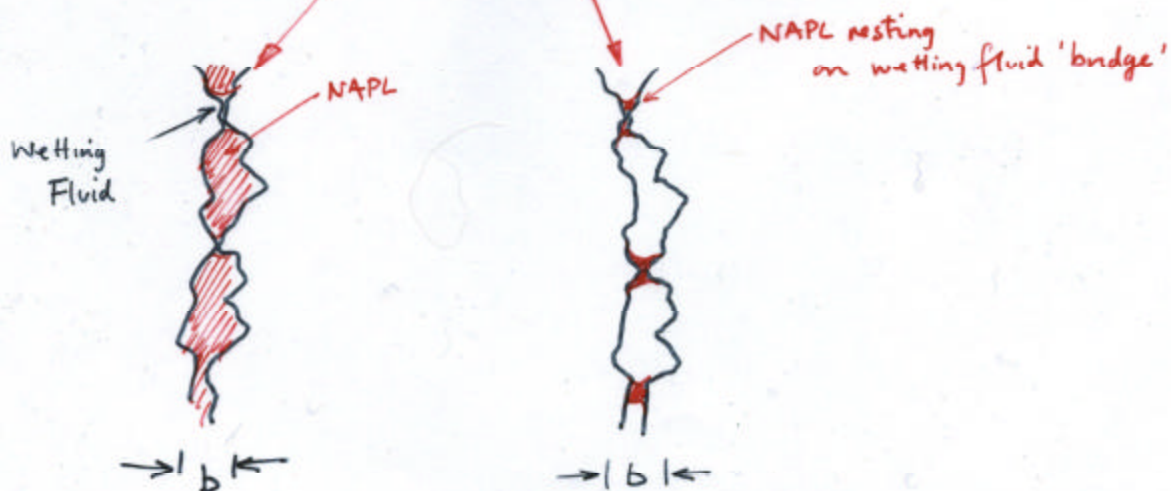
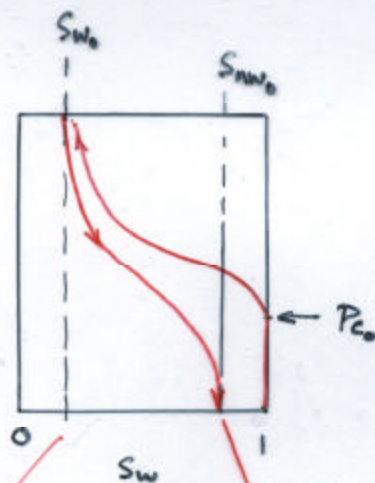
$$h_c = \frac{2\sigma}{\gamma_w b}$$

$$P_{c0} = \frac{2\sigma}{b}$$

$$P_{c0} \propto \frac{\sigma}{b}$$

REAL $P_c = f(S_w)$

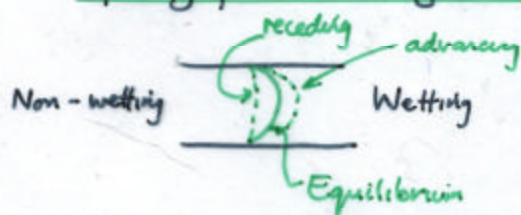
$$\sigma \approx 7 \times 10^{-2} \text{ N/m.}$$



2.4 DRAINAGE AND IMBIBITION

Capillary pressure is hysteretic

⊖ changes with direction of displ.

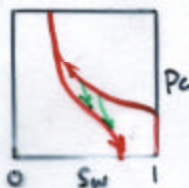


⊖ For two fluids (liquids)

For Fluid-gas



Raindrop



∴ Not a unique function of S_w , but depends on path.

∴ conclude that we cannot determine P_c from S_w , alone

Drainage - Saturated with wetting fluid initially - displace with non-wetting

Imbibition - Saturated with non-wetting fluid initially -

place wetting fluid on surface → "spontaneous" imbibition
to equilibrium condition where capillary
forces equal those due to gravity.

Unsaturated flows - Water = wetting; air = non-wetting

S_{NW_0} = entrapped air in sample

$S_{NW} \leq S_{NW_0}$ the non-wetting fluid is non continuous
∴ does not flow.

Drying and wetting scanning curves are hysteretic.



FIG. 9.2.8. Contact angle (θ) in a capillary tube in a stationary state, in a displacement of a nonwetting liquid by a wetting one (θ_1) and in a displacement of a wetting liquid by a nonwetting one (θ_2).

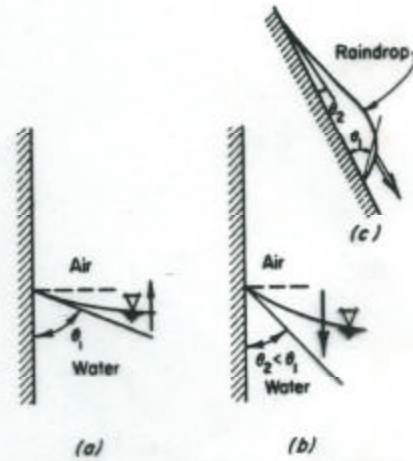


FIG. 9.2.9. Hysteresis in contact angle (rain drop effect).

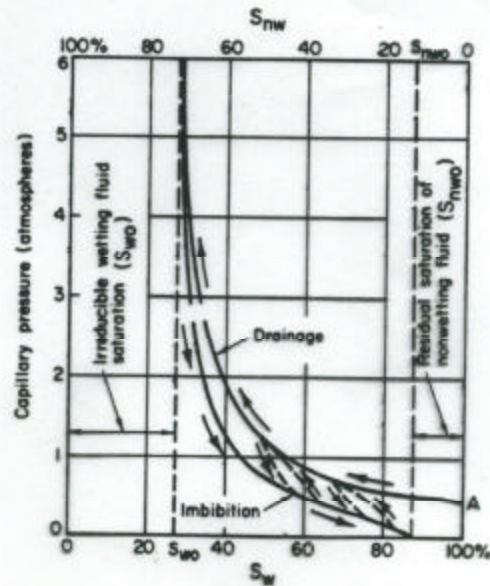


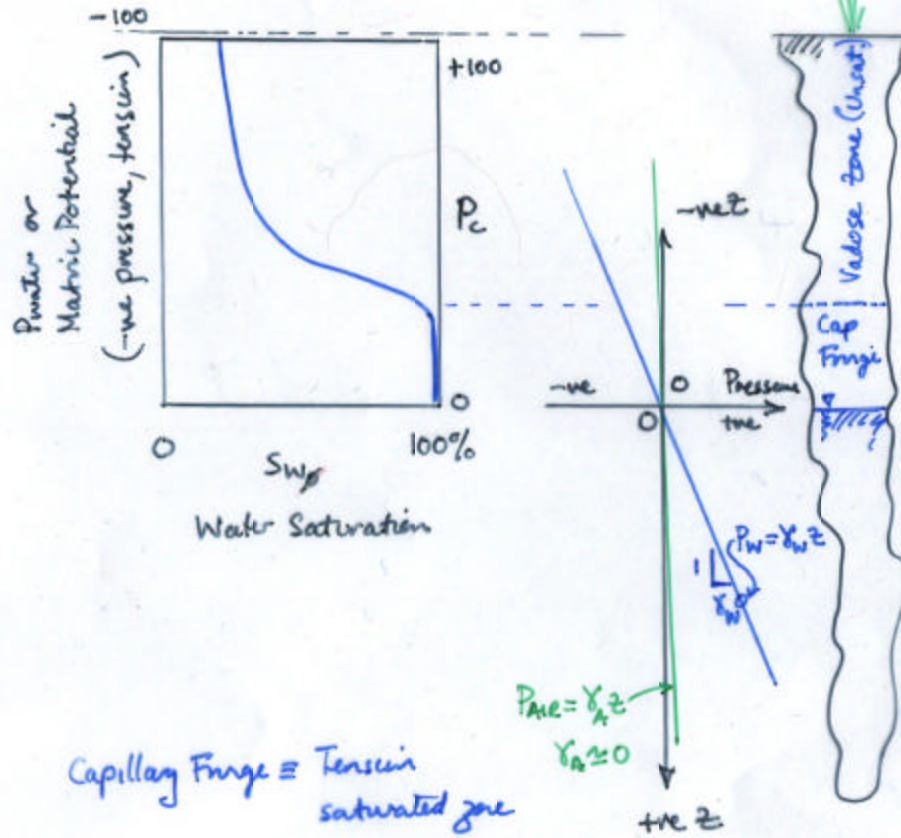
FIG. 9.2.10. Typical capillary pressure—wetting fluid saturation curves illustrating hysteresis.

STATIC SYSTEMS

$$P_c = P_{nw} - P_w$$

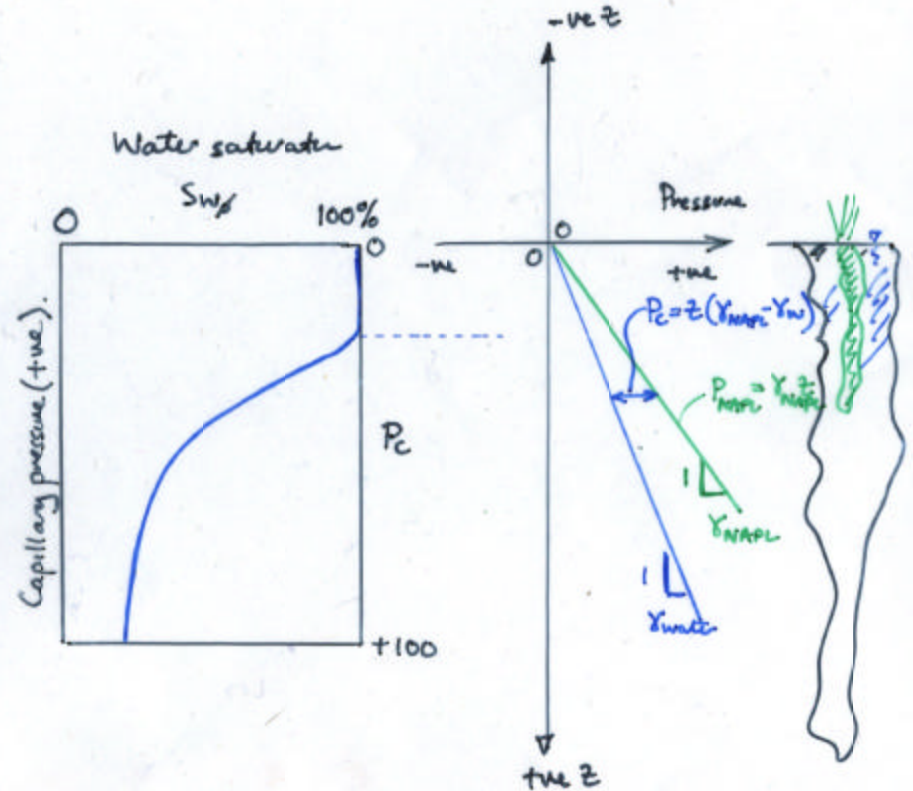
Vadose zone

$$P_c = P_{air} - P_{water}$$

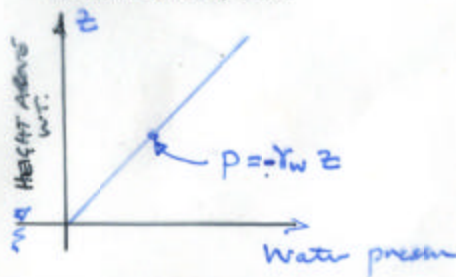
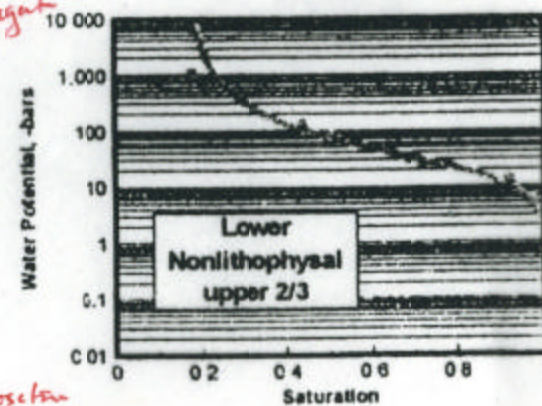
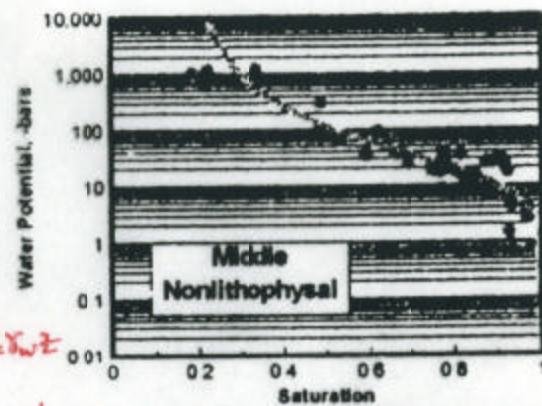
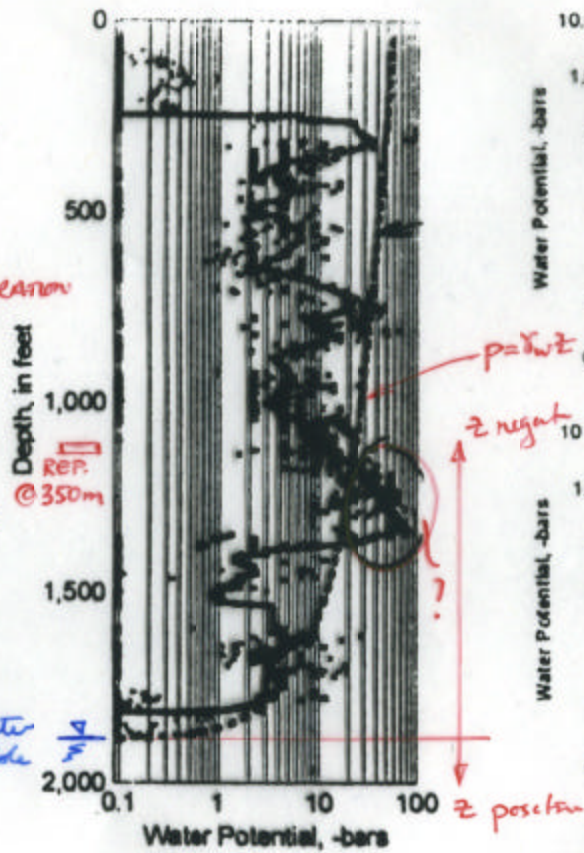
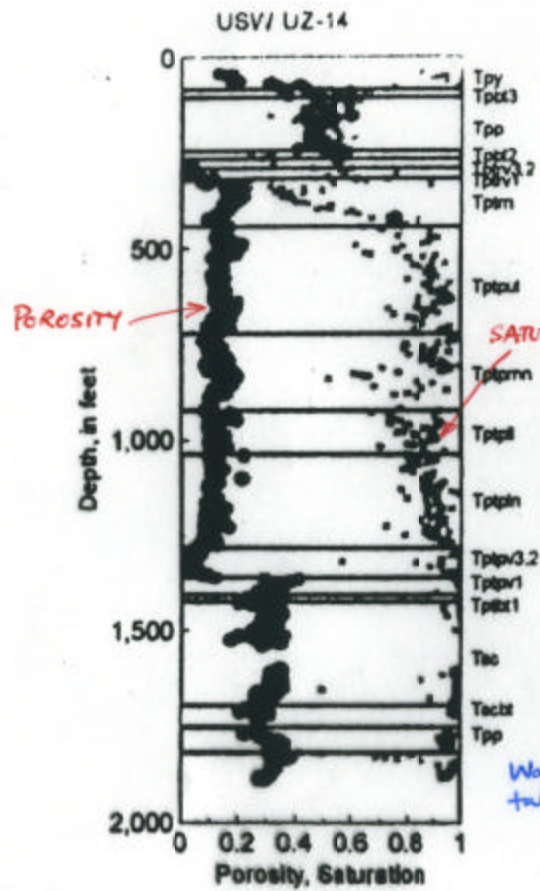


NAPL in Groundwater

$$P_c = P_{NAPL} - P_{water}$$



Yucca MTN - UZ-14

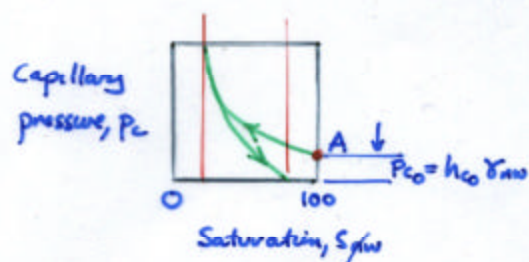


1 bar = 100 kN/m²
 ≈ 10 m of water

2.5 Non-Aqueous Fluid Penetration (Static groundwater field)

Question: How deep will a DNAPL penetrate?; Will it arrest?

Experience/evidence shows insidious penetration.



