2_1 Heat Flow in the Earth

Recap:

Budgets - US 20% of world energy use/capacity - 100 Quads = 100 EJ = 100 TCF CH4
Hydrates: 10^6 EJ
Geothermal - US-Hydrothermal 10^4 EJ; US-EGS 10^7 EJ

Movies:

Plate Tectonics: https://www.nationalgeographic.org/media/plate-tectonics/

Resources: WG2 + AG2

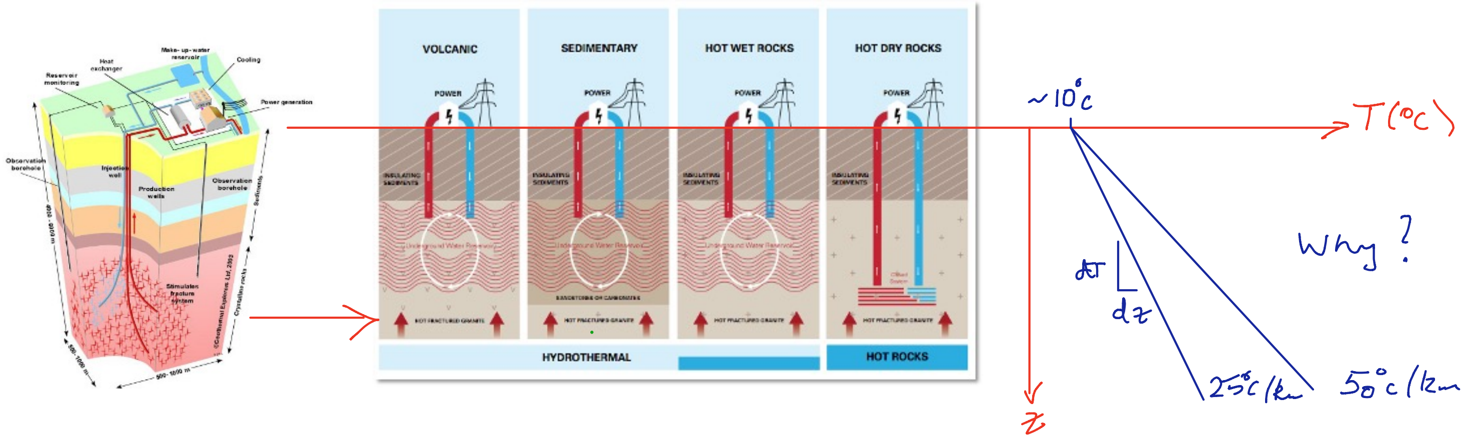
Plate Tectonics: http://www.columbia.edu/itc/ldeo/v1011x-1/jcm/Topic3/Topic3.html

Earth's Internal Heat Budget: https://en.wikipedia.org/wiki/Earth%27s_internal_heat_budget

Geothermal Gradient: https://en.wikipedia.org/wiki/Geothermal_gradient

Motivation:

1. **Motivation [10%]** Provide context for the topic. *Use of relevant public domain videos* are a useful method for this. Why is this particular topic or sub-topic important in the broad view of geothermal energy engineering?



Scientific Questions:

2. **Scientific Questions to be Answered/Outline [10%]** What questions arise from the motivation. What are the sub-topical areas that address these scientific questions.

Origin of Earth's Geothermal gradient?

Why high and low in different locations?

Is heat flux sufficient to resupply or is heat reserve sufficiently large?

Origin of the Earth's Heat/Geothermal Gradient:

3. For Each Sub-Topic:

a. Detailed Explanation of the Topic [40%] Describe the physical principles in detail and at a pace that is tutorial for an audience.

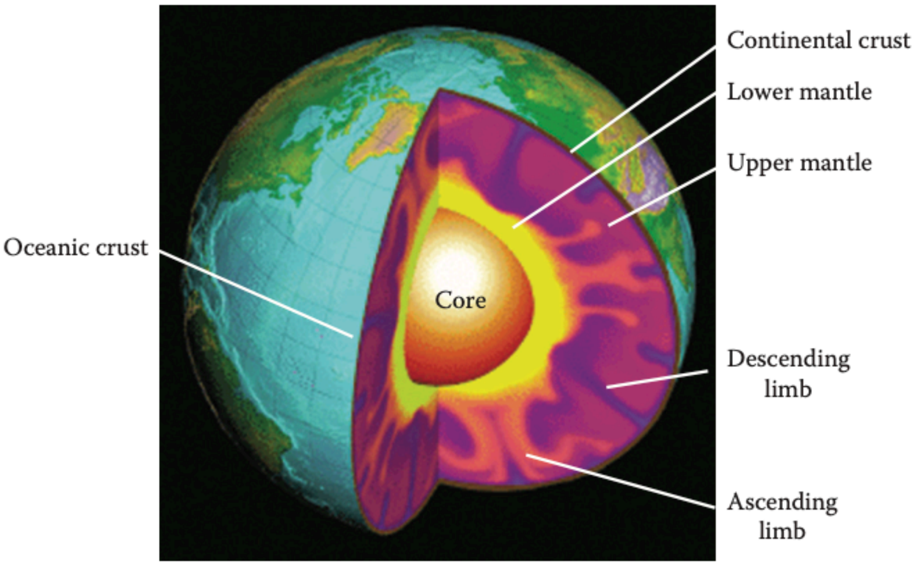
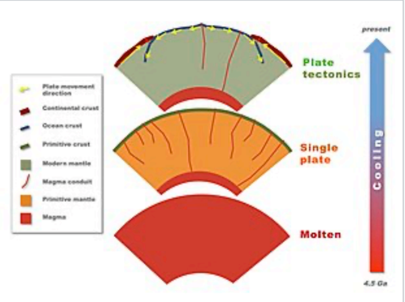


FIGURE 2.1 Interior of the earth, shown in a cutaway that depicts the outer edge of the liquid core (reflecting orange sphere), the lower mantle (yellow), the upper mantle (pink and purple), and the crust. Ascending limbs of convection cells are shown as the orange-tinted plumes extending from the lower mantle through the upper mantle to the base of the crust. Descending limbs of convection cells are shown as the darker purple features extending into the mantle from the base of the crust. (From United States Geological Survey, http://geomag.usgs.gov/about.php.)

Formation of the Earth - ~4,560 Ma - accretion from solar nebula - spherically differentiated

Moon - likely a product of meteoritic impact - tectonically dead.



Earth's tectonic evolution over time from a molten state at 4.5 Ga,[8] to a single-plate lithosphere,[20] to modern plate tectonics sometime between 3.2 Ga[21] and 1.0 Ga[22]

Radius of Earth - ~4,000 miles / ~6,400 km

Core is solid iron (r=1,200km) and molten to (r=3,480 km)
Differentiated by density
T ~5,700K to 4,000K

Lower then upper mantle - liquid but highly viscous

Rigid/solid plates ~70km

Heat supply: 40% from core / 60% from long-lived radioactive isotopes

TABLE 2.1
Heat Generation of the Primary Heat Producing Elements

Material	K	U	Th
Heat production (W/kg of element)	3.5×10^{-9}	96.7×10^{-6}	26.3×10^{-6}

Source: Beardsmore, G.R. and Cull, J.P., *Crustal Heat Flow: A Guide to Measurement and Modeling*, Cambridge University Press, Cambridge, 2001.

