[10-11]

Flow in Pipes

Pipe Flow [10-11]

$$\tau_w = \frac{\rho V^2}{8} f; \quad h_L^{major} = f\left(\frac{l}{D}\right)\frac{V^2}{2g}; \quad h_P = \frac{Power}{\gamma Q}$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + h_P = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + \sum h_L^{major} + \sum h_L^{minor}$$

$$h_L^{minor} = K_L \frac{V^2}{2g}; \quad l_{eq}^{minor} = \frac{K_L D}{f}; \quad K_L = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

Non-circular: Laminar: $[f = \frac{C}{\mathbf{Re}_h}; D_h = \frac{4A}{P}]$ Turbulent: [Use Moody; $f = \varphi(\frac{\varepsilon}{D_h})$]

Series: $h_L = h_{L_1} + h_{L_2} + \ldots + h_{L_n}$; Parallel: $h_{L_1} = h_{L_2} = \ldots = h_{L_n}$

Flow meters: $Q = CA\sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$; $\beta = \frac{D_2}{D_1}$

Transitional range

Laminar flow : $f = \frac{64}{Re}$; Complete turbulent flow : $f = \phi\left(\frac{\varepsilon}{D}\right)$

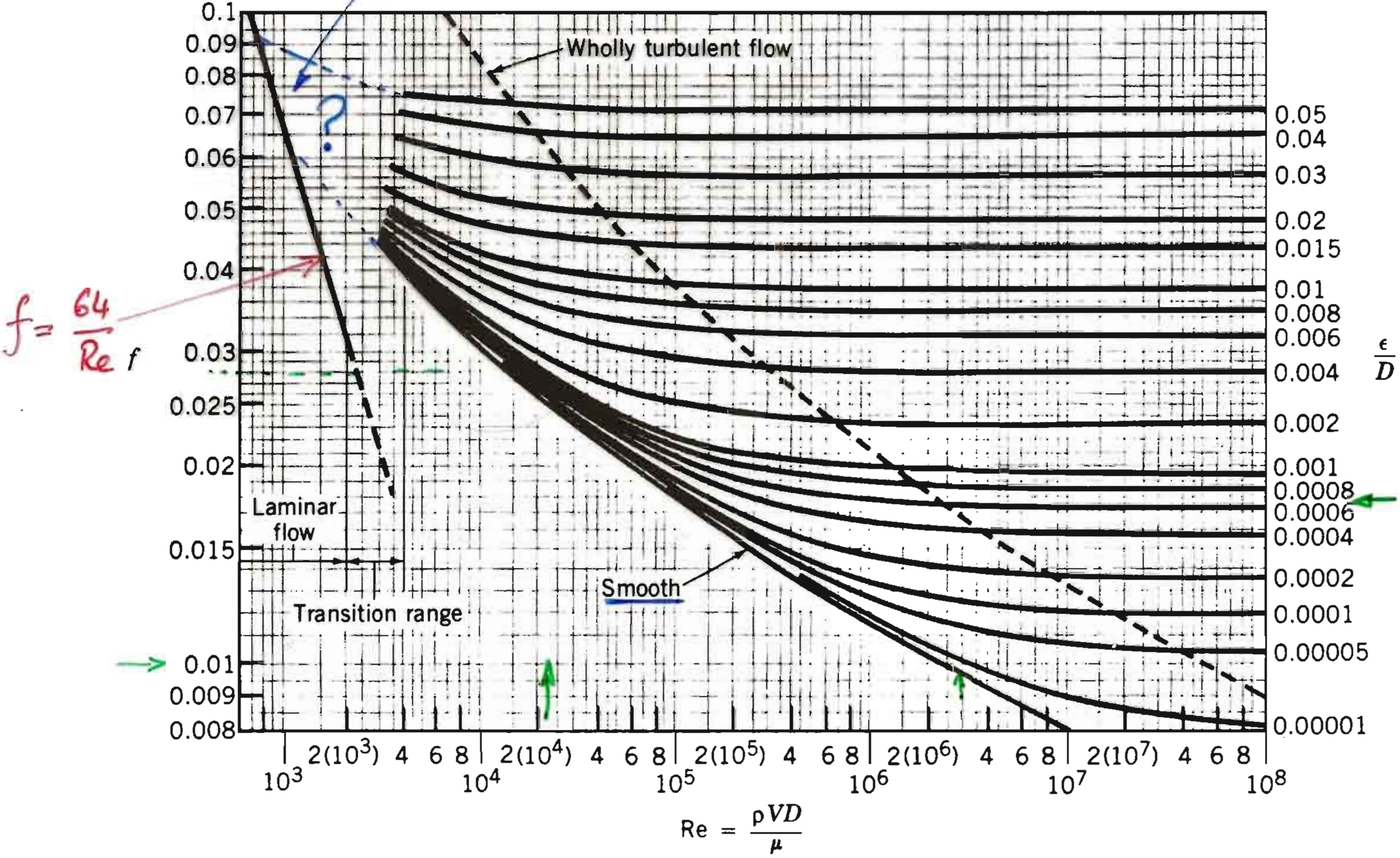


FIGURE 8.23 Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart (Data from Ref. 7 with permission).

Colebrook Formula (Non-laminar range, only)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$\left(\text{Laminar} \quad f = \frac{64}{Re}\right)$$

$$\boxed{h_L = f \frac{l}{D} \frac{V^2}{2g}}$$

BASIC EQUATIONS

(1)
$$\boxed{\frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L}$$

$\alpha_1 = 1$ for turbulent flow

h_p = head provided by pumps

h_L = head loss

(2)

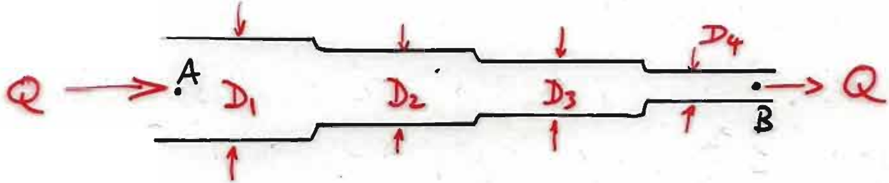
$$h_L = \sum f \frac{l}{D} \frac{V^2}{2g}$$ for "major" loss pipe sections

$$h_L = \sum K_L \frac{V^2}{2g}$$ for bends, elbows, valves etc.... "minor" losses

$$\boxed{f = \phi\left(Re\,;\,\frac{\varepsilon}{D}\right)}$$

MULTIPLE PIPE SYSTEMS

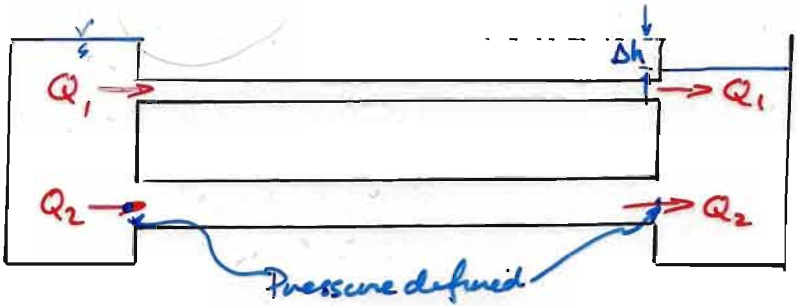
SERIES



$$Q_1 = Q_2 = Q_3 = Q_4 = Q$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3} + h_{L_4}$$