P_{c0} = entry pressure

Note - wish to penetrate at minimum saturation, not to saturate.

i.e. Penetrate @ low saturation

Consider without porous medium

Non-wet

Wetting



No penetration unless instability develops



$$P_{nw} = \gamma_{nw}(h + \Delta h)$$

$$P_w = \gamma_{nw}h + \gamma_w \Delta h$$

$$\text{If } \gamma_w < \gamma_{nw}$$

then $P_w < P_{nw}$

$\therefore$ Instability develops and pressure difference builds as Δh increases

i.e. Accelerating instability

In porous medium

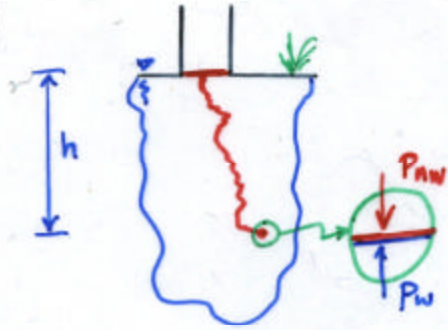


$$P_{c0} = h_{c0} \gamma_{nw}$$

Fluid will penetrate into porous medium. Two possibilities.

- 1) Flows until all large void space is filled, and stops?
- 2) Flows, but large void space is sufficiently connected to drain reservoir? ✓

Can flow stop?



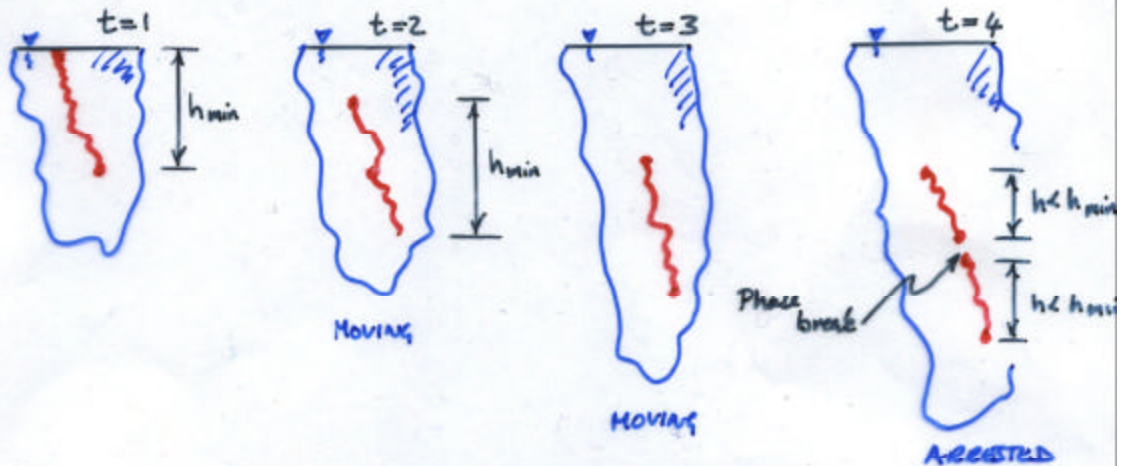
Will stop if $P_{nw} \leq P_w + P_{co}$ (1)

$$h \gamma_{nw} \leq h \gamma_w + h_c \gamma_{nw} \quad (2)$$

$$h \frac{\gamma_{nw}}{\gamma_w} - h = h_c \frac{\gamma_{nw}}{\gamma_w} \quad (3)$$

Only limitation is that "phase must be
"continuous."

$$h_{min} = \frac{h_c}{(1 - \gamma_w / \gamma_{nw})} = \frac{P_{co} / \gamma_{nw}}{(1 - \gamma_w / \gamma_{nw})} \quad (4)$$



Equation (4) shows: h_{min} largest if densities of fluids are close —
as γ_{nw} becomes denser, then less high column
of non-wetting fluid needed to generate P_{co} .

Practical Implications

1. Will only stop if (a) Fills "large voids" space and no connected "large void" space remains.
- (b) Flow "neck-down" and h_{min} is split
- (c) Fluid system is not "static". What is flow direction.

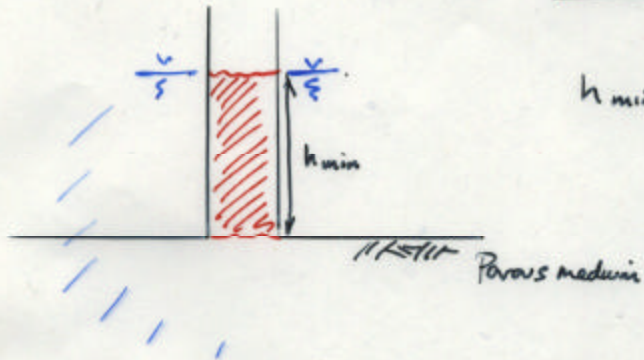
Important parameter is the connected pore space !!

This is difficult to evaluate with conventional test methods.

MAKE SURE YOU UNDERSTAND THE UNDERLYING ASSUMPTIONS

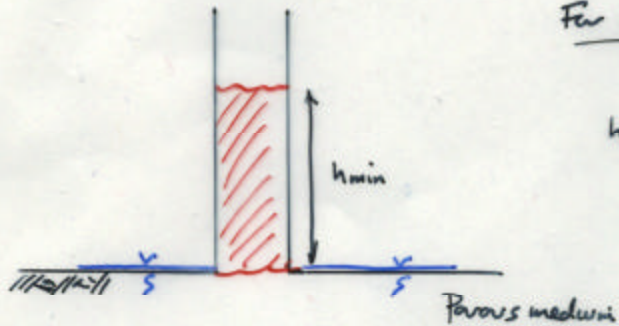
BEFORE YOU APPLY THESE

For invasion:



$$h_{min} = \frac{P_{co}/\gamma_{nw}}{(1 - \gamma_w/\gamma_{nw})} = \frac{h_c}{(1 - \gamma_w/\gamma_{nw})}$$

For invasion:



$$h_{min} = P_{co}/\gamma_{nw} = h_c$$

Table 4-2. Relationships between capillary pressure, gravity, and hydraulic forces useful for estimating conditions of DNAPL movement (from Kueper and McWhorter, 1991; WCGR, 1991; and Mercer and Cohen, 1990).

Condition	Equation
(a) Capillary pressure exerted on the surface of a nonwetting NAPL sphere	$P_c = P_{NAPL} - P_w = (2\sigma \cos \phi) / r$
(b) Capillary pressure exerted on the surface of NAPL in a fracture plane where b is the fracture aperture	$P_c = P_{NAPL} - P_w = (2\sigma \cos \phi) / b$
Hydrostatic Conditions	
(c) Critical height of DNAPL required for downward entry of DNAPL through the capillary fringe (the top of the saturated zone)	$z_c = (2\sigma \cos \phi) / (r g \rho_w)$
(d) Critical height of DNAPL required for downward entry of DNAPL into the water-saturated base of a lagoon where DNAPL is pooled beneath water; or, below the water table, for entry of DNAPL into a layer with smaller pore openings (assuming top of DNAPL body last existed under imbibition conditions)	$z_c = (2\sigma \cos \phi) / [rg(\rho_s - \rho_w)]$
(e) Critical height of DNAPL required for entry of DNAPL into a water-saturated fracture at the base of a lagoon where DNAPL is pooled beneath water; or, below the water table, for entry of DNAPL into a water-saturated fracture having an aperture, b , smaller than the host medium pore radii; or, below the water table, for entry of DNAPL into a water-saturated fracture segment having an aperture smaller than that of the overlying host fracture segment (assuming top of DNAPL body last existed under imbibition conditions)	$z_c = (2\sigma \cos \phi) / [bg(\rho_s - \rho_w)]$
(f) Critical height of DNAPL required below the water table, for entry of DNAPL into a layer with smaller pore openings where the top of the DNAPL body is under drainage conditions	$z_c = [P_c(\text{host}) - P_c(\text{medium})] / [g(\rho_s - \rho_w)]$
(g) The stable DNAPL pool length, L_p , that can exist below the water table following initial DNAPL migration where θ is the dip angle of the base of the host medium and L_p is measured parallel to the host medium base slope	$L_p = (2\sigma \cos \phi) / [rg(\rho_s - \rho_w) \sin \theta]$
(h) The stable DNAPL pool length, L_p , within a fracture that can exist below the water table following initial DNAPL migration where θ is the dip angle of the fracture, b is the maximum fracture aperture at the leading edge of the DNAPL pool, and L_p is measured parallel to the fracture slope	$L_p = (2\sigma \cos \phi) / [bg(\rho_s - \rho_w) \sin \theta]$
Hydrodynamic Conditions	
(i) Neglecting capillary pressure effects, the critical upward hydraulic gradient, i_c , required across a DNAPL body of height z_b to prevent downward DNAPL migration in a uniform porous medium	$i_c = \Delta h / \Delta z_b = (\rho_s - \rho_w) / \rho_w$
(j) Neglecting capillary pressure effects, the minimum hydraulic head difference between the bottom and top of a DNAPL body of height z_b to prevent downward DNAPL migration in a uniform porous medium	$\Delta h = i_c \Delta z_b = z_b(\rho_s - \rho_w) / \rho_w$
(k) Neglecting capillary pressure effects, the critical hydraulic gradient, i_c , required to prevent the downward movement of DNAPL along the top of a dipping (angle = θ) capillary barrier (i.e., in sloping fractures, bedding planes, or within a sloping coarse layer above a fine grained layer) with i_c measured parallel to the slope	$i_c = [(\rho_s - \rho_w) \sin \theta] / \rho_w$
(l) The critical horizontal hydraulic gradient, i_c , which must exist across a DNAPL pool of length L beneath the water table to overcome capillary resistance and mobilize DNAPL in the pool (to calculate i_c for a pool of DNAPL in a horizontal fracture, replace r with the fracture aperture, b)	$i_c = (2\sigma \cos \phi) / (r \rho_w L)$
(m) The critical upward hydraulic gradient, i_c , required to arrest the downward migration of DNAPL through an aquitard of thickness, Δz , where ΔP_c is the capillary pressure of DNAPL pooled at the top of the aquitard minus the threshold entry (displacement) pressure of the aquitard	$i_c = \Delta h / \Delta z = [(\rho_s - \rho_w) \rho_w] + [\Delta P_c / (\rho_w \Delta z)]$

2.6 EVALUATION OF CAPILLARY PRESSURE CURVES

Theoretical evaluation - Difficult except for uniform rods/spheres

$$\text{Use } P_c = (2\sigma/r) \cos\theta$$

eg. Collins (1961) for packed rods

R = radius

Limited S_w range since result valid only until adjacent fluid interfaces contact.

Laboratory Methods

$$P_c = P_c(S_w)$$

- a) Displacement methods: Establish successive states of hydrostatic equilibrium
b) Dynamic methods: Establish successive states of hydraulic steady flow

Both with a wetting and non-wetting fluid.

Displacement methods

Porous diaphragm method:

1. Wetted cone inside non-wetting fluid.

Underlain by diaphragm disc - permeable but not to the non-wetting fluid.

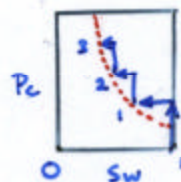
2. Increase pressure of non-wetting (displacing) fluid and wait for equilibrium (saturation)

Trace curve with: 1, 2, 3, etc.

3. Determine saturation from displaced fluid or X-ray/CT scan.

Limitations: 1. Length of time for equilibrium. 10-20d per point?
2. Is all pore space being accessed? Does it matter?

Advantage: Uses 'real' fluids (of interest).



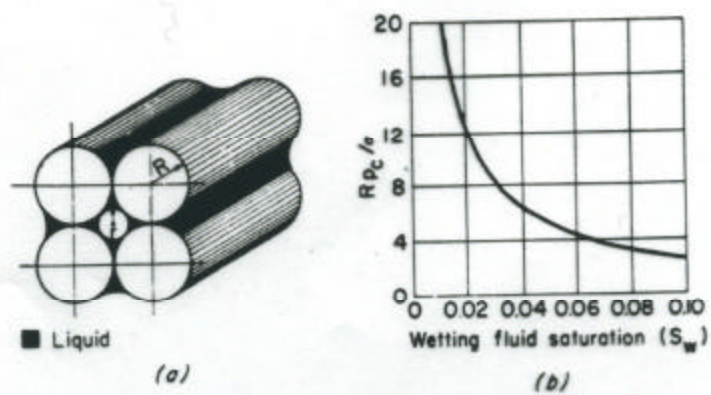


FIG. 9.2.13. Capillary pressure in a cubic packing of circular rods with liquid-air interfaces (after Collins, 1961).

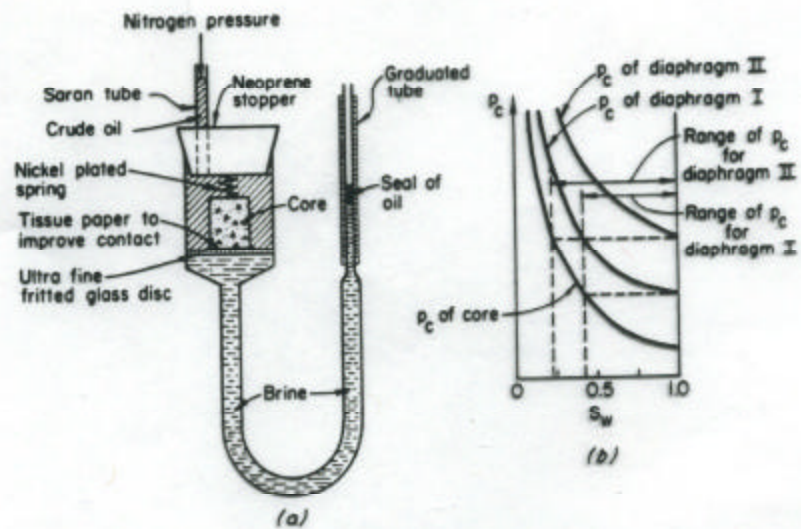


FIG. 9.2.14. Schematic diagram of a porous diaphragm device for capillary pressure determination (Welge and Bruce, 1947).

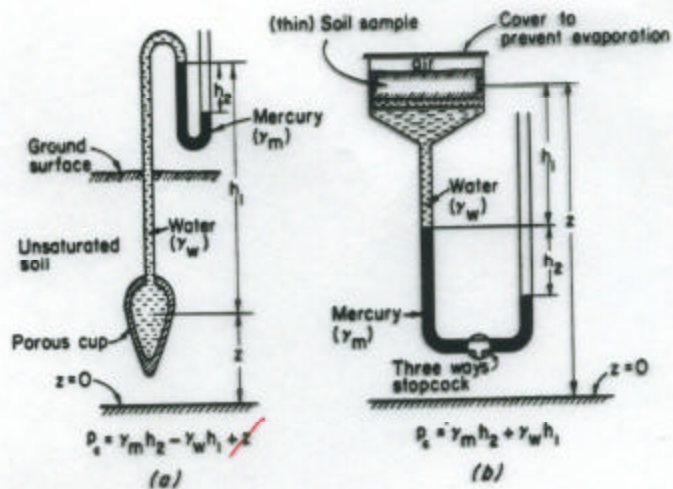
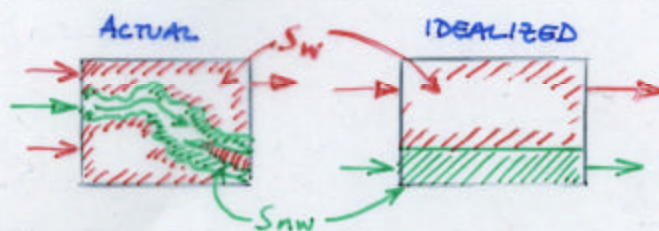


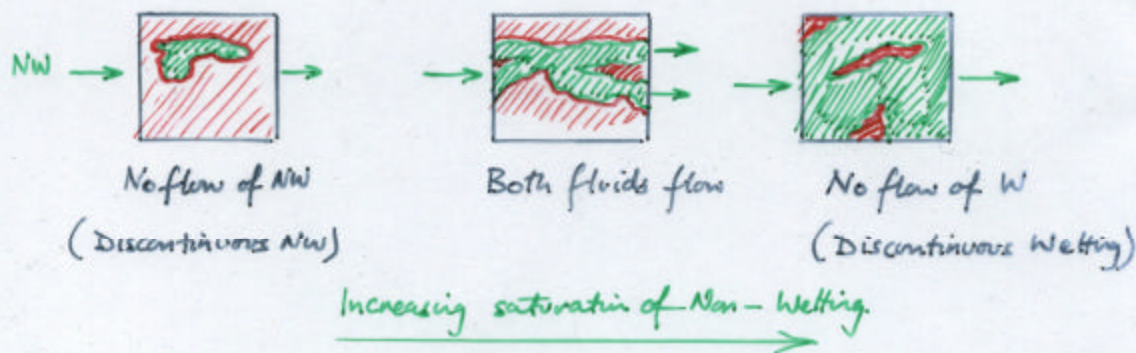
FIG. 9.4.4. The use of mercury tensiometer manometers. (a) Measurement of capillary pressure in the field, (b) the Haines apparatus for determining $p_s = p_c(S_w)$.