Plate Tectonics

Plate Tectonics: http://www.columbia.edu/itc/ldeo/v1011x-1/jcm/Topic3/Topic3.html

Continental and Oceanic crust - only interested in continental?
Evolution over time - continental drift
Current plates and their boundaries - constructive and destructive.

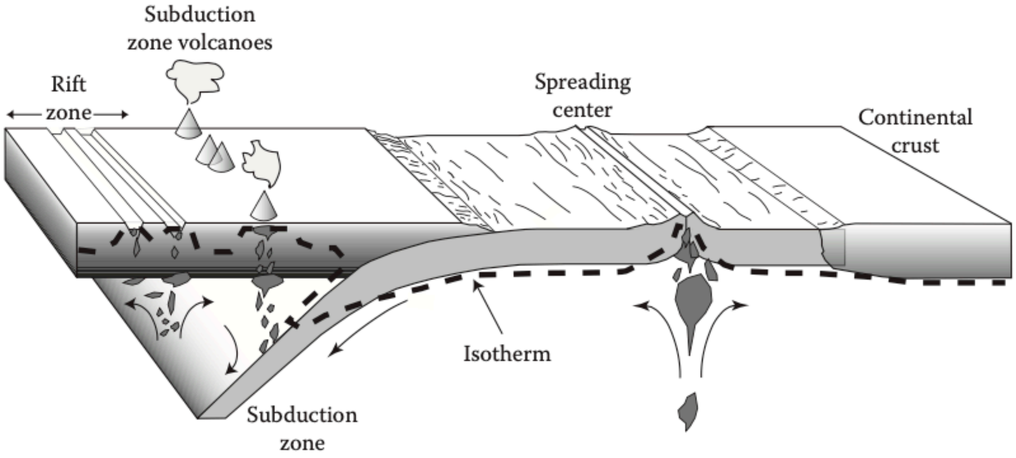


FIGURE 2.5 Schematic diagram showing the configuration of the main elements that compose plate tectonic structures. The arrows indicate local motion of convecting mantle material. The gray, irregular masses represent magma bodies as they ascend from the mantle into the crust. Note that the bulk of magma that occurs in the earth is found at spreading centers, subduction zone volcanoes, or rift zones.

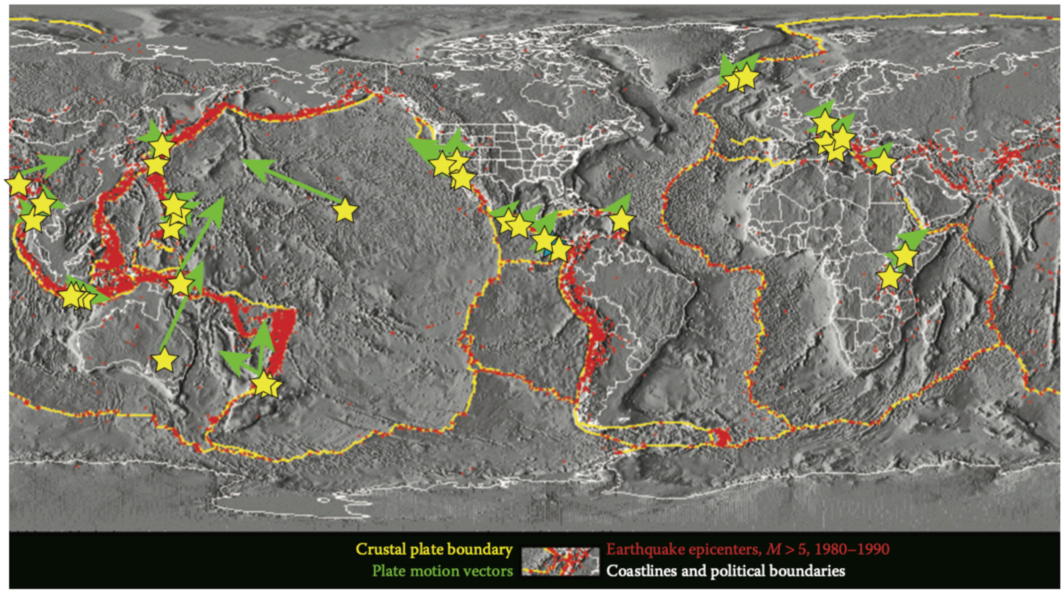


FIGURE 2.6 Global map showing the locations of earthquakes (red dots) that indicate plate boundaries (yellow lines), political boundaries (in white), and the locations of the world's geothermal power plants (stars). The directions of some plate motions are shown by the green arrows, with the length of the arrow corresponding to relative velocity of plate motion. Note the strong correlation between power plant sites and plate boundaries. There are many more power plants than stars because many sites have several power plants. The global map, earthquake data, and boundaries are from the National Oceanic and Atmospheric Administration Plates and Topography Disc and the power plant sites from the International Geothermal Association website (http://iga.igg.cnr.it/geo/geoenergy.php).

Internal Heat Budget

Earth's Internal Heat Budget: https://en.wikipedia.org/wiki/Earth%27s_internal_heat_budget

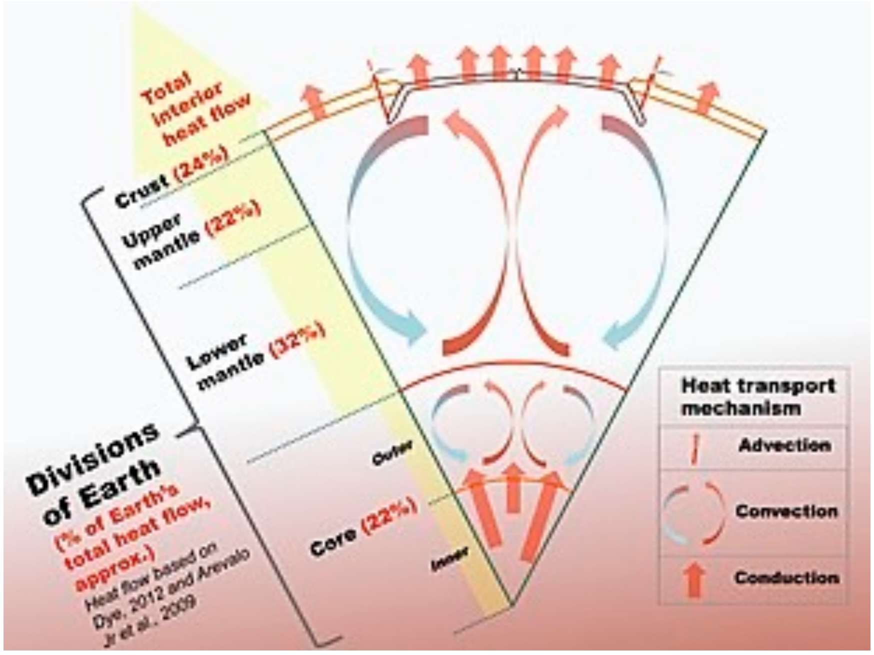
Contributions from:

Radioactive decay ~60%	~15-14 TW
Proimordial heat (core) ~40%	~12-30 TW

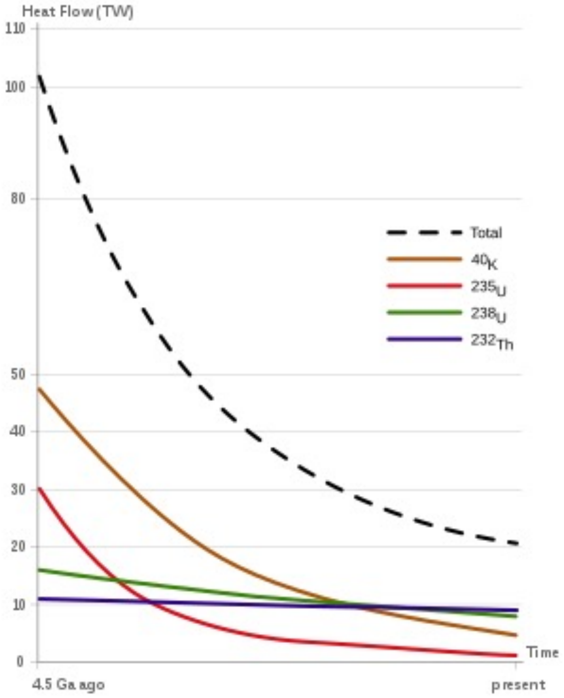
Total flow: ~47 TW (note fossil budget ~10-15 TW) 47 TW ~ 92 mW/m^2

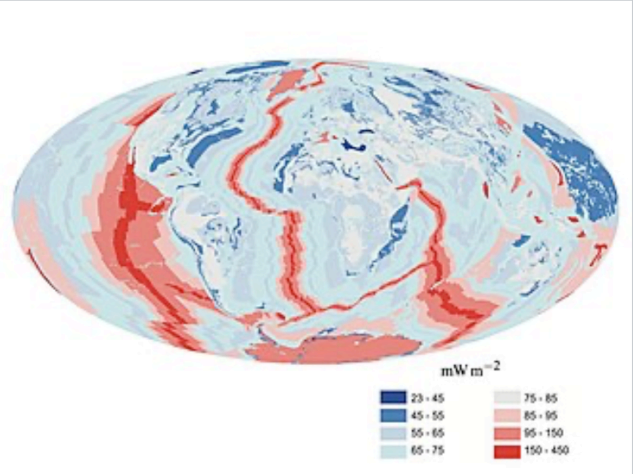
Overall heat flow:

Result of heat flow - on surface



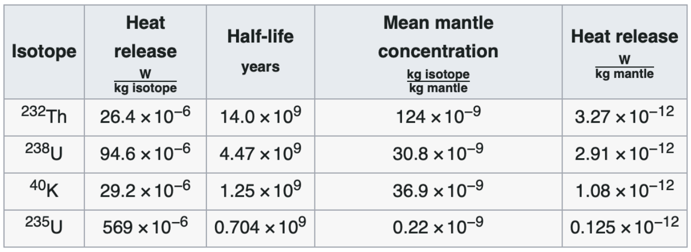
Contribution from radioactive decay





Global map of the flux of heat, in mW/m^2, from Earth's interior to the surface.[1] The largest values of heat flux coincide with mid ocean ridges, and the smallest values of heat flux occur in stable continental interiors.

An estimate of the present-day major heat-producing isotopes[2]



Isotope	Heat release $\frac{W}{kg\ isotope}$	Half-life years	Mean mantle concentration $\frac{kg\ isotope}{kg\ mantle}$	Heat release $\frac{W}{kg\ mantle}$
^{232}Th	26.4×10^{-6}	14.0×10^{9}	124×10^{-9}	3.27×10^{-12}
^{238}U	94.6×10^{-6}	4.47×10^{9}	30.8×10^{-9}	2.91×10^{-12}
^{40}K	29.2×10^{-6}	1.25×10^{9}	36.9×10^{-9}	1.08×10^{-12}
^{235}U	569×10^{-6}	0.704×10^{9}	0.22×10^{-9}	0.125×10^{-12}

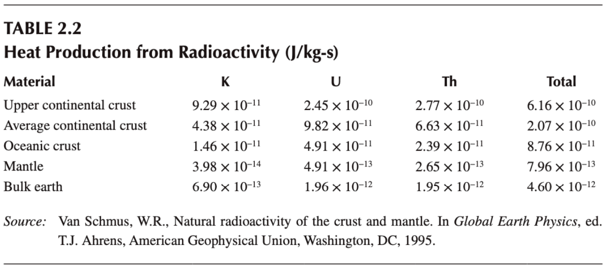


TABLE 2.2

Heat Production from Radioactivity (J/kg-s)

Material	K	U	Th	Total
Upper continental crust	9.29×10^{-11}	2.45×10^{-10}	2.77×10^{-10}	6.16×10^{-10}
Average continental crust	4.38×10^{-11}	9.82×10^{-11}	6.63×10^{-11}	2.07×10^{-10}
Oceanic crust	1.46×10^{-11}	4.91×10^{-11}	2.39×10^{-11}	8.76×10^{-11}
Mantle	3.98×10^{-14}	4.91×10^{-13}	2.65×10^{-13}	7.96×10^{-13}
Bulk earth	6.90×10^{-13}	1.96×10^{-12}	1.95×10^{-12}	4.60×10^{-12}

Source: Van Schmus, W.R., Natural radioactivity of the crust and mantle. In *Global Earth Physics*, ed. T.J. Ahrens, American Geophysical Union, Washington, DC, 1995.

Glassley – Heat flow map

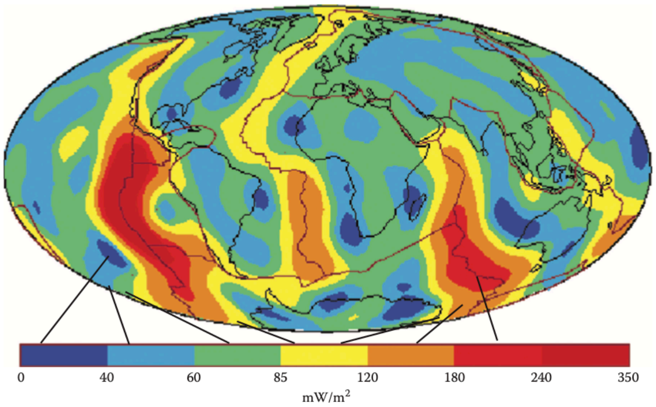
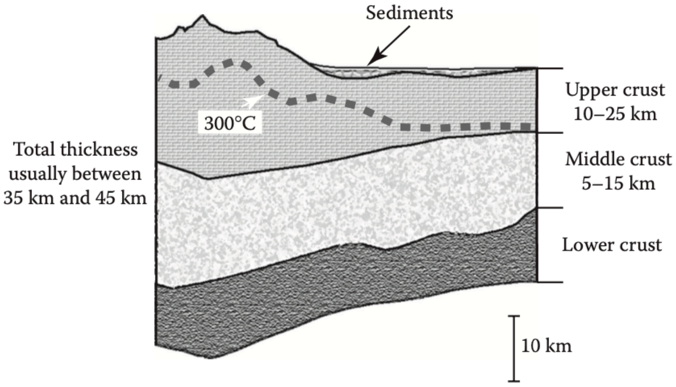


FIGURE 2.7 Low resolution global map showing the distribution of heat flow at the surface. Compare this figure with that in Figure 2.6 to see the relationship between plate boundaries, geothermal power plants, and heat flow. (From International Heat Flow Commission, http://www.geophysik.rwth-aachen.de/IHFC/heatflow.html.)