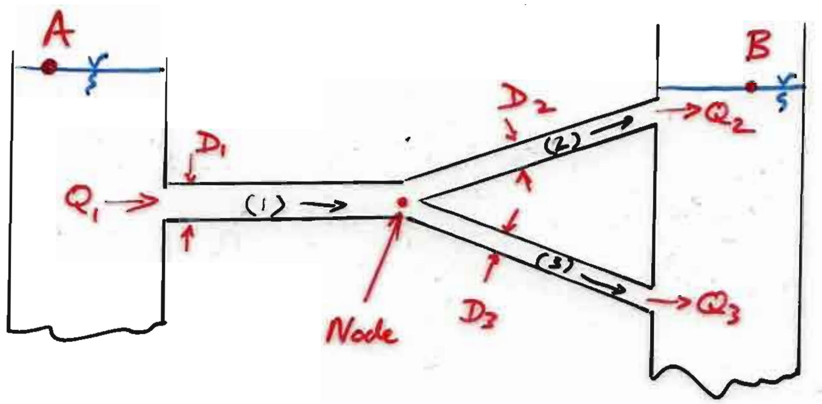
PARALLEL



$$Q = Q_1 + Q_2 - - - -$$

$$h_{L_1} = h_{L_2} = \ldots \text{etc.} = \Delta h$$

PIPE "LOOP" SYSTEMS



Pipes (1) and (2):
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2} \quad (1)$$

Pipes (1) and (3):
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3} \quad (2)$$

From these, the equations are identical $\therefore$ $\boxed{h_{L_2} = h_{L_3}}$

Physically: Energy conditions at Node are a single value HGL_N.
Flowing to final energy in tank B they are also at the same energy (but different from (HGL_N)) ie HGL_B.

From (2) energy equations (above) have 3 unknowns V_1, V_2, V_3

need extra equation, continuity

$$Q_1 = Q_2 + Q_3$$

$$V_1 A_1 = V_2 A_2 + V_3 A_3 \quad (3)$$

3 equations and 3 unknowns!!

8.125

8.125 The flowrate between tank A and tank B shown in Fig. P8.125 is to be increased by 30% (i.e., from Q to $1.30Q$) by the addition of a second pipe (indicated by the dotted lines) running from node C to tank B. If the elevation of the free surface in tank A is 25 ft above that in tank B, determine the diameter, D, of this new pipe. Neglect minor losses and assume that the friction factor for each pipe is 0.02.

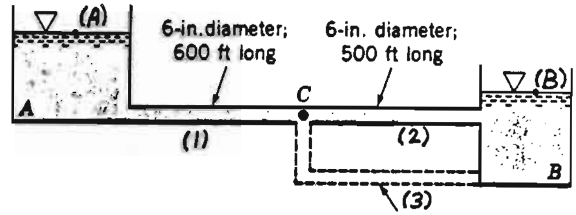


FIGURE P8.125

With the single pipe: $\dfrac{p_A}{\gamma}+\dfrac{V_A^2}{2g}+z_A=\dfrac{p_B}{\gamma}+\dfrac{V_B^2}{2g}+z_B+f_1\dfrac{\ell_1}{D_1}\dfrac{V_1^2}{2g}+f_2\dfrac{\ell_2}{D_2}\dfrac{V_2^2}{2g}$ (1)

where $p_A=p_B=0$, $V_A=V_B=0$, $z_A=25\text{ ft}$, $z_B=0$, and $V_1=V_2$ (since $D_1=D_2$).

Thus, $z_A=f_1\dfrac{(\ell_1+\ell_2)}{D_1}\dfrac{V_1^2}{2g}$, or $25\text{ ft}=(0.02)\dfrac{(600+500)\text{ft}}{(\frac{6}{12}\text{ft})}\dfrac{V_1^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$

or $V_1=6.05\frac{\text{ft}}{\text{s}}$ Hence, $Q=A_1V_1=\frac{\pi}{4}(\frac{6}{12}\text{ft})^2(6.05\frac{\text{ft}}{\text{s}})=1.188\frac{\text{ft}^3}{\text{s}}$ ONE PIPE

With the second pipe $Q=1.30(1.188\frac{\text{ft}^3}{\text{s}})=1.54\frac{\text{ft}^3}{\text{s}}$

$Q_1=Q_2+Q_3$

Thus, $Q_1=1.54\frac{\text{ft}^3}{\text{s}}=Q_2+Q_3$ or $V_1=\dfrac{Q_1}{A_1}=\dfrac{1.54\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{6}{12}\text{ft})^2}=7.84\frac{\text{ft}}{\text{s}}$