3. SIMULTANEOUS FLOW OF TWO IMMISCIBLE FLUIDS

- Capillarity and capillary pressures govern the "equilibrium" penetration of fluids — static behavior
- Once penetrated, the individual phases may transfer and be transported
 - Pure phase (free product)
 - Dissolved form (later).

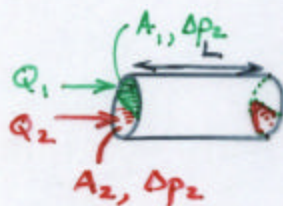


- a) Note that flow within phase is not subject to capillarity (capillarity acts at fringes, only)
- b) Each fluid establishes its own "tortuous" path → stable channels



3.1 MOTION EQUATIONS

Apply Darcy's Law



Establish steady flow at volumetric flow rates, Q_1 and Q_2 .

$$\left. \begin{array}{l} \square \quad q_1 = \left(\frac{k_1}{\mu_1} \right) \frac{\Delta p_1}{L} \\ \square \quad q_2 = \left(\frac{k_2}{\mu_2} \right) \frac{\Delta p_2}{L} \end{array} \right\} \quad q_\alpha = \left(\frac{k_\alpha}{\mu_\alpha} \right) \frac{\Delta p_\alpha}{L} \quad ; \quad q_\alpha = \frac{Q_\alpha}{A_\alpha}$$

Δp_α = effective pressure drop of α^{th} fluid
 k_α = "effective" permeability of medium to fluid α .

k_α depends on: a) Porous medium (pore size & distribution and fractures).
 b) Saturation, S_w and S_{ow} .

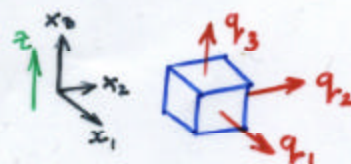
* Relative "permeabilities"

$$k_{r1} = \frac{k_1}{k} (S_1) \quad ; \quad k_{r2} = \frac{k_2}{k} (S_2)$$

- Relative to single phase permeability of a fully saturating fluid.
- Determine from "equilibrium" laboratory flow tests under different saturations.

$$\begin{array}{c} \text{Effective permeability} \nearrow k_1 = k_{r1} \underset{\substack{\nearrow \\ \text{True permeability (D^2)}}}{k} \\ \text{Relative permeability} \nearrow k_{r1} \end{array}$$

FLUID MOTION EQUATION (DARCY'S LAW)



i = flow direction, x_1, x_2, x_3 . ("z" is // to x_3)

$$q_{i1} = - \frac{k_1}{\mu_1} \left(\frac{\partial p_1}{\partial x_j} + \rho_1 g \frac{\partial z}{\partial x_j} \right) = - k \frac{k_{r1}}{\mu_1} \left(\frac{\partial p_1}{\partial x_j} + \rho_1 g \frac{\partial z}{\partial x_j} \right)$$

$$q_{i2} = - \frac{k_2}{\mu_2} \left(\frac{\partial p_2}{\partial x_j} + \rho_2 g \frac{\partial z}{\partial x_j} \right) = - k \frac{k_{r2}}{\mu_2} \left(\frac{\partial p_2}{\partial x_j} + \rho_2 g \frac{\partial z}{\partial x_j} \right)$$

May also use piezometric head, h , for constant ρ_1 and ρ_2 , but must be defined separately for each fluid, h_α

$$h_\alpha = \frac{p_\alpha}{\rho_\alpha g} + z \quad ; \quad \alpha = 1, 2$$

$$\begin{aligned} q_{i1} &= - k \frac{k_{r1}}{\mu_1} \rho_1 g \frac{\partial h_1}{\partial x_j} \\ q_{i2} &= - k \frac{k_{r2}}{\mu_2} \rho_2 g \frac{\partial h_2}{\partial x_j} \end{aligned}$$

Similarity between:

$$q = - k \frac{\rho g}{\mu} \frac{dh}{dx_j}$$

permeability (L^2)

$$= - K \frac{dh}{dx_j}$$

Hydraulic Conductivity (L/T)

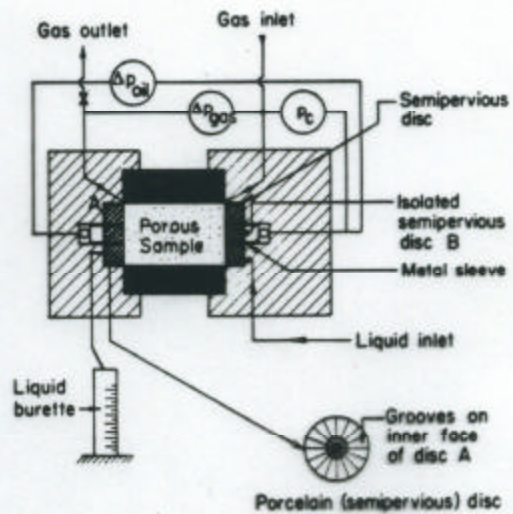


FIG. 9.2.15. Hassler's apparatus for relative permeability determination (after Osoba et al., 1951).

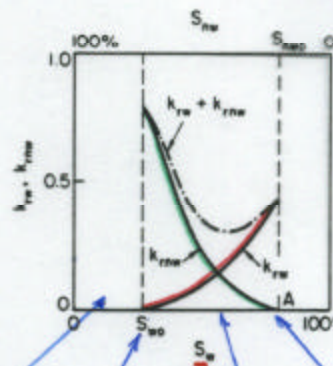


FIG. 9.3.1. Typical relative permeability curves (e.g., wetting fluid = water, nonwetting fluid = oil).

Non-wetting



3.2 RELATIVE PERMEABILITY

$$\square \quad k_{r_{nw}} + k_{r_w} \neq 1 \quad \Rightarrow \quad k_{r_{nw}} + k_{r_w} < 1$$

Most effective transmission is at 100% saturation (if accessible). Interference.

- $\square$ Usually $k_{r_{nw}}$ closer to 1 than k_{r_w}
- $\square$ Steep decline of k_{r_w} with increasing S_{nw} indicates larger pores occupied first by nonwetting phase.

Nonwetting phase occupies larger pores preferentially due to capillary pressure arguments.

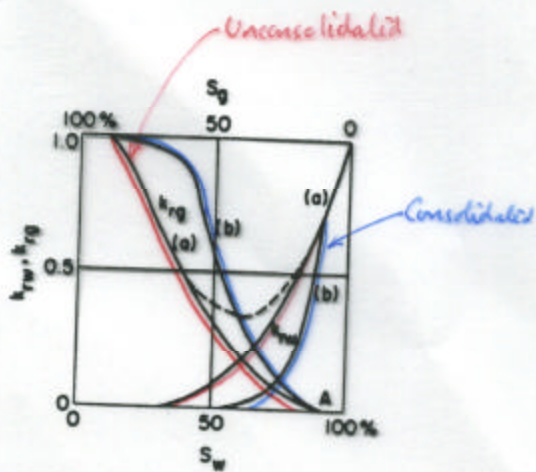
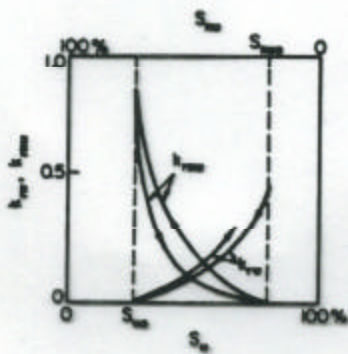
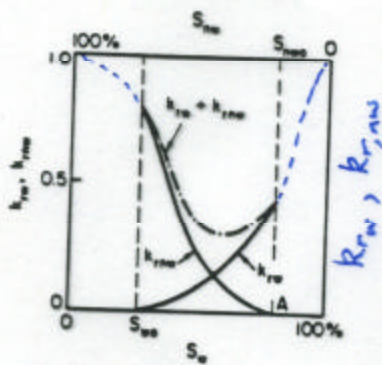
$\square$ k to wetting fluid is always larger for open pored unconsolidated material.

$\square$ k to non-wetting fluid is always smaller for open-pored unconsolidated material.

Hysteresis:

1. Wetting fluid surrounds grains and non-wetting fluid $\therefore$ may move nw fluid even if no pressure gradient in nw fluid.
2. Since change in saturation requires change in wetted grain surface - wettability is hysteretic. $\therefore$ permeabilities are hysteretic.

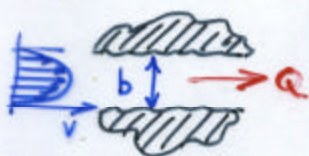




Curves switch over
- new fluid saturates
largest pores first.

PERMEABILITY/CONDUCTIVITY OF FRACTURES

Flow in fractures



$\bar{v}$ = average velocity.

$$\bar{v} = -\frac{g b^2}{12 \nu} \frac{dh}{dx}$$

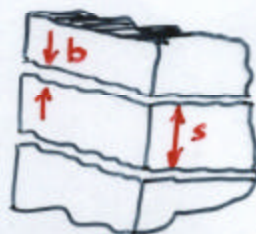
ν = kinematic viscosity of fluid

$$\nu = \frac{\mu}{\rho}$$

Equivalent flow rate per unit width:
for single fracture

$$Q = b \frac{g b^2}{12 \nu} \frac{dh}{dx}$$

Multiple fractures arranged in parallel:



Total of N fractures per unit height:

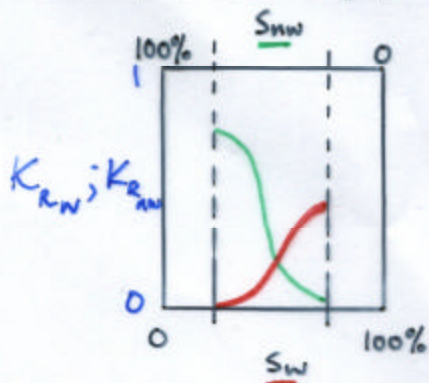
$$N = 1/s$$

$$Q = \underbrace{\frac{g b^3}{12 \nu}}_{K_b} \frac{1}{s} \frac{dh}{dx}$$

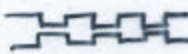
Equivalent conductivity for multiple sets: $K_b = \frac{g b^3}{6 \nu s}$

Enables b to be evaluated if K known (measured).

Relative permeability of fractures



- Similar behavior to porous medium

- Distribution of apertures 
Large apertures saturate first

Capillary pressure or head $h_c = \frac{2\sigma}{b \gamma_w}$

FRACTURE PERMEABILITIES AND
CAPILLARY PRESSURES

$$k = \frac{b^2}{12} \quad \text{single fracture permeability} \quad (1)$$

$$k_b = \frac{b^3}{12s} \quad \text{bulk permeability}$$

Permeabilities and capillary pressures related.

$$h_c = \frac{2\sigma}{b\gamma_w} \quad \sim \quad h_c \gamma_w = \frac{2\sigma}{b} = p_{c_b} \quad (2)$$

From (1) and (2) $p_{c_b} = \frac{2\sigma}{\sqrt{12k}}$

General relation for fractures and porous media

$$p_c \propto \sqrt{\frac{1}{k}}$$

eg. Lamé's 'J' function -

$$J = \frac{p_c}{\sigma} \sqrt{\frac{k}{n}}$$

$$\therefore p_c = J \sigma \sqrt{\frac{n}{k}}$$

3.3 Mass Conservation in Multiphase Flow

Continuity equation:
$$\frac{\partial}{\partial t}(n_s \rho_\alpha) + \frac{\partial}{\partial x_i}(\rho_\alpha q_{ix}) = 0 \quad \alpha = 1, 2$$

 $\sim \alpha = 1, 2, 3.$

For an incompressible fluid and medium $\frac{\partial}{\partial t}(n \text{ and } \rho) = 0$

Substitute q_α from relative permeability relation:

Results in 4 equations:

$$n \frac{\partial s_1}{\partial t} - \frac{\partial}{\partial x_i} \left[k \frac{k_{r1}}{\mu_1} \left(\frac{\partial p_1}{\partial x_j} + \rho_1 g \frac{\partial z}{\partial x_j} \right) \right] = 0$$

$$n \frac{\partial s_2}{\partial t} - \frac{\partial}{\partial x_i} \left[k \frac{k_{r2}}{\mu_2} \left(\frac{\partial p_2}{\partial x_j} + \rho_2 g \frac{\partial z}{\partial x_j} \right) \right] = 0$$

$$s_1 + s_2 = 1$$

$$p_2 - p_1 = p_c(s_1)$$

Solve for 4 unknowns: s_1, s_2, p_1, p_2

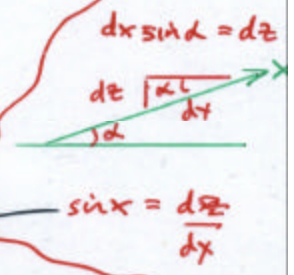
$$\text{with } h_1 = z + \frac{p_1}{\rho_1 g} \quad ; \quad h_2 = z + \frac{p_2}{\rho_2 g}$$

Solve using numerical techniques.

3.5 BUCKLEY-LEVESETT (1942) EQUATIONS

Assume: Neglect gravity, capillarity, liquid compressibility.

Homogeneous reservoir of thickness, b , inclined @ α° .



$$\begin{aligned} q_w &= - (k_w/\mu_w) (\partial p_w / \partial x + \rho_w g \sin \alpha) \\ q_{nw} &= - (k_{nw}/\mu_{nw}) (\partial p_{nw} / \partial x + \rho_{nw} g \sin \alpha) \end{aligned} \quad \left. \vphantom{\begin{aligned} q_w &= - (k_w/\mu_w) (\partial p_w / \partial x + \rho_w g \sin \alpha) \\ q_{nw} &= - (k_{nw}/\mu_{nw}) (\partial p_{nw} / \partial x + \rho_{nw} g \sin \alpha) \end{aligned}} \right\} \begin{array}{l} \text{Relative permeability} \\ \text{and Darcy's Law.} \end{array}$$

$$n \partial S_w / \partial t + \partial q_w / \partial x = 0 \quad (1)$$

$$n \partial S_{nw} / \partial t + \partial q_{nw} / \partial x = 0 \quad (2)$$

$$S_w + S_{nw} = 1 \quad (3)$$

$$p_c = p_{nw} - p_w \quad (4)$$

4 eqns for 4 unknowns.

Initial and boundary conditions

$$t \leq 0$$

$$0 \leq x \leq L$$

$$S_{nw} = 1 - S_{w0}$$

Irreducible wetting saturation

$$t > 0$$

$$x = 0$$

$$q_w = \text{constant}; \quad q_{nw} = 0$$

Wetting fluid injected to displace non-wetting fluid (oil) at irreducible wetting saturation $S_{nw} = 1 - S_{w0}$

Injected @ constant rate at $x = 0$.

Unconsolidated $\rightarrow$ open pores
 $\therefore$ high k to wetting fluid
 low k to non-wetting

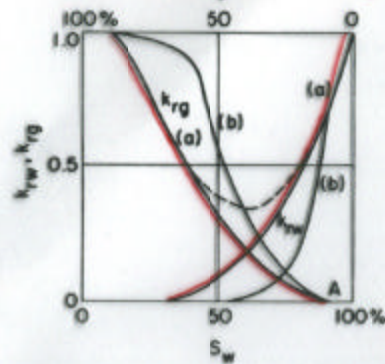


FIG. 9.3.2. Typical relative permeability to gas and water. (a) Unconsolidated sand. (b) Consolidated sand (Botset, 1940).

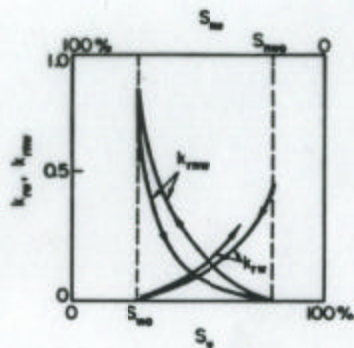


FIG. 9.3.3. Effect of hysteresis on relative permeability.