Glassley – Crustal heat flow



Blackwell – SMU – heat flow map

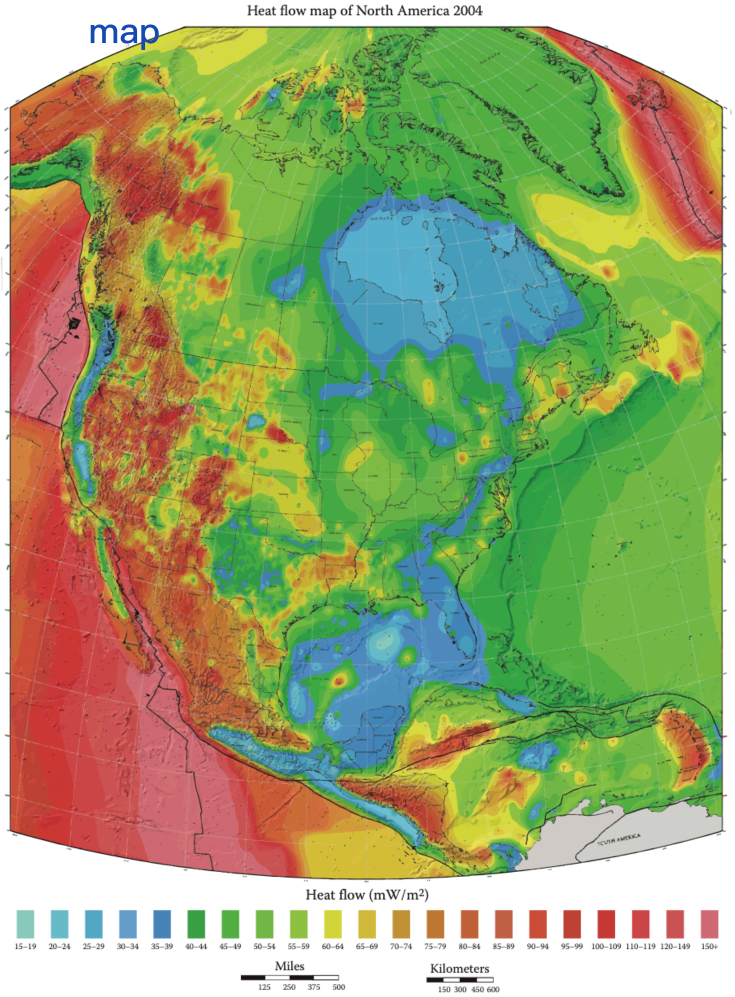
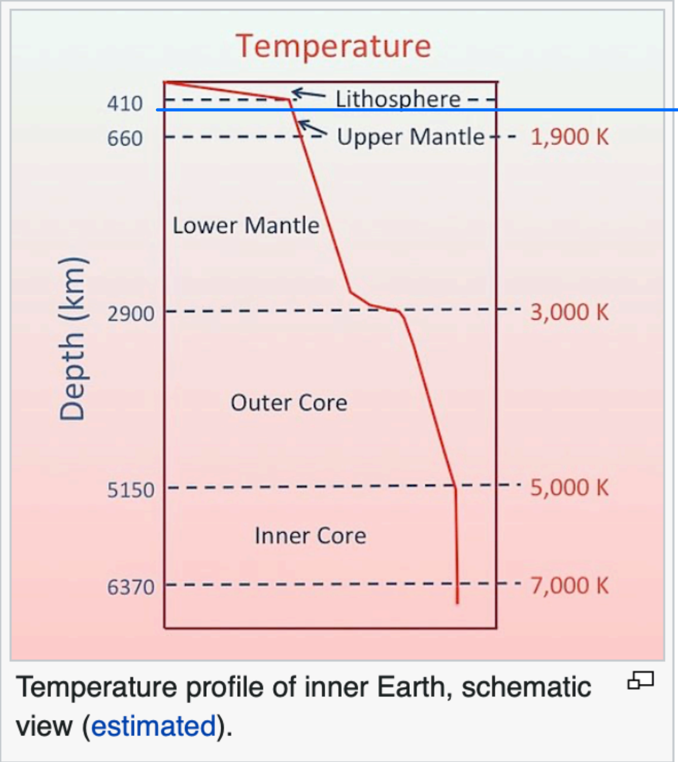


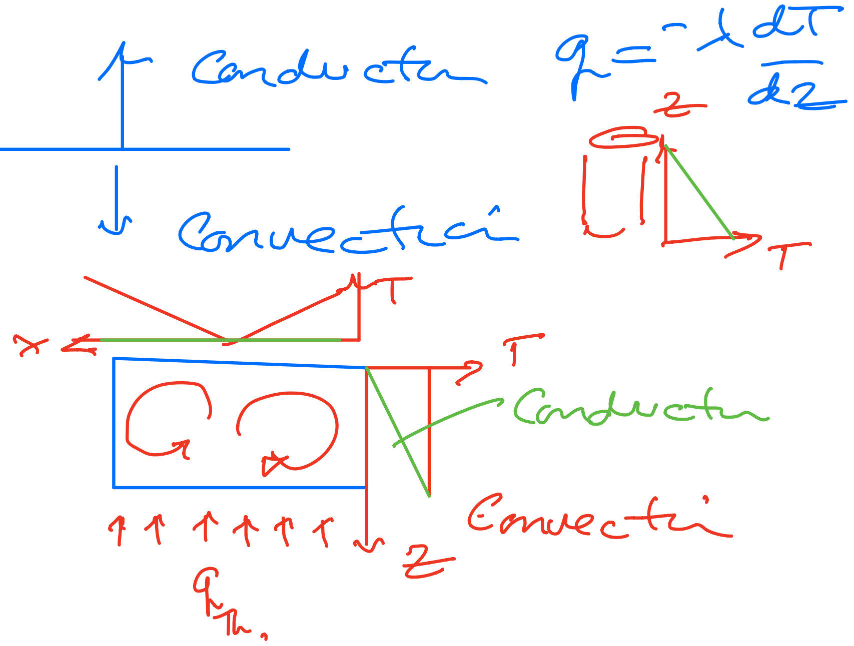
FIGURE 2.9 Heat flow map of North America 2004. (From Geothermal Laboratory, Southern Methodist University. http://smu.edu/geothermal/2004NAMap/Geothermal_MapNA_7x10in.gif.)

Heat Flow - Influence on Geothermal Gradient

Geothermal Gradient: https://en.wikipedia.org/wiki/Geothermal_gradient



Temperature profile of inner Earth, schematic view (estimated).



$$q = -\lambda \frac{dT}{dz}$$

Conduction

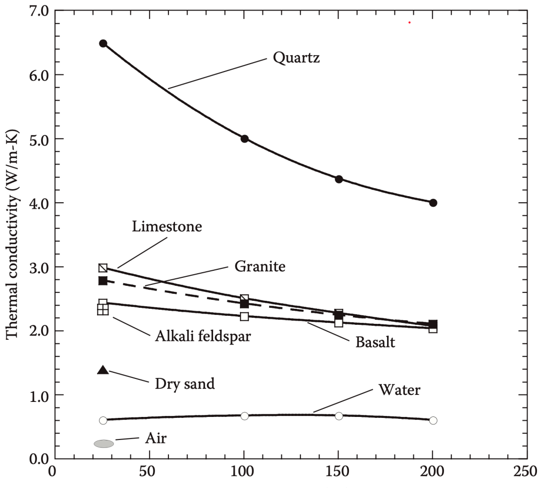
TABLE 2.3
Thermal Conductivity of Some Common Materials (W/m-K)

Material	25°C	100°C	150°C	200°C
Quartz[a]	6.5	5.01	4.38	4.01
Alkali feldspar[b]	2.34	–	–	–
Dry sand[a]	1.4	–	–	–
Limestone[a]	2.99	2.51	2.28	2.08
Basalt[a]	2.44	2.23	2.13	2.04
Granite[a]	2.79	2.43	2.25	2.11
Water[c]	0.61	0.68	0.68	0.66

Sources: [a] Clauser, C. and Huenges, E., Thermal conductivity of rocks and minerals. In *Rock Physics and Phase Relations*, ed. T.J. Ahrens, American Geophysical Union, Washington, DC, 1995

[b] Sass, J.H., *Journal of Geophysical Research*, 70, 4064–4065, 1965

[c] Weast, R.C., *CRC Handbook of Chemistry and Physics*, CRC Press, Boca Raton, FL, 1985.

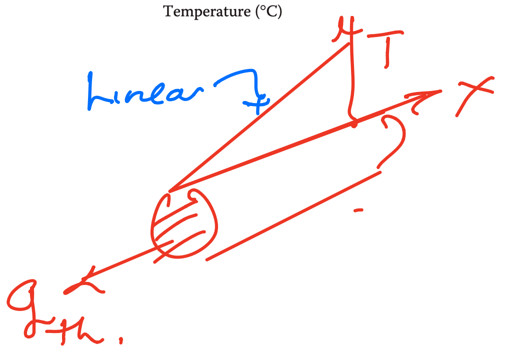


Fourier's Law

$$q = -\lambda \frac{dT}{dx}$$

$$\lambda \doteq W/(m.K)$$

$$\therefore q \doteq \frac{W}{m.\cancel{K}} \frac{\cancel{K}}{m} \doteq \frac{W}{m^2}$$



Transient Conduction

Conservation of Energy: $\frac{d}{dx}q_x + \rho c \frac{dT}{dt} = 0$

$$\boxed{\lambda \frac{d^2T}{dx^2} = \rho c \frac{dT}{dt}}$$ Diffusion Equation.

Coefficients

Thermal conductivity (λ) Water ~ 0.5 W/m.K

Rock ~ 1-5 W/m.K

Density (ρ) Water ~ 1000 kg/m³

Rock/Magma ~ 2700 kg/m³

Specific Heat Capacity (C)

C_p ≐ @ constant pressure

C_v ≐ @ constant volume

$$\rho C_p = \rho C_v + \alpha^2\left(\frac{VT}{\beta}\right)$$

where:

- α is the coefficient of thermal expansion
- β is the coefficient of compressibility
- *V* is the molar volume
- *T* is the absolute temperature (K)

To first order for rock: $K = \frac{\lambda}{c\rho} \sim 30\ m^2/yr.$

(Thermal diffusivity).

Rock (granite) $C_p = C_v \sim$ 918 J/kg.K

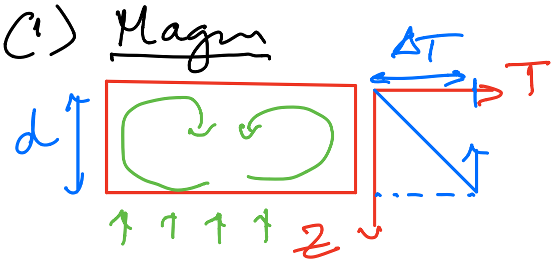
Water 4187 J/kg.K.

Convection

Free-Convection: http://www.columbia.edu/itc/ldeo/v1011x-1/jcm/Topic3/Topic3.html

Note: Two types -
(i) Free-convection in the mantle
(ii) Free-convection of water in a porous medium

(i) Magma



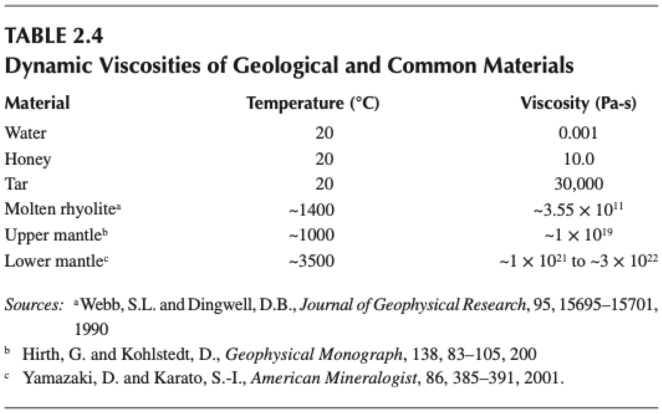


TABLE 2.4
Dynamic Viscosities of Geological and Common Materials

Material	Temperature (°C)	Viscosity (Pa-s)
Water	20	0.001
Honey	20	10.0
Tar	20	30,000
Molten rhyolite[a]	~1400	~3.55 × 10¹¹
Upper mantle[b]	~1000	~1 × 10¹⁹
Lower mantle[c]	~3500	~1 × 10²¹ to ~3 × 10²²

Sources: [a] Webb, S.L. and Dingwell, D.B., *Journal of Geophysical Research*, 95, 15695–15701, 1990

[b] Hirth, G. and Kohlstedt, D., *Geophysical Monograph*, 138, 83–105, 200

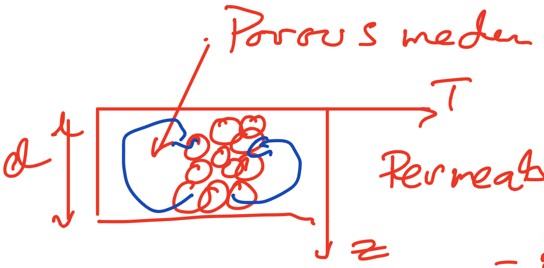
[c] Yamazaki, D. and Karato, S.-I., *American Mineralogist*, 86, 385–391, 2001.

$$Ra = \frac{\text{Buoyant F}}{\text{Viscous F}} = \frac{\alpha \Delta T g \rho d^3}{\mu \kappa}$$