Inject water @
 $x=0$
 $t=0$

Inject water as wetting fluid

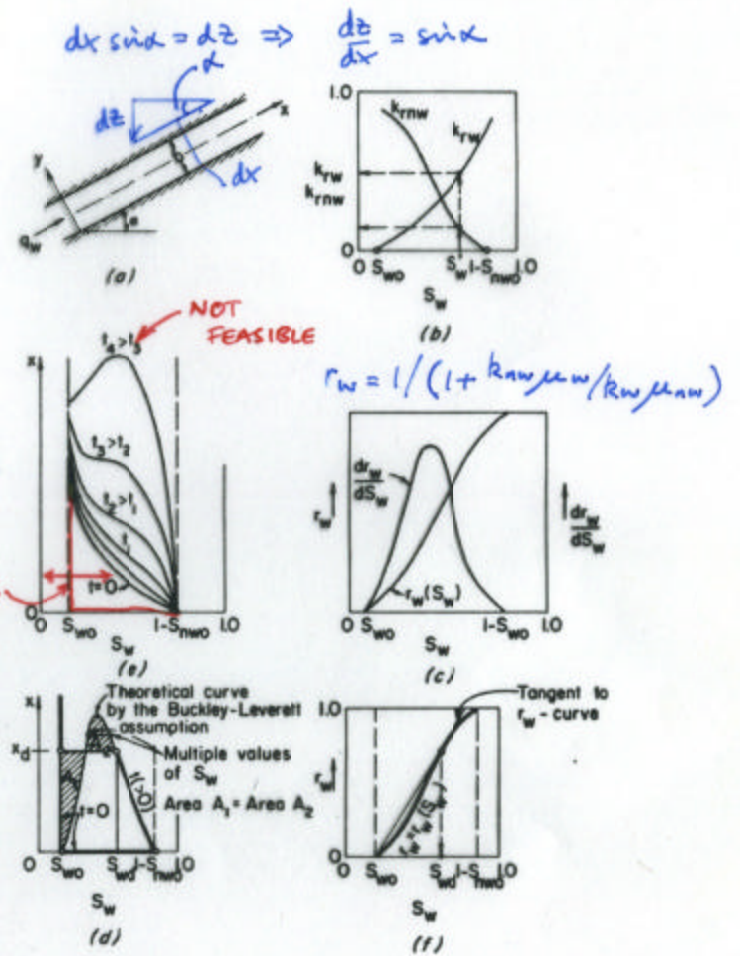


FIG. 9.3.4. The Buckley-Leverett solution for a linear immiscible displacement.

B-L Equations (cont'd)

Enables evaluation of front movement, see figure 9.3.4.(e) $M = \frac{V_w}{V_{nw}}$

$$r_w = 1 / (1 + k_{rw}/k_{rnw}) = [1 / (1 + 1/\text{Mobility ratio})]$$

Note since capillarity neglected @ t_4 , two saturations exist.

$\therefore$ only applicable to high flow rates

- where capillary effects are masked

||

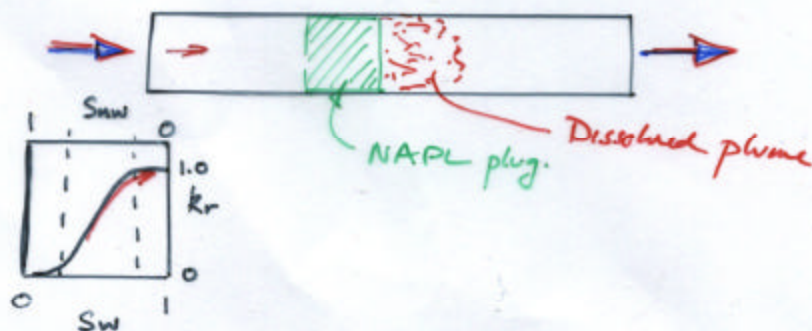
$$M = \frac{k_{rw}/k_{rnw}}{k_{rnw}/k_{rnw}} = \frac{V_w}{V_{nw}}$$

General Comments

- ① Techniques developed for petroleum industry
Difficult interpretation for saturations less than
irreducible saturation. This is perhaps
most important region for groundwater contamination

□ Need to incorporate:
Dissolution effects.

- ② For pump-and-treat need to understand two effects
- Role of k_r in controlling water flow rates
through NAPL plug. Plug does not move
since fingerlike water saturation
 - Role of dissolution as $S_{nw} \downarrow$



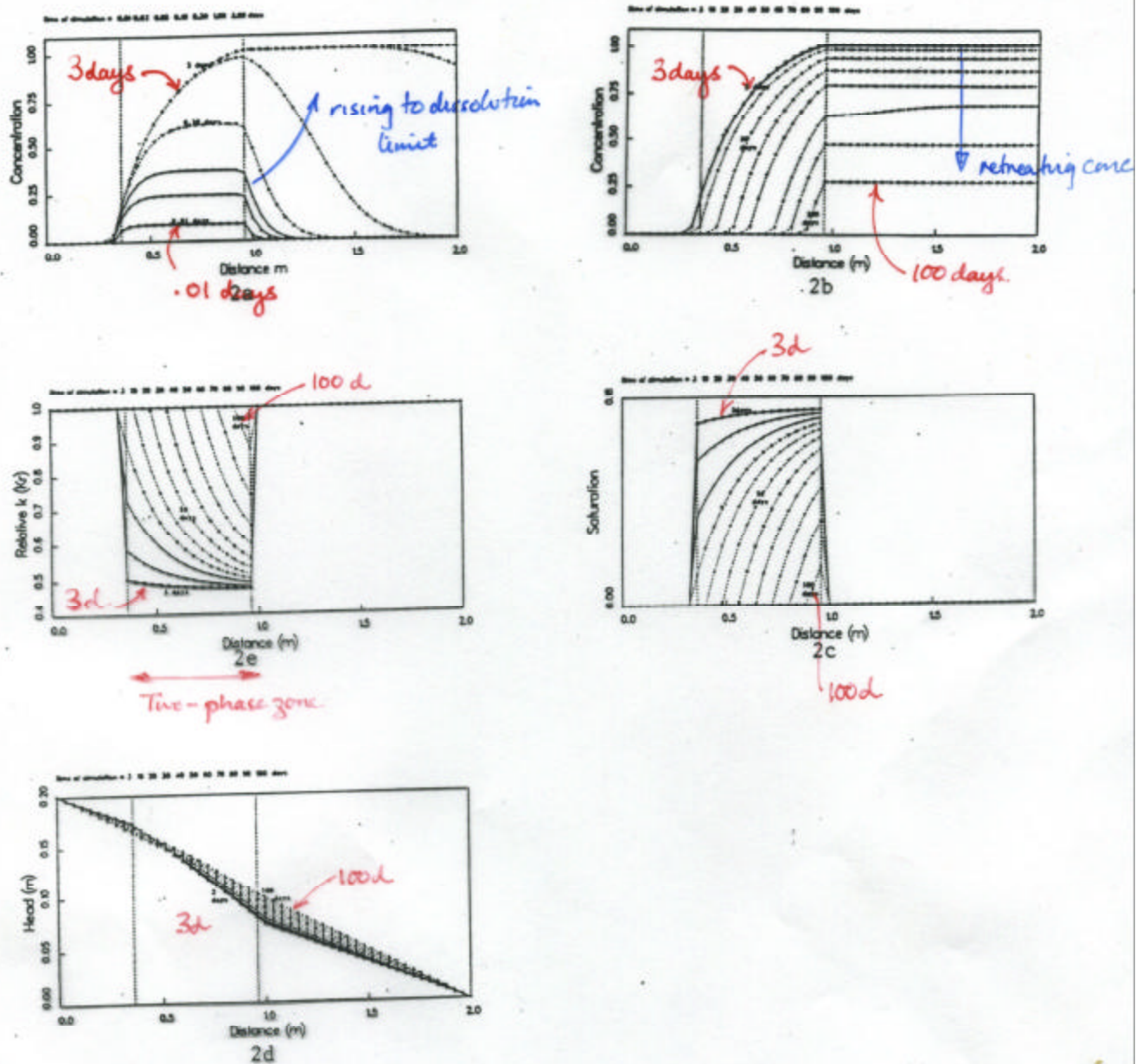
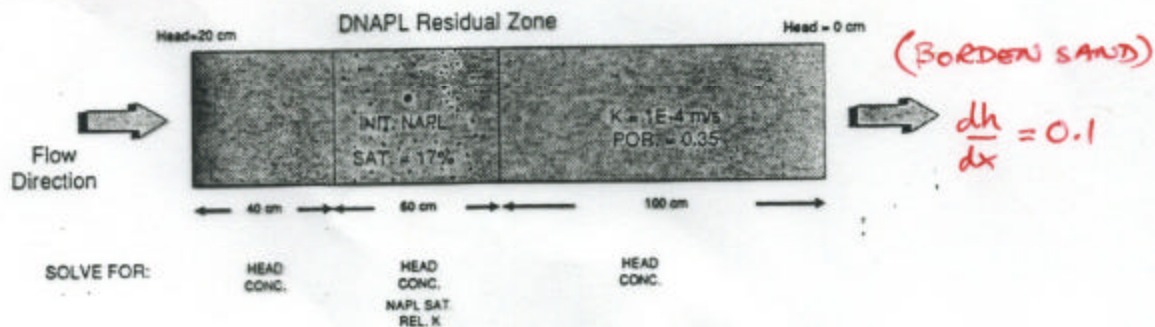


Fig. 2 - Results with Mass Transfer Coefficient = $1 \times 10^{-4} \text{ l/s}$ (a) Conc. at early times (b) Conc. at late times (c) DNAPL Saturation (d) Head distribution (e) Relative permeability.

COLUMN EXPERIMENT



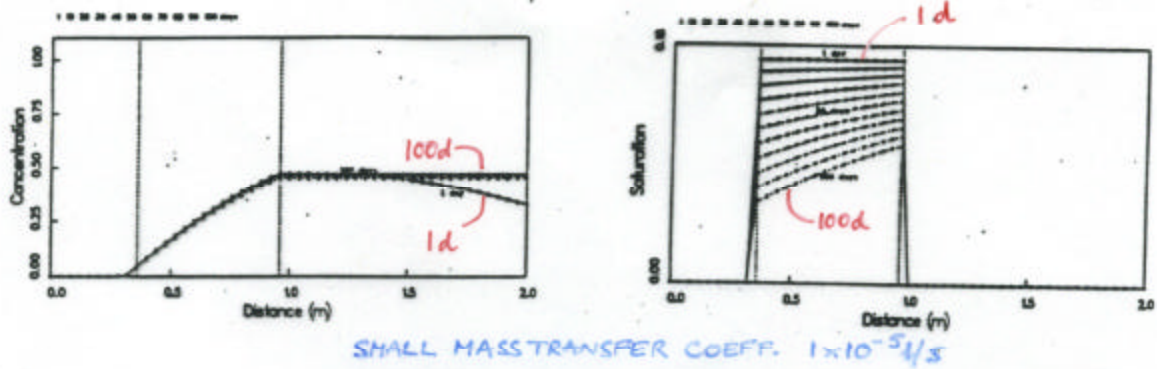


Fig. 3 - Results with Mass Transfer Coefficient = $1 \times 10^{-5} 1/s$

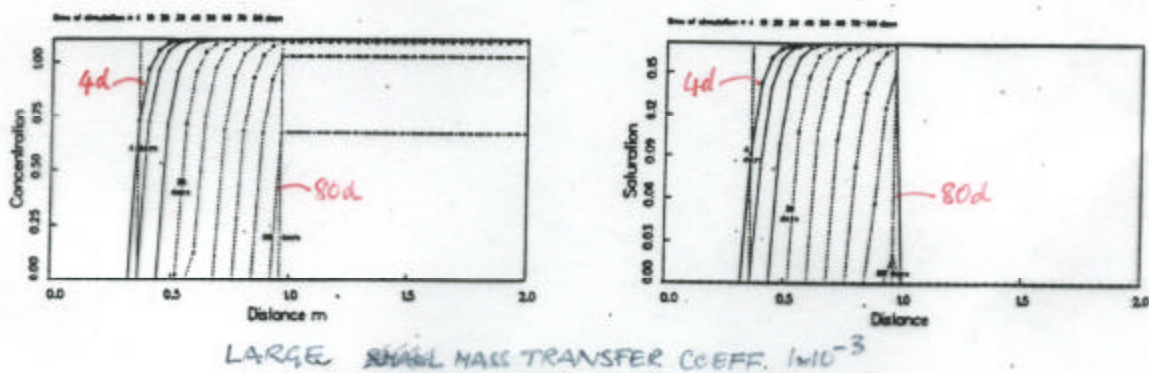


Fig. 4 - Results with Mass Transfer Coefficient = $2 \times 10^{-5} 1/s$

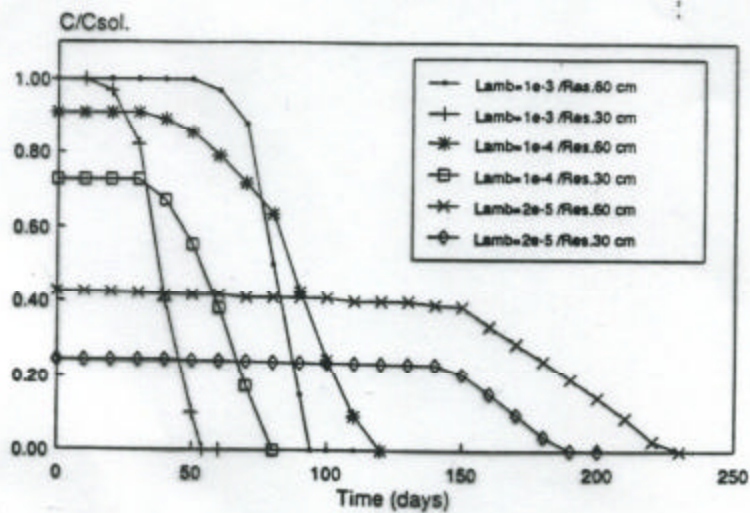


Fig. 5 - Concentration observed at the end of the residual saturation zone for different Mass Transfer Coefficients and lengths of the residual saturation zones

3.6 Non-Aqueous Fluid Penetration (Moving Groundwater)

- Two situations: ① NAPL saturates medium $\rightarrow$ No capillary forces
 ② NAPL and water

3.6.1 NAPL Saturated

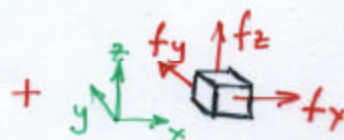
What pressure gradient to move fluid?
 What direction of movement?



$$dx dy dz = dV$$



BODY FORCE

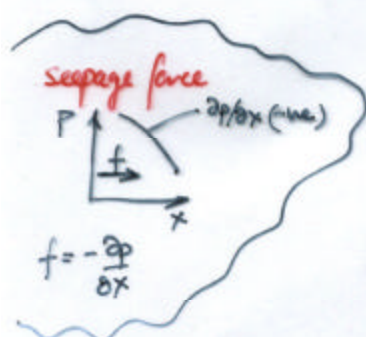


SEEPAGE FORCE

=



RESULTANT



$$dV \rho g \begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix} + dV \begin{Bmatrix} -\partial/\partial x \\ -\partial/\partial y \\ -\partial/\partial z \end{Bmatrix} P = \underline{F}$$

$$\left[\rho g \begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix} - P \begin{Bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{Bmatrix} \right] dV = \underline{F}$$

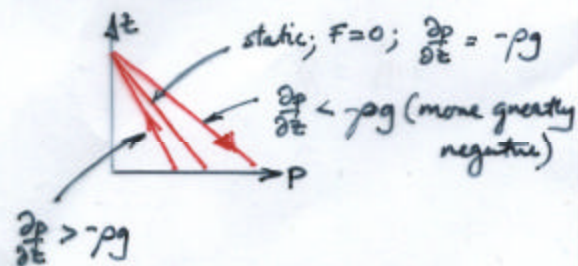
equilibrium $\Rightarrow F=0$

$$\frac{\partial p}{\partial z} = -\rho g$$

$\underline{F}$ is the resultant force vector. Force per unit volume.

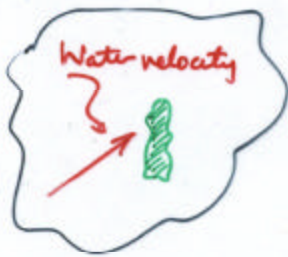
Define the direction of force and $\therefore$ direction of movement.

$F_z = 0$	$\partial p / \partial z = -\rho g$	static fluid
$F_z = +ve$	$\partial p / \partial z < -\rho g$	$\uparrow$ flow
$F_z = -ve$	$\partial p / \partial z > -\rho g$	$\downarrow$ flow



Also define lateral direction of flow.

3.6.2 NAPL and water



Will the gradient move the NAPL

Require to overcome capillary pressure, P_c

P_c acts uniformly in all directions
and must be overcome

Need to know size of NAPL pocket.

τ acts over length, L .

L is typically unknown



3.7 BEHAVIOR OF LNAPLS

3.7.1 Migration of LNAPLS

Surface spill: 1. Penetrate vadose zone under gravity
with influence of capillary forces.
Vadose zone water wet.
NAPL is non-wetting.