$Ra > 1000$ → Free convection

α = Thermal expansion coeff

(ii) Porous medium



Permeability, k

$$v = -\frac{k}{\mu}\frac{dp}{dx}$$

Rayleigh-Darcy No, Ra

$$\text{Replace: } \frac{g\rho \boxed{d^2}}{\mu} \longrightarrow \frac{g\rho \boxed{k}}{\mu}$$

$$Ra = \frac{\alpha \Delta T g \rho \boxed{d\,k}}{\mu \kappa} \doteq \frac{(1/\cancel{K})\,\cancel{K}\,.\,\cancel{m}/s^{\cancel{2}}\ kg/\cancel{m^3}\ \cancel{m^3}}{Pa.s.\ m^{\cancel{2}}/\cancel{s}}$$

Plate Tectonics - Distribution of Geothermal Resources

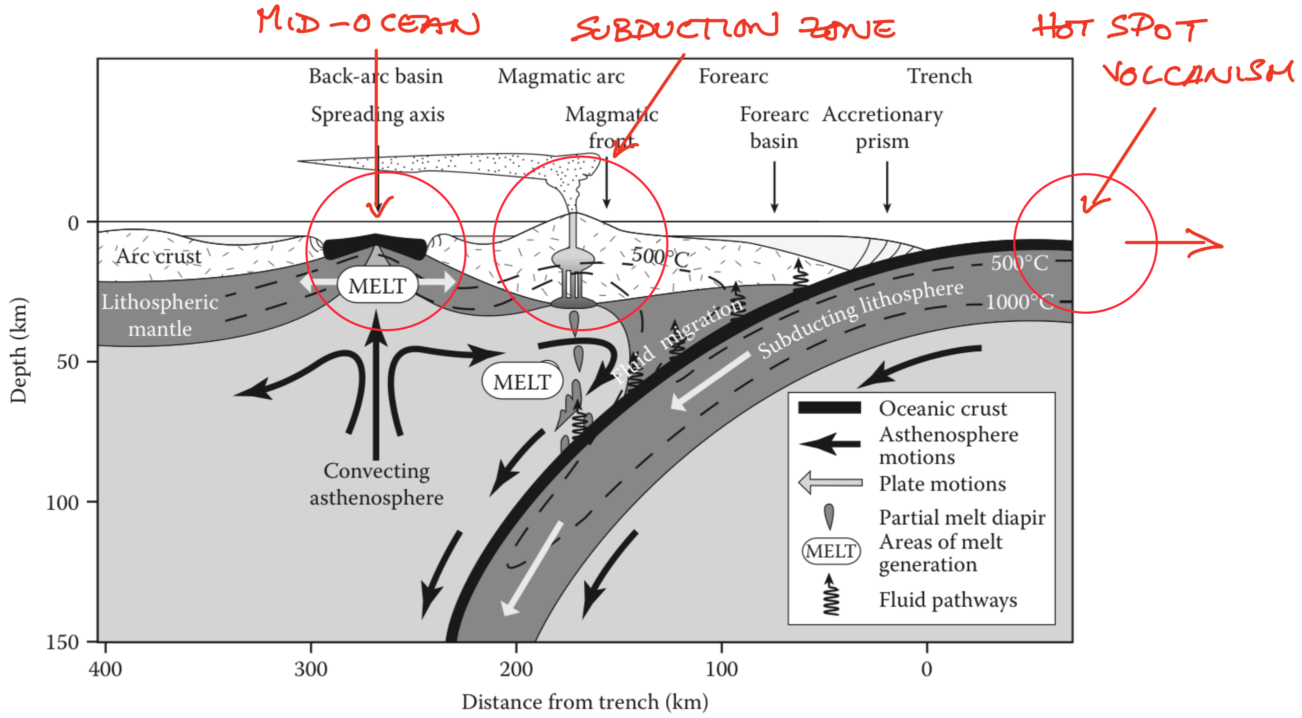


FIGURE 2.8 Schematic cross section through a subduction zone similar to that in Japan. (Modified from DuHamel, J., 2009. Wry heat—Arizona history Chapter 5: Jurassic time. http://tucsoncitizen.com/wryheat/tag/subduction/.)

Three Principal Environments

Destructive boundaries - subduction zones - e.g. The Geysers, CA
Constructive boundaries - mid-ocean ridges - e.g. Iceland
Hot spots - volcanoes - e.g. Hawaii

See prior photos by location

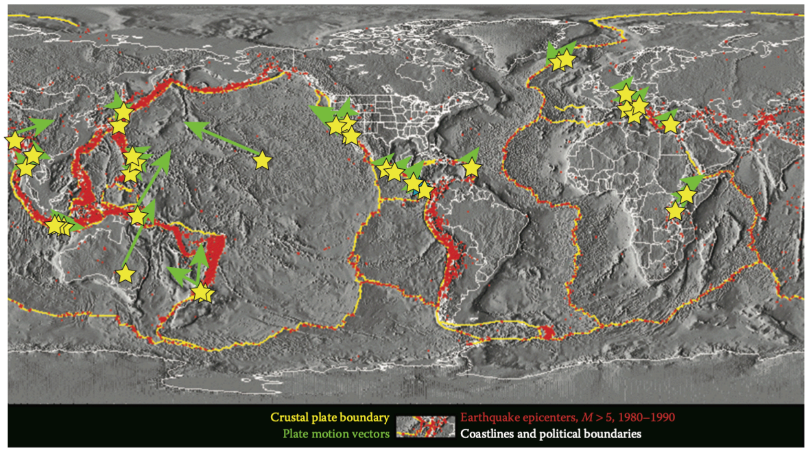


FIGURE 2.6 Global map showing the locations of earthquakes (red dots) that indicate plate boundaries (yellow lines), political boundaries (in white), and the locations of the world's geothermal power plants (stars). The directions of some plate motions are shown by the green arrows, with the length of the arrow corresponding to relative velocity of plate motion. Note the strong correlation between power plant sites and plate boundaries. There are many more power plants than stars because many sites have several power plants. The global map, earthquake data, and boundaries are from the National Oceanic and Atmospheric Administration Plates and Topography Disc and the power plant sites from the International Geothermal Association website (http://iga.igg.cnr.it/geo/geoenergy.php).