Capillary Fringe: Holds water
as continuous phase with
residual air saturation

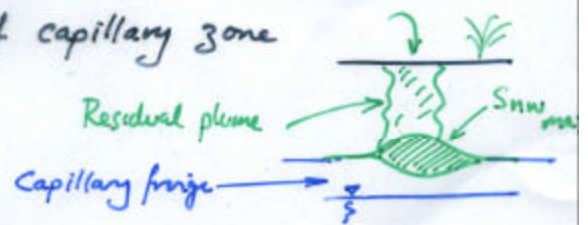
$h_c \uparrow$ with $\downarrow$ grain size $\rightarrow \downarrow$ pore size

$$\left\{ \text{i.e. } h_c = \frac{4\sigma}{d\gamma} \right\}$$

Example usual capillary rise

	<u>Grain size (mm)</u>	<u>Rise (cm)</u>
Fine gravel	2-5	2.5
Medium sand	0.2-0.5	24.6
Fine silt	0.02-0.05	200+

LNAPL will penetrate down to top of capillary zone
(if enough volume)



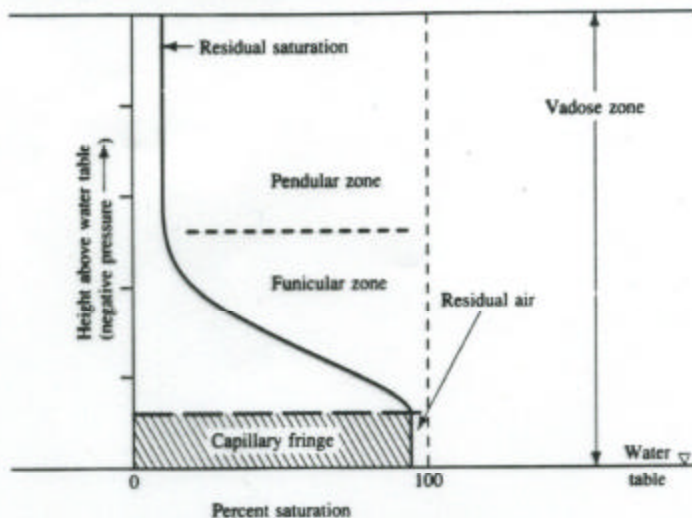


FIGURE 5.13 Vertical distribution of water in the vadose zone in the absence of nonaqueous phase liquids. Source: A. S. Abdul, *Ground Water Monitoring Review* 8, no. 4 (1988):73–81. Copyright © 1988 Water Well Journal Publishing Co.

TABLE 5.1 Visual capillary rise in unconsolidated materials (porosity of all samples is about 41%).

Material	Grain Size (mm)	Capillary Rise (cm)
Fine gravel	2–5	2.5
Very coarse sand	1–2	6.5
Coarse sand	0.5–1	13.5
Medium sand	0.2–0.5	24.6
Fine sand	0.1–0.2	42.8
Silt	0.05–0.1	105.5
Fine silt	0.02–0.05	200 +

Source: Lahman (1972).

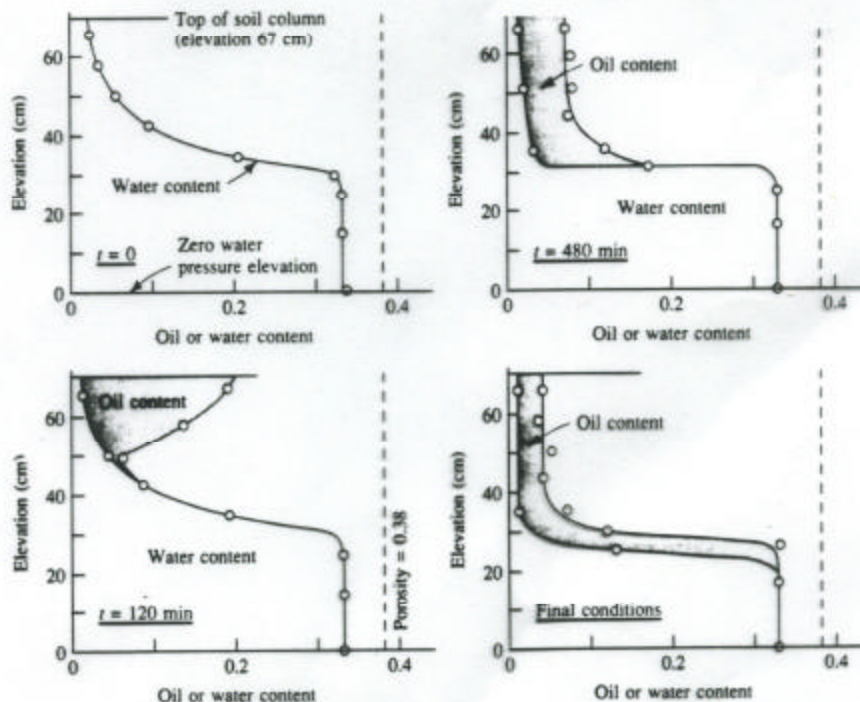


FIGURE 5.14 Changes in the vertical distribution of oil with time after a slug of oil is added to the top of a column of sand. Oil content and water content are expressed as a fraction of the total volume of the porous media. Source: D. K. Eckberg and D. K. Sunada, *Water Resources Research* 20, no. 12 (1984):1891–97. Copyright by the American Geophysical Union.

DEVELOPMENT OF AN OIL TABLE (Abdul, 1988)

Add oil incrementally to

- a) $\rightarrow$ b) develop oil fringe
- b) $\rightarrow$ c) develop oil table
- c) $\rightarrow$ d) remove capillary fringe and create oil table.

Mobile oil product may migrate "free" product

3 Components

□ Residual column

- What is saturation? - Since free phase moved through zone, residual saturation must be greater than irreducible.
- Re-mobilized as rain fall from surface $\downarrow$

Partitioning $\rightarrow$

1. Vapor phase $\rightarrow$ volatilization.
2. Dissolves in aqueous phase.

□ "Free" product - May mobilize with rising/falling water table. May flow down steep water table

□ Dissolution into groundwater

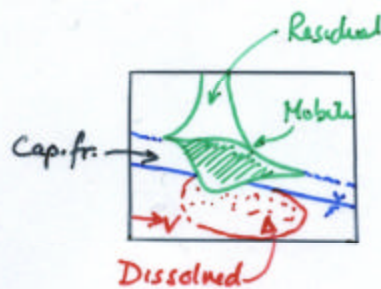
Gasoline $\rightarrow$ Benzene, Toluene, Ethylbenzene & Xylene (BTEX) soluble fractions.

Degree of partitioning depends on volatility of fractions

Which fractions will partition in $\rightarrow$ air
 $\rightarrow$ water } Henry's Law

What is partitioning rate?

$\rightarrow$ Transport processes (aqueous/gaseous).



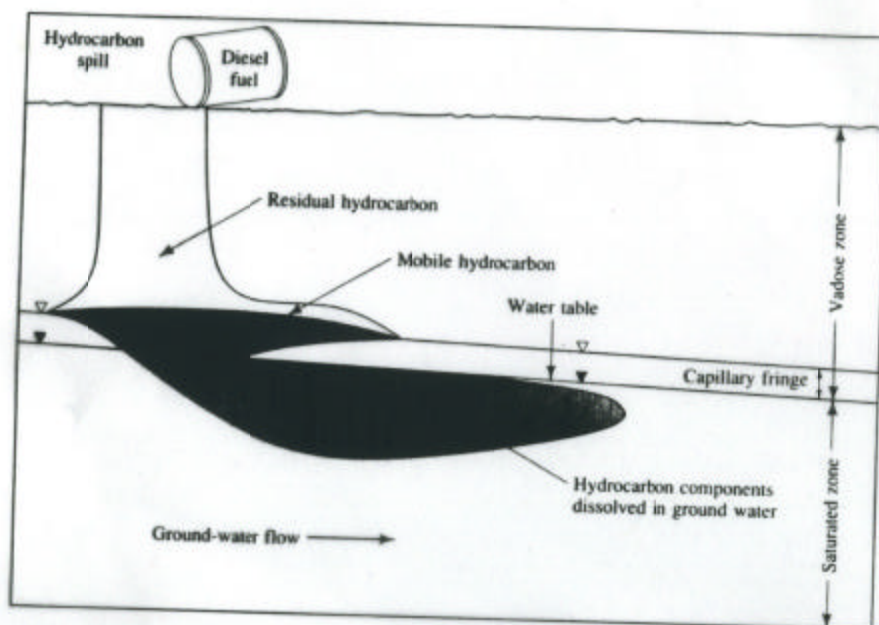


FIGURE 5.16 Subsurface distribution of an LNAPL spill.

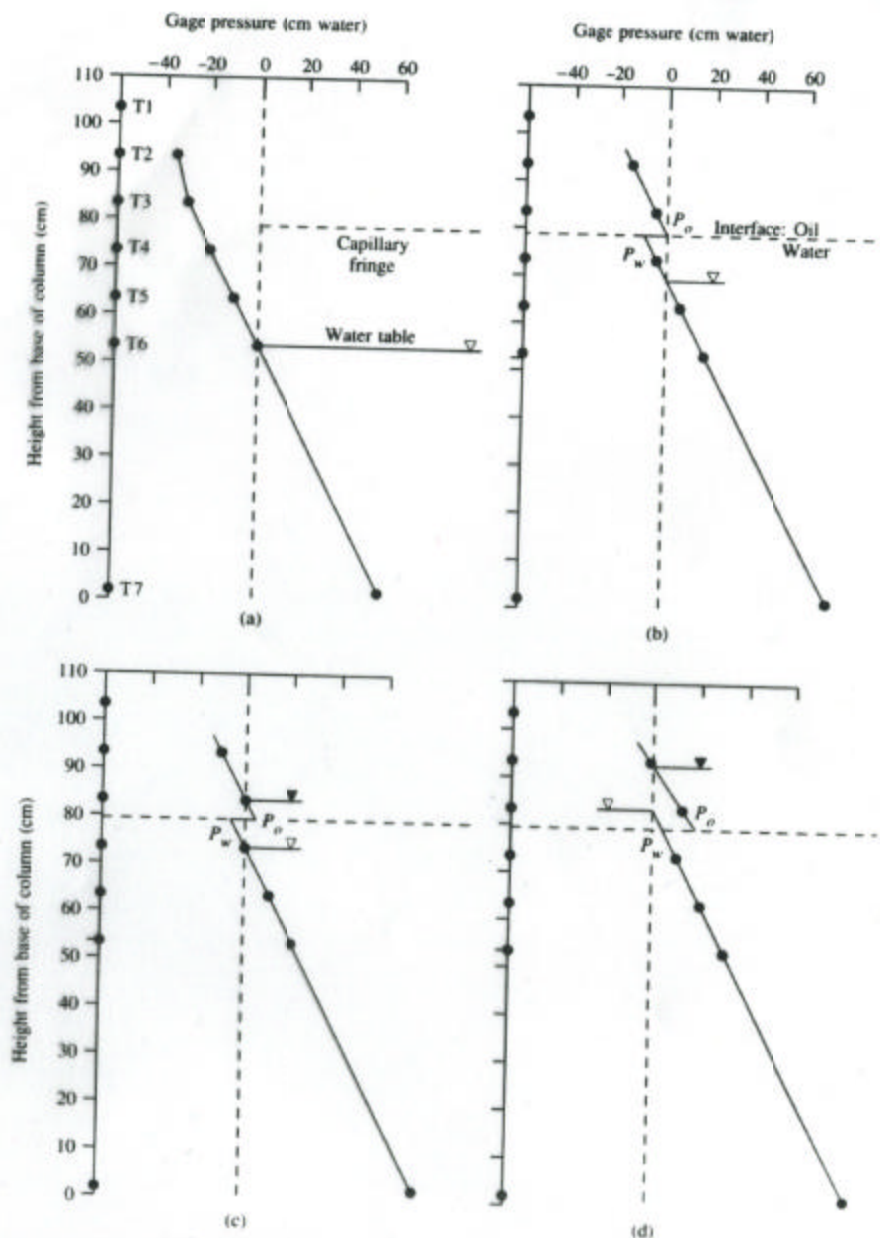


FIGURE 5.15 Hydrostatic pressure head/tension distribution in a sand column to which oil is being added to the top of the column. (a) Before the addition of the oil, (b) after addition of the oil showing the development of an oil fringe, (c) after addition of sufficient oil for an oil table to form, and (d) after sufficient mobile oil has accumulated to eliminate the water capillary fringe. Source: A. S. Abdul, *Ground Water Monitoring Review* 8, no. 4 (1988): 73–81. Copyright © 1988 Water Well Journal Publishing Co. Used with permission.

Henry's Law

Linear relationship between vapor pressure of a solute and its aqueous solution and the concentration of the solution.

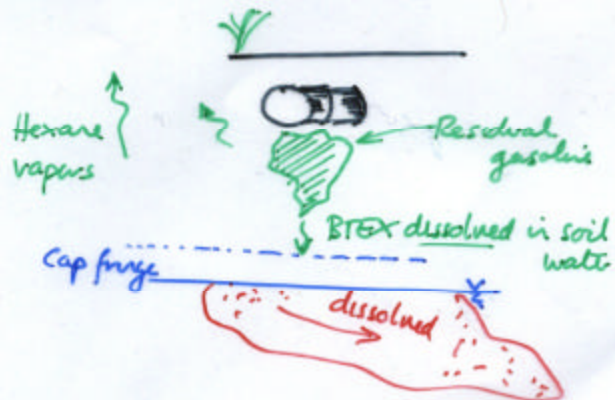
Proportionality coefficient $\equiv$ Henry's Law constant.

Also applicable to partitioning in air.

Low water-air partition coefficients (alkanes) $\rightarrow$ favor vapor phase

High water-air " " (benzene) $\rightarrow$ favor aqueous phase

Different time of release for different materials:



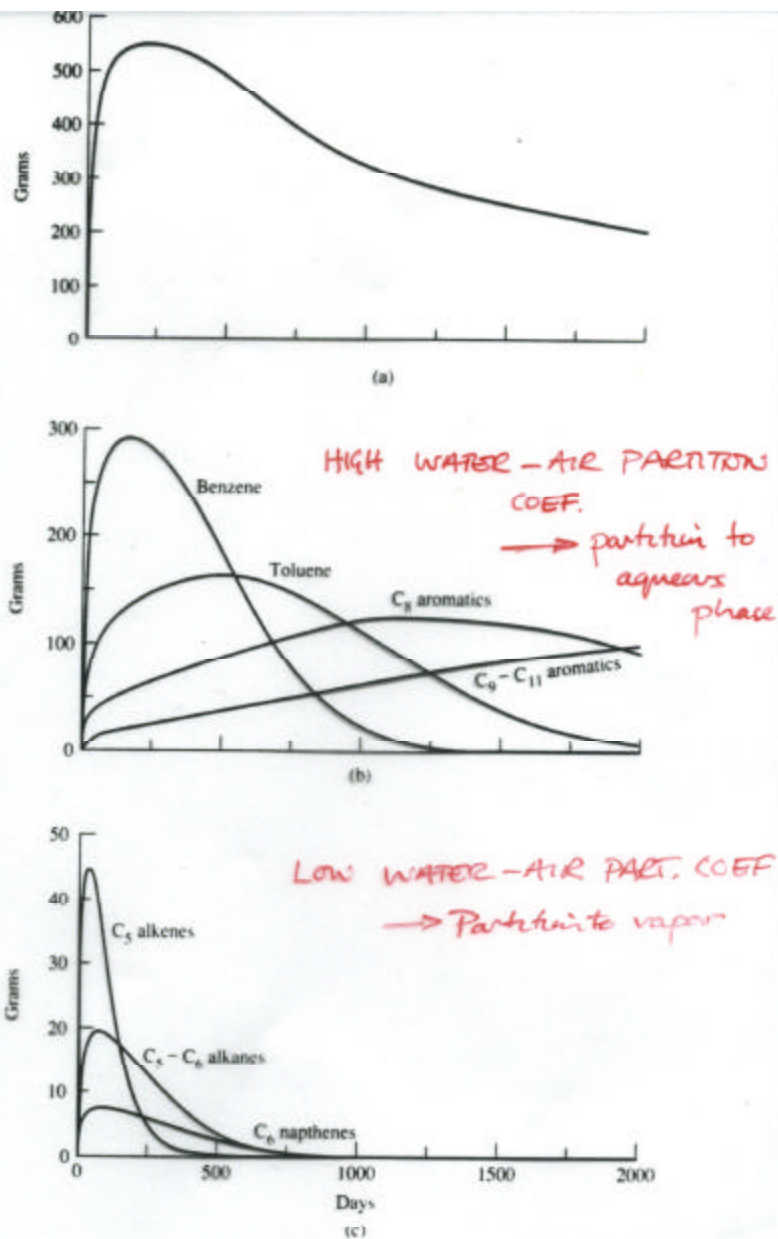


FIGURE 5.17 Mass of residual hydrocarbon in vadose zone partitioning into capillary water as a function of time, with (a) total hydrocarbons, (b) aromatic constituents, and (c) nonaromatic constituents. Source: A. L. Baehr, *Water Resources Research* 23, no. 10 (1987): 1926–38. Copyright by the American Geophysical Union.