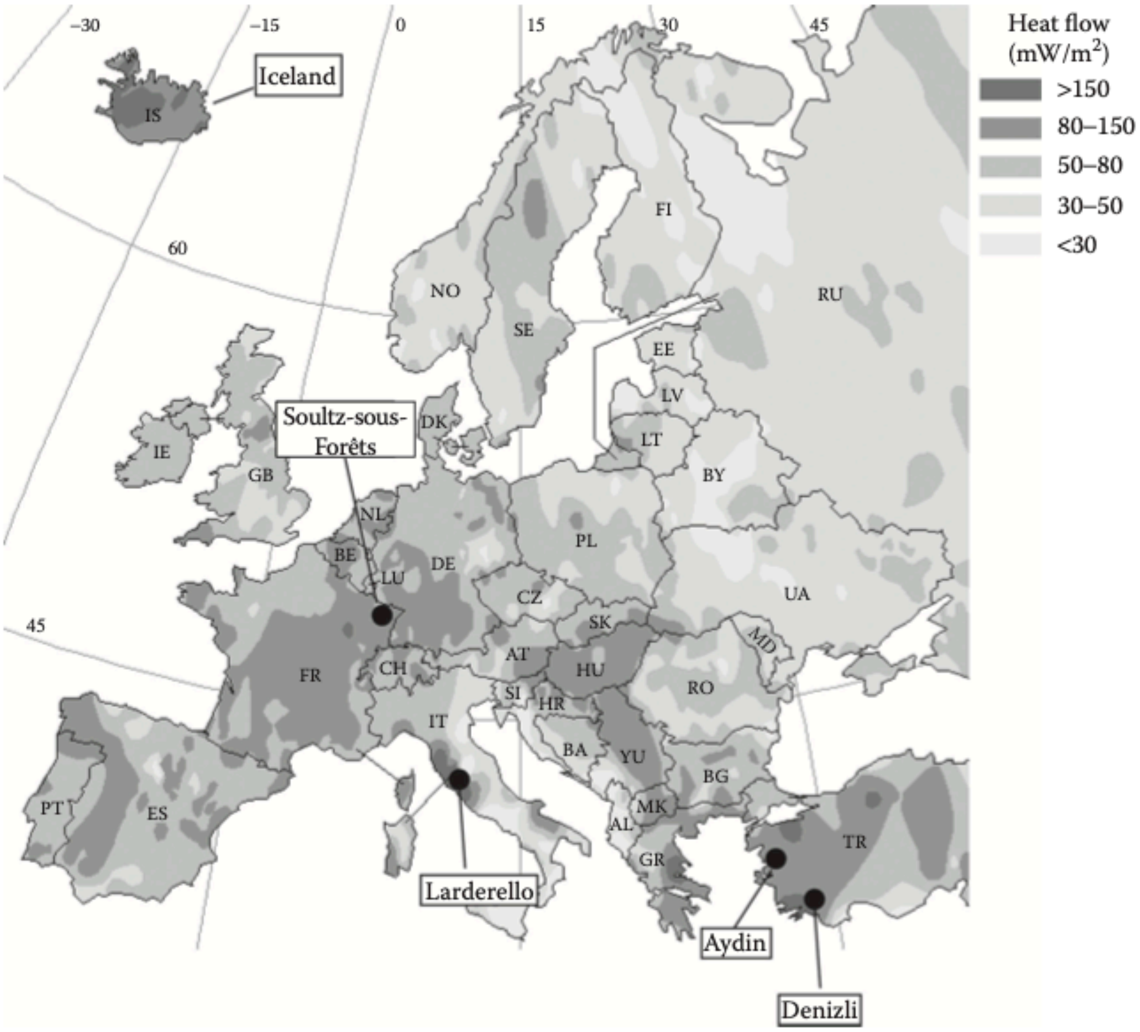
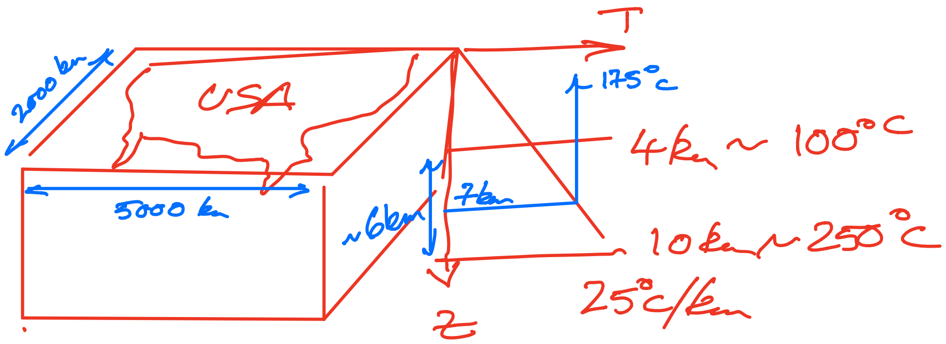


FIGURE 2.10 Heat flow map of Europe. (Modified from the European Community Nr. 17811.)

Simple Calculation

b. Example Hand-Calculation [10%] Simple calculation to demonstrate the technique.

Geothermal - US-Hydrothermal 10^4 EJ; US-EGS 10^7 EJ?? What is the origin?



$$\text{Energy} = \underset{A\times(10km-4km),\ A\sim 10{,}000\,km^2}{V}\ \overset{2500\,kg/m^3}{\rho}\ \underset{1000\,J/kg.K}{c}\ \overset{(175°C-100°C)\ \sim 100K}{\Delta T}$$

$$E = \underbrace{(10^4 \times 6)\,km^3 \cdot 2500}\,\tfrac{\cancel{kg}}{m^3} \cdot 1000\,\tfrac{J}{\cancel{kg}.\cancel{K}} \cdot 100\cancel{K} \times \frac{1\,m^3}{10^{-9}\,km^3}$$

$$= 10^4 \cdot (1.5\times10^4) \cdot 10^3 \cdot 10^2 \cdot 10^9 \sim 1.5\times10^{22}\,J$$

$$\sim 1.5\times10^4\,EJ$$

OTHER ESTIMATES? 10^7 EJ?

Compares with MIT/Tester Report (2006)

The Future of Geothermal Energy

Impact of Enhanced Geothermal Systems (EGS) on the United States in the 21st Century

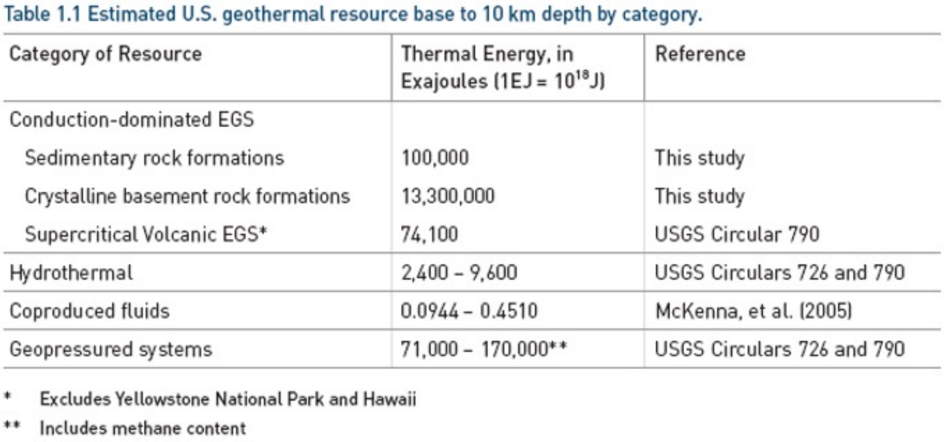


Table 1.1 Estimated U.S. geothermal resource base to 10 km depth by category.

Category of Resource	Thermal Energy, in Exajoules (1EJ = 10^{18}J)	Reference
Conduction-dominated EGS		
Sedimentary rock formations	100,000	This study
Crystalline basement rock formations	13,300,000	This study
Supercritical Volcanic EGS*	74,100	USGS Circular 790
Hydrothermal	2,400 – 9,600	USGS Circulars 726 and 790
Coproduced fluids	0.0944 – 0.4510	McKenna, et al. (2005)
Geopressured systems	71,000 – 170,000**	USGS Circulars 726 and 790

* Excludes Yellowstone National Park and Hawaii

** Includes methane content

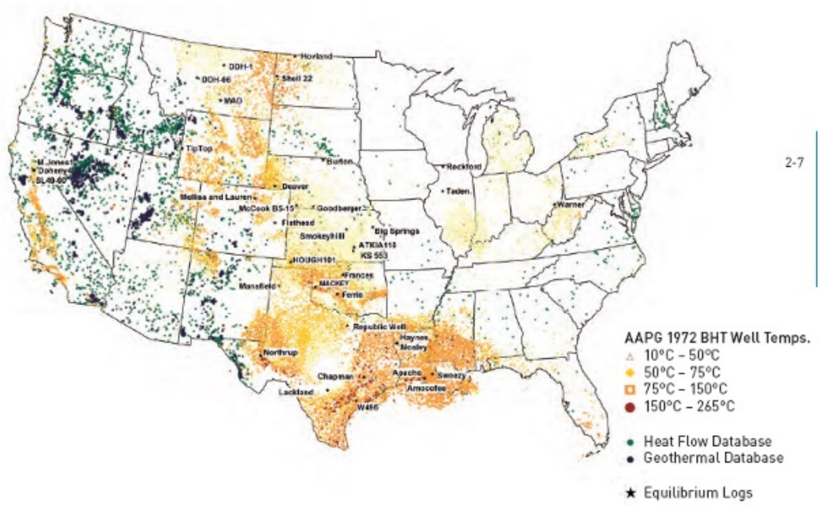
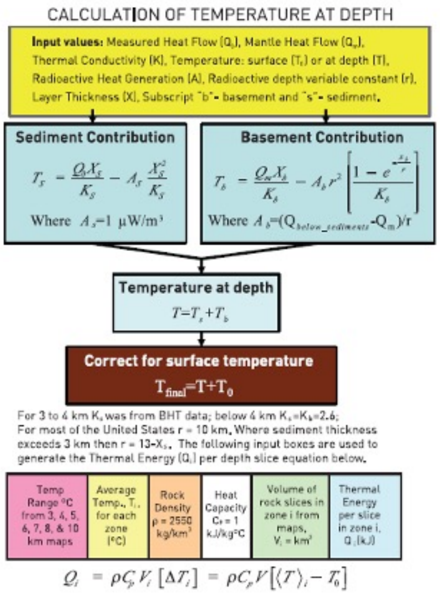


Figure 2.2 All BHT sites in the conterminous United States in the AAPG database. BHT symbols are based on depth and temperature (heat flow is not available for all of the sites, so some were not used for preparation of the Geothermal Map of North America). The named wells are the calibration points. The regional heat flow and geothermal database sites are also shown.



For 3 to 4 km K_s was from BHT data; below 4 km $K_s = K_b = 2.6$; For most of the United States r = 10 km. Where sediment thickness exceeds 3 km then r = 13-X_s. The following input boxes are used to generate the Thermal Energy (Q_i) per depth slice equation below.

Figure 2.3 Flow chart for calculation of temperature and heat content at depth. Note: 1 kW-sec = 1 kJ and angle brackets denote depth-averaging.

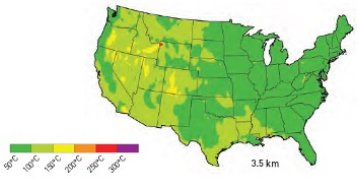


Figure 1.3 Temperatures at a depth of 3.5 km.

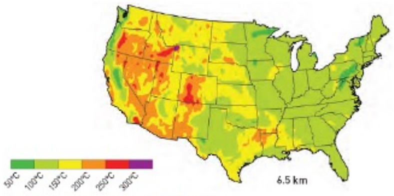


Figure 1.4 Temperatures at a depth of 6.5 km.

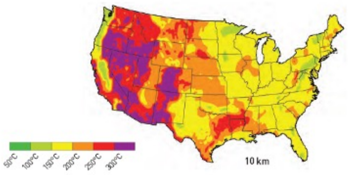
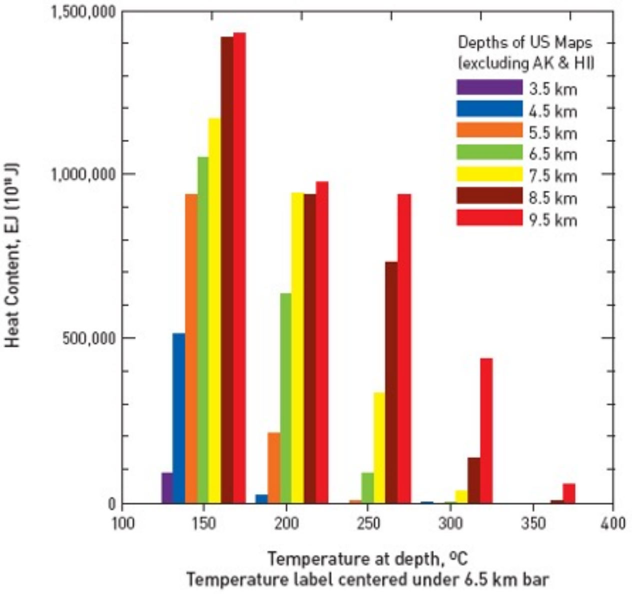


Figure 1.5 Temperatures at a depth of 10 km.



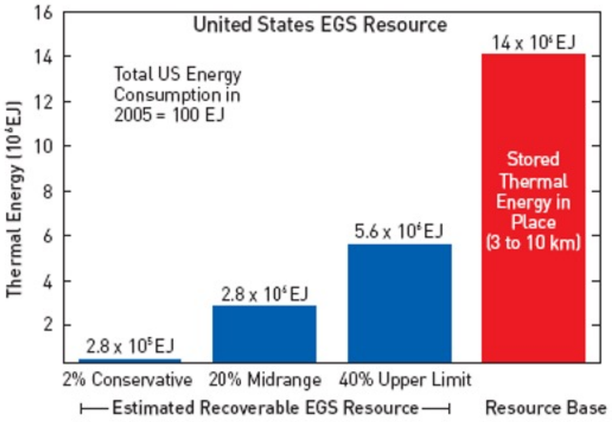


Table 1.2 Estimated land area and subsurface reservoir volumes needed for EGS development. Note: Above 100 MW_e, reservoir size scaling should be linear.

Plant size in MW_e	Surface area for power plant and auxiliaries in km²	Subsurface reservoir volume in km³
25	1	1.5
50	1.4	2.7
75	1.8	3.9
100	2.1	5.0

1. Assuming 10% heat to electric-power efficiency, typical of binary plants.

2. Introduces a factor of 4 to surface area and volumes to deal with redrilling of reservoir at 5-year intervals over a 20-year projected lifetime.

RATES OF RECOVERY?

How quickly can heat be recovered?

Case Study - N/A

c. **Case Study [10%]** If appropriate.

Conclusion

4. **Conclusion [20%] Summarize important/key points from the presentation.**

Geothermal Resources - result of heating by close (40%) and radiogenic heat (60%)
Earth budget controlled by:
- Convection at depth in molten mantle - Free convection - Rayleigh No.
- Conduction across crust - Fourier's law

High gradients at plate boundaries and hot-spot volcanic centers