TABLE 5.2 Water-air partition coefficients for selected organic compounds.

Compound	Formula	Molecular Weight	Water-Air Partition Coefficient
Aromatics			
Benzene	C_6H_6	78	5.88
Toluene	C_7H_8	92	3.85
o-Xylene	C_8H_{10}	106	4.68
Ethylbenzene	C_8H_{10}	106	3.80
Nonaromatics			
Cyclohexane	C_6H_{12}	84	0.15
1-Hexene	C_6H_{12}	84	0.067
n-Hexane	C_6H_{14}	86	0.015
n-Octane	C_8H_{18}	114	0.0079

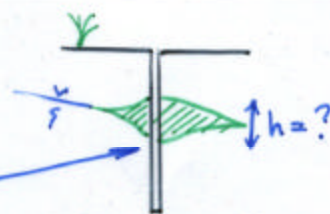
Source: A. L. Baehr, *Water Resources Research* 23, no. 10: 1928. Published 1987 by American Geophysical Union. Used with permission.

3.7.2 Thickness of Floating Product

What is true depth/thickness of NAPL

Borehole will show artificial LNAPL depth

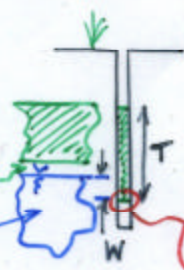
Artificial depth since.



Well thickness (of product)

Water capillary fringe

Water



$$P_o = P_w$$

$$P_o = \rho_o T g$$

$$P_w = \rho_w W g$$

Equating $\rho_o T = \rho_w W$

$\Rightarrow$

$$W = T \left(\frac{\rho_o}{\rho_w} \right)$$

Measure T, calculate W.

Volume of product $\Rightarrow$ Thickness (T-W) (ignoring cap fringe height)

Volume of porespace $A(T-W)n$

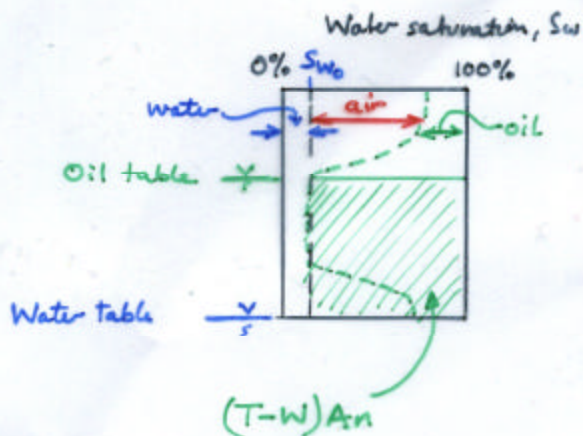
Volume of product $\sim A(T-W)n(1-S_{w0})$

S_{w0}

Not all is recoverable!!

Approx recoverable volume of

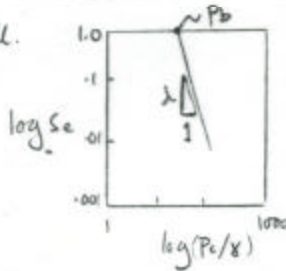
product $\sim A(T-W)n(1-S_{w0}-S_{mr0})$



Recoverable Volume

Brooks-Corey (1966) soil.

Recall.



S_e = effective saturation

$$S_e = \frac{(S_w - S_{w0})}{(1 - S_{w0})}$$

where

P_a^{ow} = the Brooks-Corey air-organic displacement pressure

P_a^{ow} = the Brooks-Corey organic-water displacement pressure

g = the acceleration of gravity

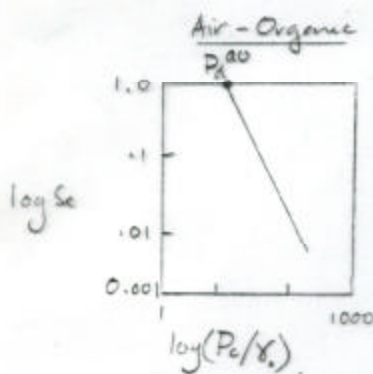
$\log S_e$ air-org

$\log S_e$ organic-

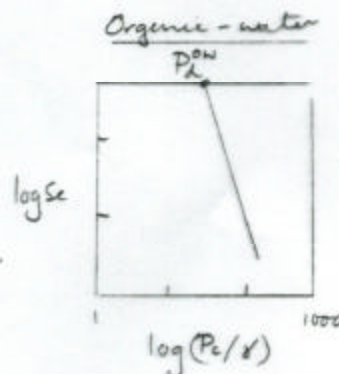
$\log (P_c/\gamma)$

$$* D_a^{ow} = D_a^{ow} - \frac{P_a^{ow}}{\rho_o g} \quad \text{height of oil capillary fringe (5.29)}$$

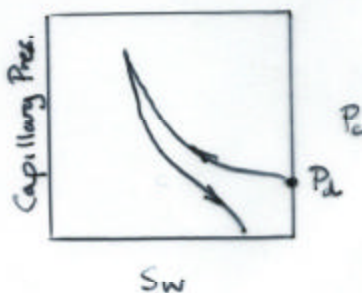
$$* D_a^{ow} = D_a^{ow} - \frac{P_a^{ow}}{(\rho_w - \rho_o)g} \quad \text{height of W + water capillary fringe (5.30)}$$



Wetting = organic
Non-wetting = air



Wetting = water
Non-wetting = organic



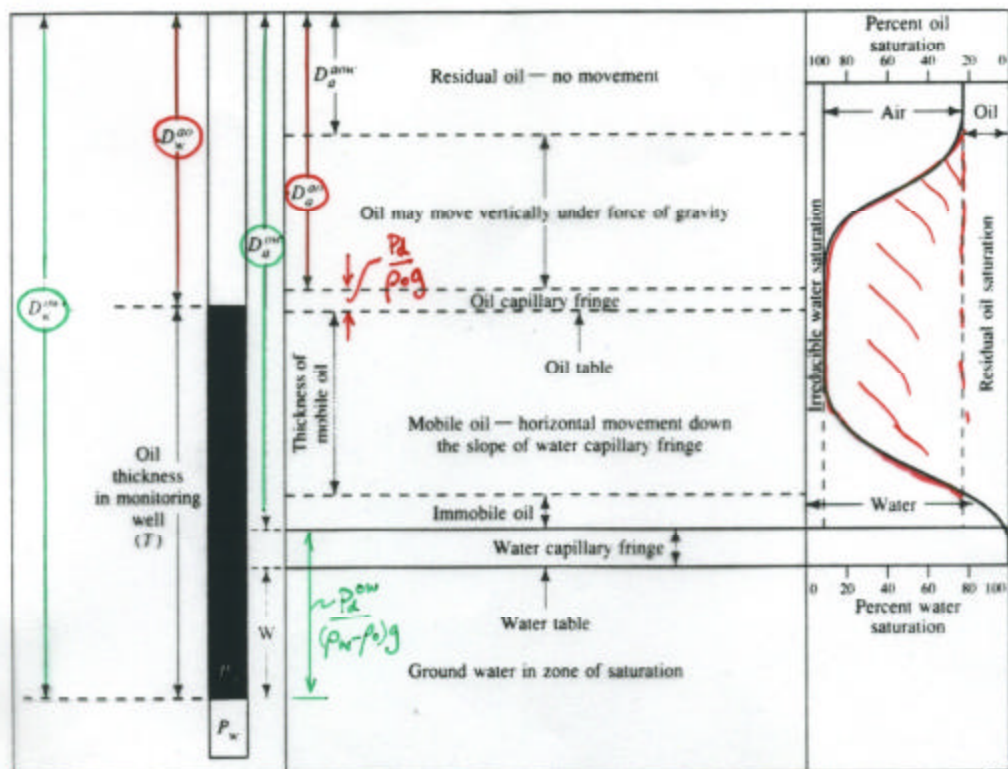


FIGURE 5.19 Comparison of distribution of mobile oil in an aquifer with the thickness of floating oil in a monitoring well for the case where a water capillary fringe exists below the zone of mobile oil.

TERMINOLOGY:

D_w^{ow} ← organic-water interface

D_w ← Measured in the well

Since as figure $D_w^{ow} = D_w^{ao} + T$, then

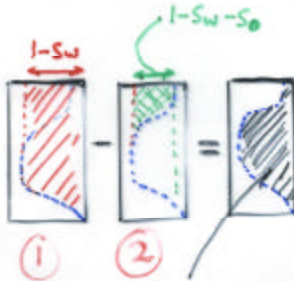
Equation 5.30 may be rewritten as

$$D_a^{ow} = (D_w^{ao} + T) - \frac{P_d^{ow}}{(\rho_w - \rho_o)g} \quad (5.31)$$

If any of the organic liquid exists at a positive pore pressure, then D_a^{ow} will be greater than D_w^{ao} and from Equation 5.31,

$$T \geq \frac{P_d^{ow}}{(\rho_w - \rho_o)g} \quad (5.32)$$

SUM OIL COMPONENTS



If the organic liquid is all under tension in the capillary zone, then there will be no mobile organic layer and no organic liquid will collect in the monitoring well. Under these conditions, Equations 5.29, 5.30, 5.31, and 5.32 are not applicable. However, as soon as free organic liquid appears in the aquifer, it will collect to a depth of at least $P_d^{ow}/(\rho_w - \rho_o)g$.

The total volume of nonresidual organic liquid in the vadose zone is given by

$$V_o = n \left\{ \int_{D_a^{ow}}^{D_w^{ao}} (1 - S_w) dz - \int_{D_a^{ow}}^{D_w^{ao}} [(1 - (S_w + S_o))] dz \right\} \quad (5.33)$$

where

V_o = the volume of organic liquid per unit area

n = the porosity

S_w = the water-saturation ratio

S_o = the organic liquid saturation ratio

z = the vertical coordinate measured positively downward

D_a^{ow} = a value determined from Equation 5.30

D_w^{ao} = a value determined from Equation 5.29

D_a^{ow} = the top of the zone where nonresidual oil occurs

RELATE FLUID

PRESSURES

TO SATURATIONS

Based on work by Lenhard and Parker (1987, 1988), the fluid-content relations are

$$S_o - S_w = (1 - S_{wi}) \left(\frac{P_c^{ao}}{P_d^{ao}} \right)^{-\lambda} + S_{wi}, \quad P_c^{ao} > P_d^{ao} \quad (5.34a)$$

$$S_o + S_w = 1, \quad P_c^{ao} < P_d^{ao} \quad (5.34b)$$

$$S_w = (1 - S_{wi}) \left(\frac{P_c^{ow}}{P_d^{ow}} \right)^{-\lambda} + S_{wi}, \quad P_c^{ow} > P_d^{ow} \quad (5.35a)$$

$$S_w = 1, \quad P_c^{ow} < P_d^{ow} \quad (5.35b)$$

where

S_{wi} = the irreducible water saturation

λ = the Brooks-Corey pore-size distribution index

In addition,

$$P_c^{ao} = \rho_o g (D_w^{ao} - (P_d^{ao}/\rho_o g) - z) + P_d^{ao} \quad (5.36)$$

$$P_c^{ow} = g(\rho_w - \rho_o) \left[D_w^{ow} - \frac{P_d^{ow}}{(\rho_w - \rho_o)g} - z \right] + P_d^{ow} \quad (5.37)$$

Integration of Equation 5.33 for $D_a^{nw} > 0$, using Equations 5.34, 5.35, 5.36, and 5.37, yields the following. For λ not equal to 1,

VOLUMES IF
RESIDUAL
ZONES
PRESENT

$$V_o = \frac{n \phi (1 - S_{wi}) D}{1 - \lambda} \left[\lambda + (1 - \lambda) \left(\frac{T}{D} \right) - \left(\frac{T}{D} \right)^{1-\lambda} \right] \quad (5.38a)$$

For λ equal to 1,

$$V_o = n(1 - S_{wi}) [1 - D(1 + \ln T)] \quad (5.38b)$$

where

$$D = \frac{P_d^{nw}}{(\rho_w - \rho_o)g} - \frac{P_d^{so}}{\rho_o g}$$

$$T = D_w^{nw} - D_w^{so} \geq \frac{P_d^{nw}}{(\rho_w - \rho_o)g}$$

If organic liquid above the residual saturation exists all the way to the land surface, then D_a^{nw} does not exist. Under this condition integration of Equation 5.33 yields the following. For λ not equal to 1,

VOLUMES IF
RESIDUAL
ZONES
ABSENT

$$V_o = n(1 - S_{wi}) \left\{ (T - D) - \frac{P_d^{so}}{\rho_o g (1 - \lambda)} \left[1 - \left(\frac{\rho_o g D_w^{so}}{P_d^{so}} \right)^{1-\lambda} \right] \right. \\ \left. + \frac{P_d^{nw}}{(\rho_w - \rho_o)g (1 - \lambda)} \left[1 - \left(\frac{(\rho_w - \rho_o)g D_w^{nw}}{P_d^{nw}} \right)^{1-\lambda} \right] \right\} \quad (5.39a)$$

For λ equal to 1,

$$V_o = n(1 - S_{wi}) \left[(T - D) - \frac{P_d^{nw}}{(\rho_w - \rho_o)g} \ln D_w^{nw} + \frac{P_d^{so}}{\rho_o g} \ln D_w^{so} \right] \quad (5.39b)$$

$$\text{Approx. volume (simple)} = V \approx n(1 - S_{w0} - S_{nw0})(T - W)$$

Reasons not to be able to recover free product:

1. Lenses of low conductivity

3.7.3. Rise and Fall of Water Table

Fall of water table — "Free" product drops

Rise of water table — Residual LNAPL is trapped below
water table and available $\rightarrow$ dissolution

Rate of movement controlled by:

- Darcy's law, $k/k/\mu$
- Density (driving flow)
- Capillarity

Residual volumes controlled by soil retention capacity

Gravel 5 L/m^3
Silty sand 40 L/m^3 } due to capillarity

Say $n = 30\%$

$$1\text{ m}^3 = 1000\text{ L}$$

porosity @ $30\% \rightarrow 300\text{ L}$

$$\therefore \text{saturation } S_{nw0} = \frac{5}{300} \text{ to } \frac{40}{300}$$

$$S_{nw0} = 1.6\% \text{ to } 13\%$$

- Free product may be drained by open wells and trenches
- Bound product must be removed by $\rightarrow$ volatilization/venting
 $\rightarrow$ dissolution

Note: cannot apply high fluid suction pressures
due to short circuiting.

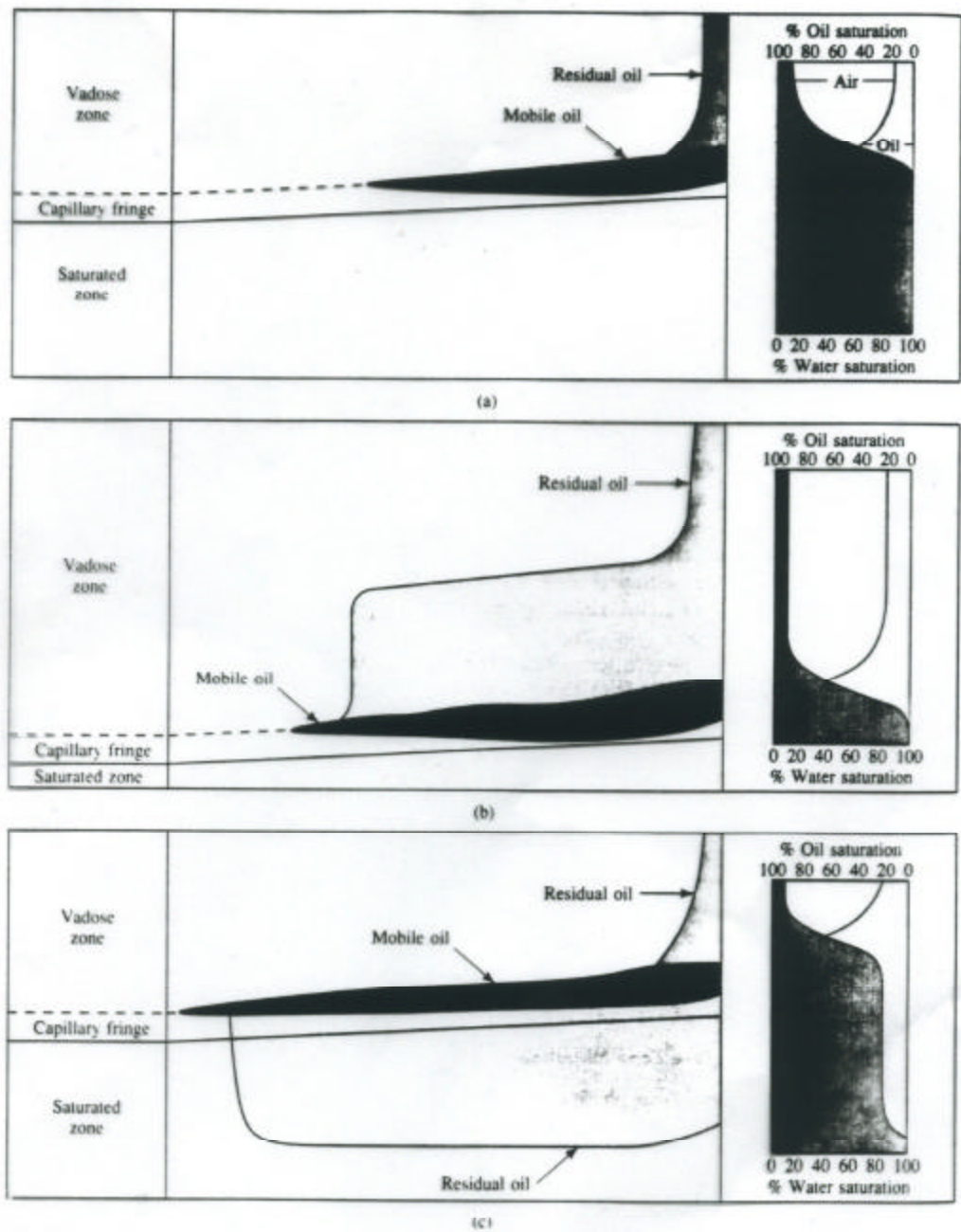


FIGURE 5.22 Effect of a falling and then rising water table on the distribution of mobile and residual phases of an LNAPL.

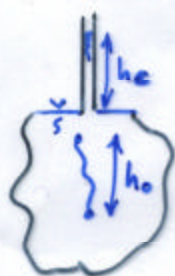
3.8 Behavior of DNAPLs

3.8.1 Vadose zone migration

- Similar characteristics to LNAPL but $\uparrow$ density $\rightarrow$ more penetrative (water table)
- Preferentially chooses large pores and continuous pore throat channels
- Displaces air and reaches capillary zone.
- Displaces water @ capillary zone.

3.8.2 Vertical movement in saturated zone

Static fluid: Hobson's Formula - defines required critical height for penetration, h_o



$h_o \neq h_c$

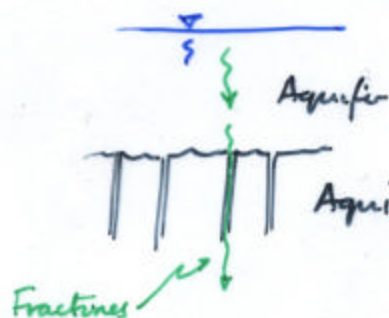
$$h_o = \frac{+2\sigma \cos \theta (1/r_t - 1/r_p)}{g(\rho_w - \rho_o)}$$

r_p = pore radius
 r_t = throat radius

$$h_c = \frac{P_b}{\gamma_o}$$

Rhombohedral packing, $r_p = .212d$
 $r_t = .077d$ } d = grain diameter

Same as $h = \frac{h_c}{(1 - \rho_o/\rho_w)}$ if $h_c = \frac{2\sigma \cos \theta}{r^* \gamma_w}$
as h_c includes $(\theta, \sigma, r^*, \gamma_w)$



Aquifer - will it penetrate?

Capillary tube: $h_c = \frac{4\sigma \cos \theta}{d \gamma_w}$

Fracture: $h_c = \frac{2\sigma \cos \theta}{b \gamma_w}$

Monitoring wells:

- Place at aquifer base to collect flow from mobile DNAPL -
Will separate in well into individual components.
 - water at saturation (potentially).
- DNAPL level is intermediate within zone of mobile water and DNAPL
- Deep wells record pulse DNAPL depth.

Vertical distribution of DNAPL

Accumulation depends on grain size (actually pore size) distribution $\rightarrow$ since controls capillary pressure distribution.

Small pores/fractures: Low $K \rightarrow$ thin pure DNAPL layer
thick DNAPL + water

Large pores/fractures: High $K \rightarrow$ thick pure DNAPL layer
thin DNAPL + water.

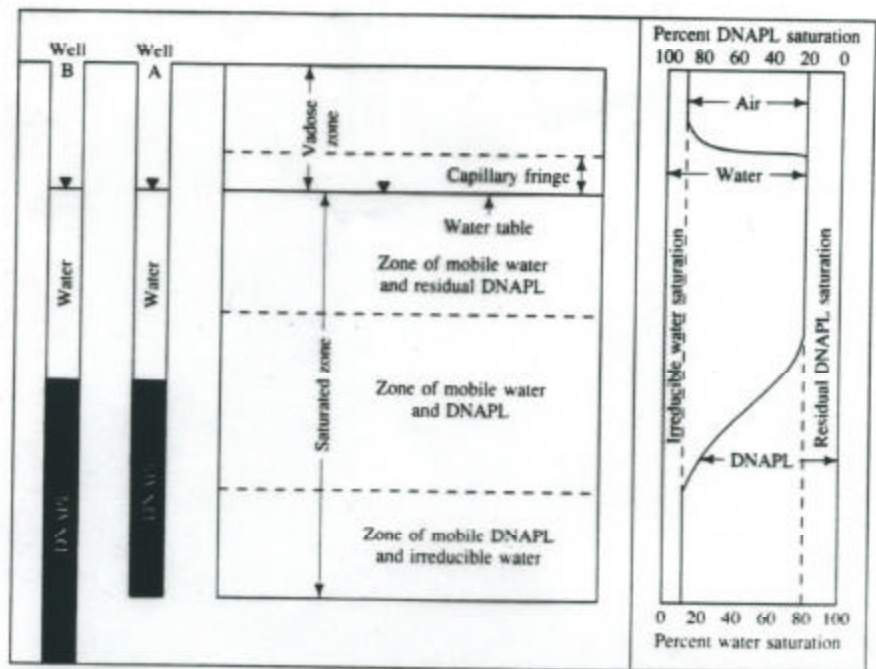


FIGURE 5.23 Zones of a DNAPL and the relationship of mobile DNAPL and nonmobile DNAPL to the DNAPL saturation; relationship of mobile DNAPL thickness to thickness of DNAPL is measured in a monitoring well.

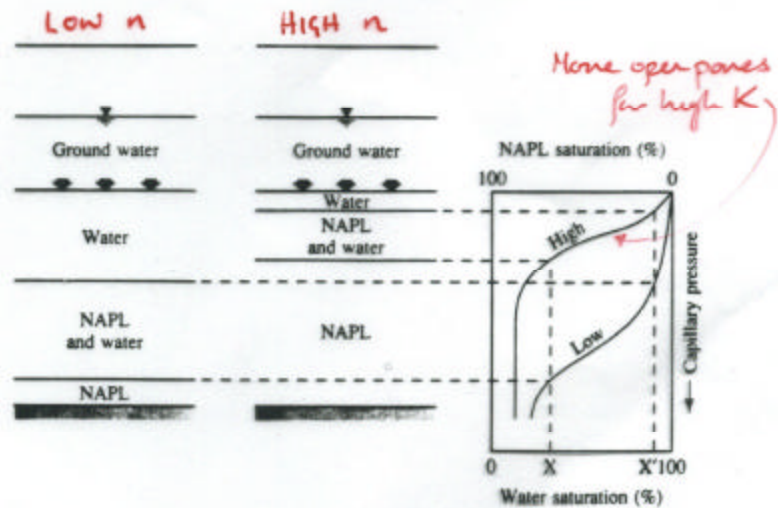


FIGURE 5.24 Effect of high and low permeability (and porosity) on the distribution of mobile DNAPL at the bottom of an aquifer; the arrows indicate level of original injection of the DNAPL. Source: J. F. Villiaume, *Ground Water Monitoring Review* 5, no. 2 (1985): 60-74. Copyright © 1985 Water Well Journal Publishing Co.

3.8.3. Horizontal Movement in Saturated Zone

- Difficult to estimate potential for movement

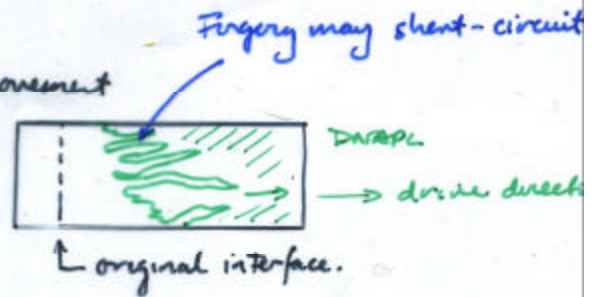
- Water drive will produce fingering

Classical fingering

1) Due to instability

2) Accentuated by heterogeneity

- heterogeneity has overriding effect in most shallow aquifers.



Gradient required to move DNAPL (horizontally)

$$V_p = \frac{2\sigma}{L_o \left(\frac{1}{r_t} - \frac{1}{r_p} \right) r^*}$$

L_o = length of continuous DNAPL phase
 r_t = throat ; r_p = pore radii

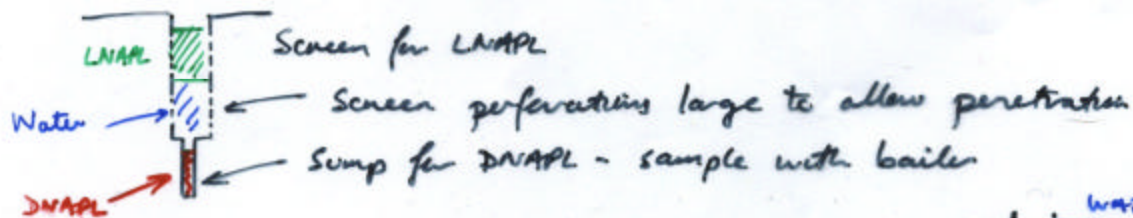
eg. to determine radius of influence of capture well:

$$\frac{dp}{dr} = \frac{1}{r_w} \frac{dh}{dr}$$

Difficulty is estimating L_o .

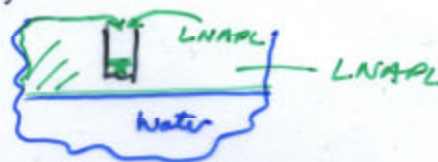
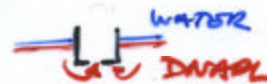


3.9 MONITORING LNAPL & DNAPL



DNAPL - Bottom loading bailer

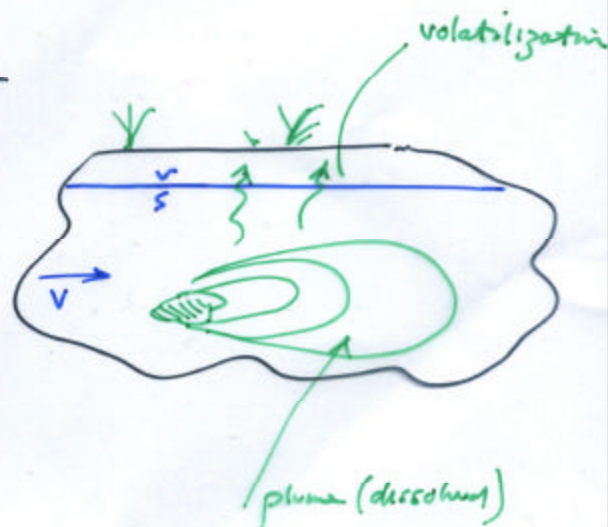
LNAPL - Top loading bailer scooping from upper surface



- Allow fluids to separate in sample $\rightarrow$ decant.
- Partial emulsion induced by sampling process.

Fractured bedrock sampling most difficult -

- Pooled DNAPL locations
- Infer source
- Drilling may remobilize "free" product by dilating fractures:



4. UNSATURATED FLOW

- Contaminants traverse vadose zone $\left\{ \begin{array}{l} \text{Pesticides (deliberate application)} \\ \text{Miscible} \\ \text{Dissolved} \end{array} \right.$

- Clay mineralogy

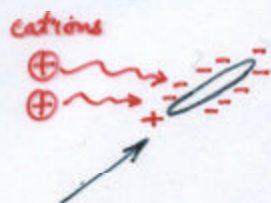
Reactive clays in many soils (clay $< 2 \mu\text{m}$)

Weathering products - Aluminium, Silica, Oxygen

Kaolinite (china clay) $5-20 \text{ m}^2/\text{g}$

Montmorillonite - high surface area $700 \text{ m}^2/\text{g}$.

- Why important -



(Net +ve charge @ ends)

- Electrical double layer captures colloids (unbalanced -ve surface)

attracts +ve cations

$\therefore$ good buffers or attenuators

- Electrostatic double layer affects hydrology

Swelling clays swell and seal pathways.

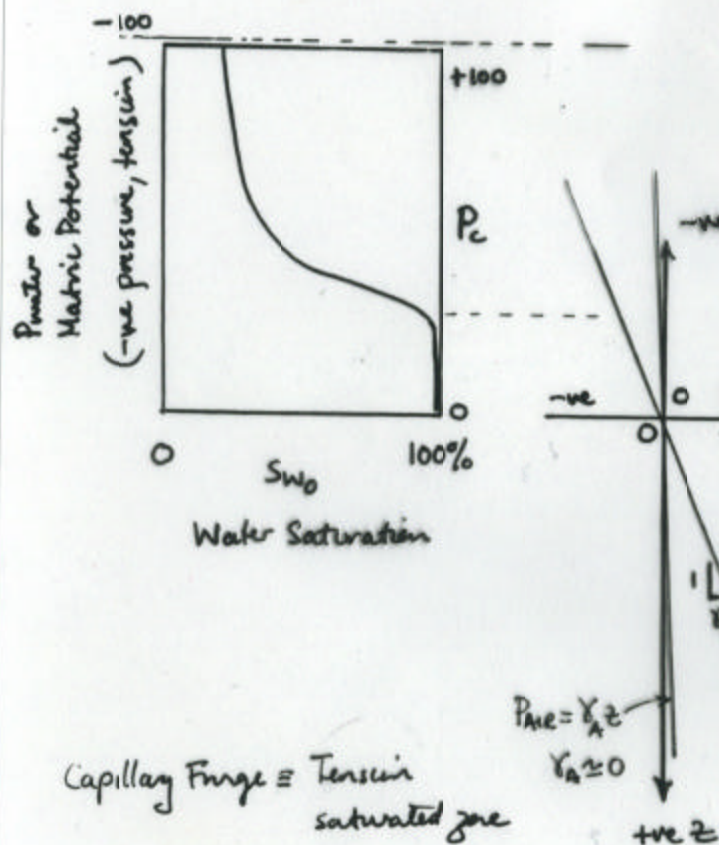
Water salinity $\uparrow \rightarrow$ swelling $\rightarrow$ $\downarrow$ conductivity

Process reverses if flushed by fresh water.

STATIC SYSTEMS

Vadose zone

$$P_c = P_{air} - P_{water}$$

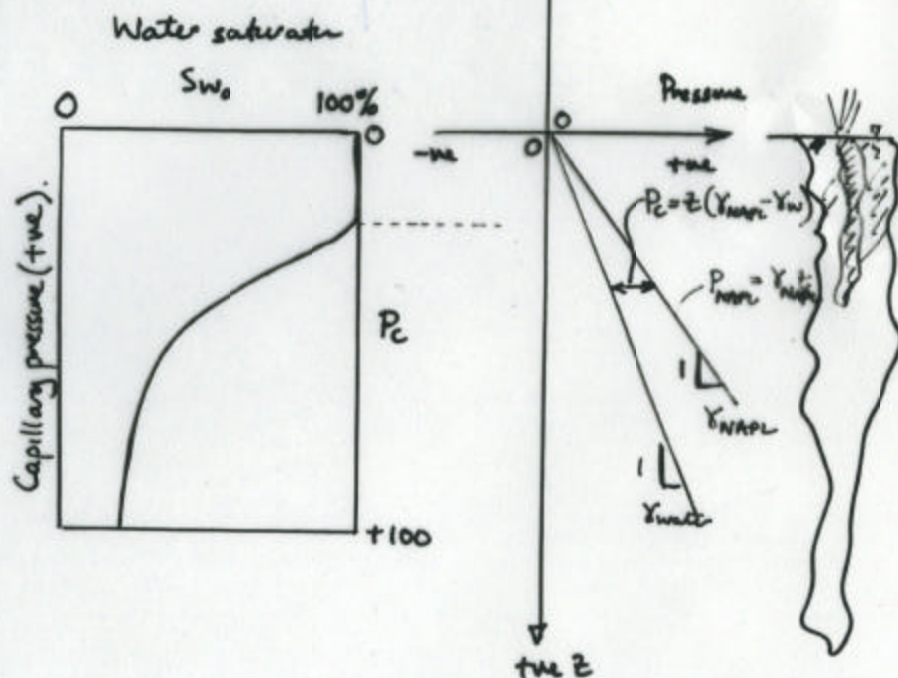


$$P_c = P_{nw} - P_w$$

NAPL in Groundwater

$$P_c = P_{NAPL} - P_{water}$$

$$\begin{aligned} v_d = q &= -K \frac{dh}{dx} = -k_r K(\theta) \frac{dh}{dx} \\ &= k_r \frac{k}{\mu} \left(\frac{\partial p}{\partial x} + \rho g \frac{\partial z}{\partial x} \right) \end{aligned}$$



4.1 Flow of water in the unsaturated zone

- Neglect osmotic pressures
(flow due to concentration gradients)
- Neglect electrochemical processes

$$h = \underbrace{\psi(\theta)}_{\frac{P}{\gamma_w}} + z + \frac{v^2}{2g}$$

h = total head
 z = elevation head
 $\psi(\theta)$ = matric potential
i.e. matric fluid pressure
 θ = moisture content

$$\psi(\theta) = \frac{P(\theta)}{\gamma_w}$$

Matric pressure, $P(\theta)$ is a unique function of moisture content, θ .

Two popular relations (empirical) for $P(\theta)$ -vs- θ
or $P(s)$ -vs- s_w

1. Brooks and Corey (1966).
2. van Genuchten (1980).

4.1.1. Brooks & Corey (1966)

May define for P_c -vs- Θ or P_c -vs- S_w .

$$\Theta = \Theta_{w_0} + (\Theta_s - \Theta_{w_0}) \left(\frac{P}{P_b} \right)^{-\lambda} \quad \left\{ \begin{array}{l} \Theta_{w_0} = \text{irreducible water content} \\ \Theta_s = \text{saturated water content} \\ P_b = \text{bubbling pressure} \\ P = \text{capillary pressure} \end{array} \right.$$

$$\frac{\Theta}{\Theta_s} = \frac{\Theta_{w_0}}{\Theta_s} + \left(\frac{\Theta_s - \Theta_{w_0}}{\Theta_s} \right) \left(\frac{P}{P_b} \right)^{-\lambda} \Rightarrow S = S_{w_0} + (1 - S_{w_0}) \left(\frac{P}{P_b} \right)^{-\lambda}$$

$$S_e \rightarrow \frac{(S - S_{w_0})}{(1 - S_{w_0})} = \left(\frac{P}{P_b} \right)^{-\lambda}$$

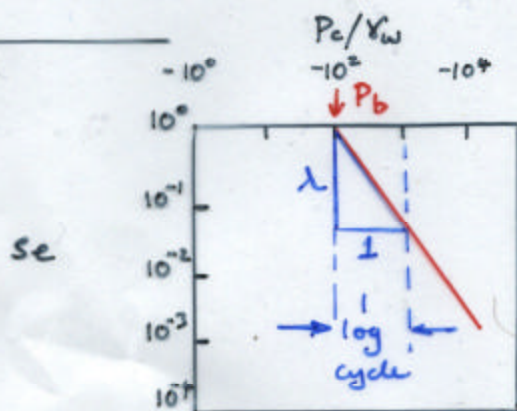
This is an empirical relation: $S_e = \left(\frac{P}{P_b} \right)^{-\lambda}$

$$\log S_e = -\lambda \log (P/P_b)$$

$$\log S_e = -\lambda [\log P - \log P_b]$$

Limiting conditions:

For $P \equiv P_b$ Then $\log S_e = 0$; $S_e = 1$



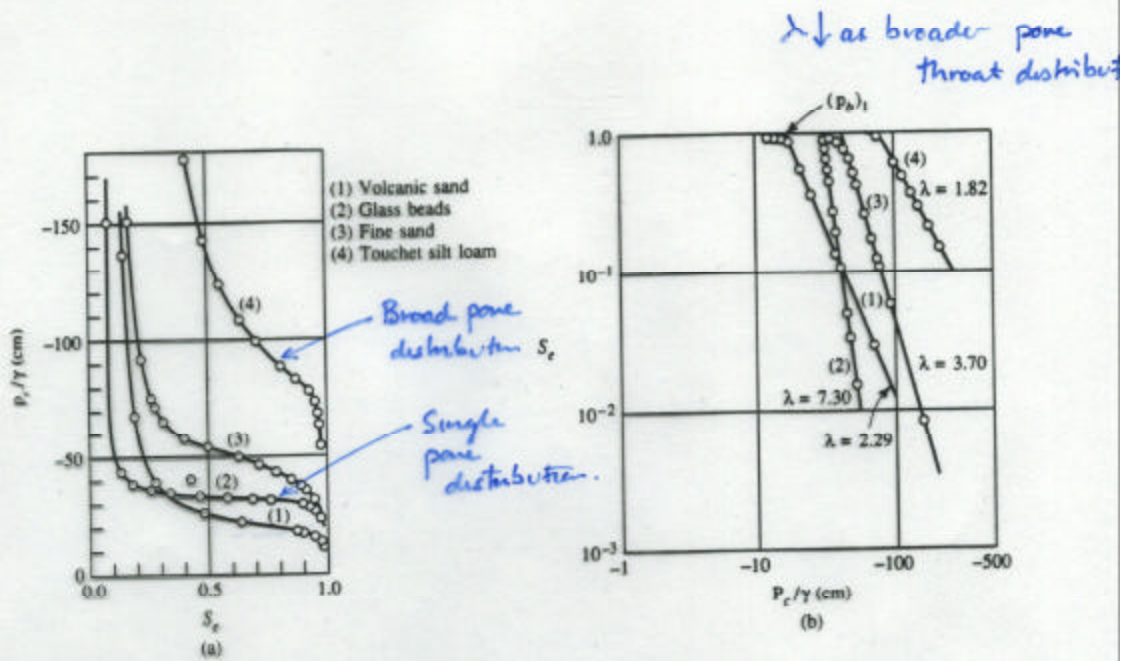
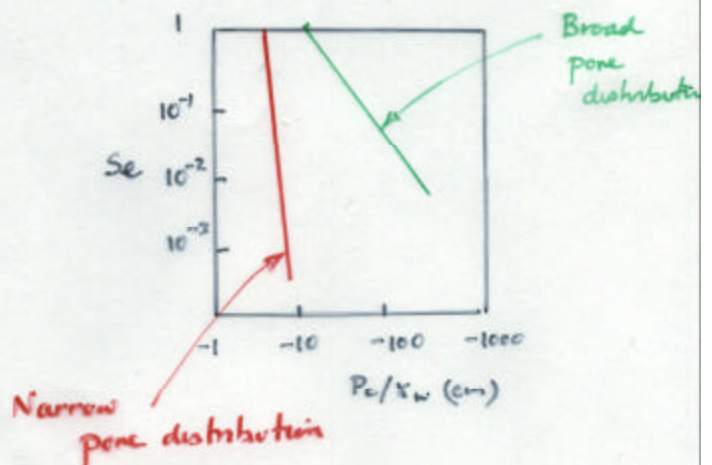
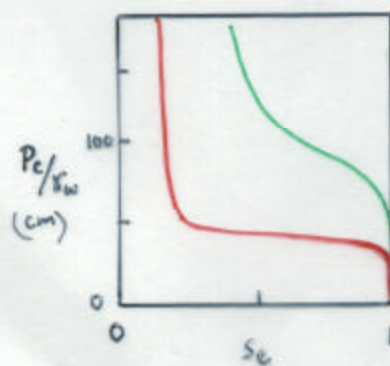


FIGURE 4.5 Capillary pressure head as a function of effective saturation for porous materials with various pore sizes. (a) Plotted on arithmetic paper and (b) plotted on log-log paper. Source: R. H. Brooks and A. T. Corey, *Proceedings, American Society of Civil Engineers, Irrigation and Drainage Division* 92, no. 182 (1966): 61-87.

BEHAVIOR DESCRIBED BY TWO PARAMETERS:

- 1) P_b - BOBBING PRESSURE
- 2) λ - SLOPE OF GRAPH OVER 1 LOG CYCLE



4.1.2 van GENUCHTEN (1980) Empirical.

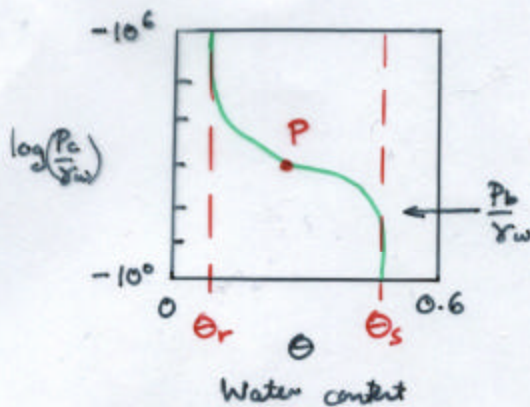
H. van Genuchten, Soil Science Society of America, Journal 44 (1980) 892-898.

Moisture Content

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha \frac{P}{\gamma_w})^n]^m}$$

$$\theta_e = \frac{1}{[1 + (\alpha \frac{P}{\gamma_w})^n]^m}$$

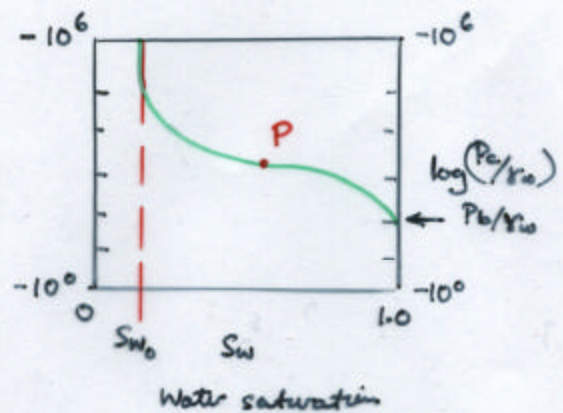
Parameters: m, n, α



Saturation

$$S = S_{w0} + \frac{(1 - S_{w0})}{[1 + (\alpha \frac{P}{\gamma_w})^n]^m}$$

$$S_e = \frac{1}{[1 + (\alpha \frac{P}{\gamma_w})^n]^m}$$



not porosity $\longrightarrow n = 1/(1-m)$

$$\alpha = \frac{\gamma_w}{P_b} (2^{1/m} - 1)^{(1-m)}$$

$$\alpha \approx \frac{1}{h_b} \approx \frac{\gamma_w}{P_b}$$

DETERMINING PARAMETERS

MEANS MOISTURE CONTENT

$$\theta_p = \frac{1}{2}(\theta_s + \theta_r)$$

$$= \frac{1}{2}(0.5 + 0.1) = 0.3$$

SLOPE AT θ_p

$$f = 0.34$$

DIMENSIONLESS SLOPE

$$f_p = \frac{f}{\theta_s - \theta_r}$$

$$f_p = \frac{0.34}{0.5 - 0.1} = 0.85$$

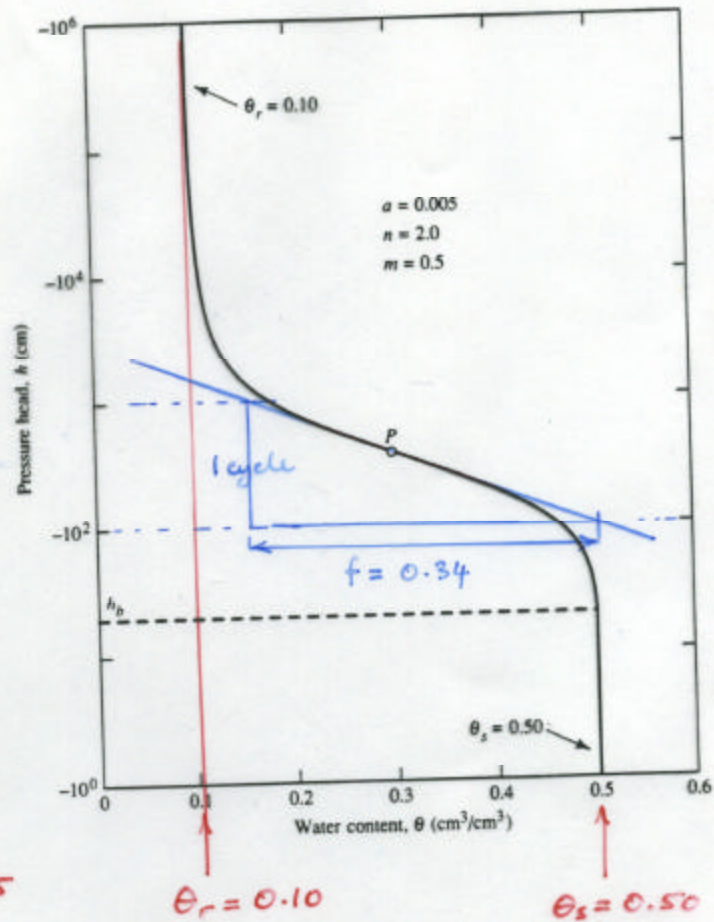
$$m = \begin{cases} 1 - \exp(-0.8 f_p) & 0 \leq f_p \leq 1 \\ 1 - \frac{0.5755}{f_p} + \frac{0.1}{f_p^2} + \frac{0.025}{f_p^3} & f_p > 1 \end{cases}$$

$$m = 0.5$$

$$n = \frac{1}{1-m} = \frac{1}{1-0.5} = 2$$

$$\alpha = \frac{\gamma_w}{P_b} (2^{1/n} - 1)^{1-m} = \frac{\gamma_w}{P_b} (1.73)$$

$$\alpha \approx \frac{1}{P_b / \gamma_w}$$



4.2 Flow of Water in the Unsaturated Zone

4.2.1 Hydraulic Conductivity

$$K(\theta) = k_r(\theta) \frac{k}{\mu_w} \rho_w g$$

k = intrinsic permeability

k_r = relative permeability

van Genuchten (1980)

Empirically relate k_r to θ

$$K(\theta) = K_{sat} S_e^{1/2} [1 - (1 - S_e^{1/m})^m]^2 \quad S_e = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)}$$

$$K(h) = K_{sat} \frac{\{1 - (\alpha h)^{n-1} [1 + (\alpha h)^n]^{-m}\}^2}{[1 + (\alpha h)^n]^{m/2}}$$

Note: $h = \frac{P}{\gamma_w}$ and P related to S_e or θ

h or P_c



S_e
or
 θ_e

Useful in complex numerical models for unsaturated flow.

VAN GENUCHTEN CURVES (1980)

Ideally, the relative conductivity, moisture content, and water capacity curves are determined directly by performing a series of tests on the soils involved in the study. However, in many cases they can be approximated using a set of measured or approximated constants and a set of empirical relationships. For example, one option for generating the curves is to use the van Genuchten functions (van Genuchten, 1980). The van Genuchten relationships are:

$$K_r = \theta_e^{0.5} \left[1 - (1 - \theta_e^{1/\gamma})^\gamma \right]^2$$

and

$$S_e = \theta_r = \left[1 + (\alpha h)^\beta \right]^{-\gamma} \quad \text{for } h < 0$$

$$\theta_r = 1 \quad \text{for } h \geq 0$$

where:

$$\theta_w = \theta_r + \theta_e (\theta_s - \theta_r)$$

$$\gamma = 1 - \frac{1}{\beta}$$

and

θ_w = moisture content (dimensionless)

$S_e = \theta_e$ = effective moisture content (dimensionless)

θ_s = saturation moisture content (dimensionless)

θ_r = residual moisture content (dimensionless)

β, γ = soil-specific exponents (dimensionless)

α = soil-specific coefficient

COMPARISON WITH "FETTER" TERMINOLOGY

$$S_e = \theta_e = \frac{(\theta_w - \theta_r)}{(\theta_s - \theta_r)}$$

$$\gamma = m$$

$$\beta = n$$

Free parameters

α represents $1/(P_0/\gamma_w)$

β represents slope of P_c -vs- S_e } $S_e = \theta_e$

Table 5.1 lists a set of saturated and residual moisture contents and the van Genuchten α and β terms for a variety of common soil types. When applying the α term, care should be taken to convert it to the proper units.

Table 5.1

Representative Soil Parameters

Soil Type	Saturated Moisture Content, θ_s	Residual Moisture Content, θ_r	α (cm ⁻¹)	β (γ)
Clay**	0.38	0.068	0.008	1.09
Clay Loam	0.41	0.095	0.019	1.31
Loam	0.43	0.078	0.036	1.56
Loam Sand	0.41	0.057	0.124	2.28
Silt	0.46	0.034	0.106	1.37
Silt Loam	0.45	0.067	0.020	1.41
Silty Clay	0.36	0.070	0.005	1.09
Silty Clay Loam	0.43	0.089	0.010	1.23
Sand	0.43	0.045	0.145	2.68
Sandy Clay	0.38	0.100	0.027	1.23
Sandy Clay Loam	0.39	0.100	0.059	1.48
Sandy Loam	0.41	0.085	0.075	1.89

** Agricultural soil, less than 60% clay

Source: Carsel and Parrish (1988)