For fluid flowing from A to B through pipes 1 and 2,

$z_A=h_{L_1}+h_{L_2}$

$z_A=h_{L_1}+h_{L_2}=f_1\dfrac{\ell_1}{D_1}\dfrac{V_1^2}{2g}+f_2\dfrac{\ell_2}{D_2}\dfrac{V_2^2}{2g}$ (see Eq.(1))

or

$$25\text{ ft}=(0.02)\frac{600\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{(7.84\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}+(0.02)\frac{500\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{V_2^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

Hence, $V_2=2.60\frac{\text{ft}}{\text{s}}$

and

$$Q_2=A_2V_2=\frac{\pi}{4}(\tfrac{6}{12}\text{ft})^2(2.60\tfrac{\text{ft}}{\text{s}})=0.511\tfrac{\text{ft}^3}{\text{s}}$$

Thus, $Q_3=Q_1-Q_2=1.54\frac{\text{ft}^3}{\text{s}}-0.511\frac{\text{ft}^3}{\text{s}}=1.03\frac{\text{ft}^3}{\text{s}}$

For fluid flowing from A to B through pipes 1 and 3,

$z_A=h_{L_1}+h_{L_3}$

$z_A=h_{L_1}+h_{L_3}=f_1\dfrac{\ell_1}{D_1}\dfrac{V_1^2}{2g}+f_3\dfrac{\ell_3}{D_3}\dfrac{V_3^2}{2g}$, where $V_3=\dfrac{Q_3}{A_3}=\dfrac{1.03\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}D_3^2}=\dfrac{1.31}{D_3^2}$

Thus,

$$25\text{ ft}=(0.02)\frac{600\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{(7.84\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}+(0.02)\frac{500\text{ ft}}{D_3}\frac{\left(\frac{1.31}{D_3^2}\right)^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

or

$$\underline{\underline{D_3=0.662\text{ ft}}}$$

Note: With the parameters given, the solution is quite sensitive to roundoff errors in the calculations

[12]

External Flows

External Flows [12]

$$D = \int dF_x = \int p \cos\theta dA + \int \tau_w \sin\theta dA$$

$$L = \int dF_y = -\int p \sin\theta dA + \int \tau_w \cos\theta dA$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}; \qquad C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

DRAG AND LIFT COEFFICIENTS

LIFT:

$$C_L = \frac{L}{\frac{1}{2}\rho u^2 A}$$

lift coefficient, C_L

DRAG:

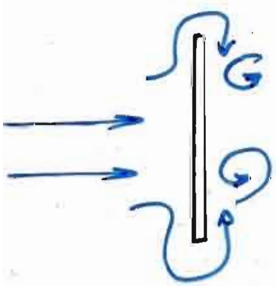
$$C_D = \frac{D}{\frac{1}{2}\rho u^2 A}$$

drag coefficient, C_D

A is a "characteristic" area:

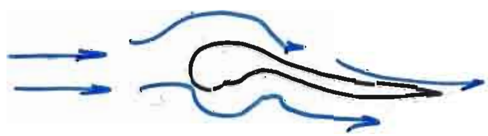
Must be chosen that represents the physical behaviour of system.

FRONTAL AREA



Pressure drag.

PLANIFORM AREA



Shear drag

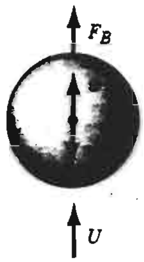
Lift

EXAMPLE 9.10

A small grain of sand diameter $D = 0.10$ mm and specific gravity $SG = 2.3$ settles to the bottom of a lake after having been stirred up by a passing boat. Determine how fast it falls through the still water.

SOLUTION

A free-body diagram of the particle (relative to the moving particle) is shown in Fig. E9.10. The particle moves downward with a constant velocity U that is governed by a balance between the weight of the particle, $\mathcal{W}$, the buoyancy force of the surrounding water, F_B, and the drag of the water on the particle, $\mathcal{D}$.



■ FIGURE E9.10

From the free-body diagram we obtain

$$\boxed{\mathcal{W} = \mathcal{D} + F_B}$$

where

$$\mathcal{W} = \gamma_{\text{sand}} \forall = SG\, \gamma_{H_2O} \frac{\pi}{6} D^3 \tag{1}$$

and

$$F_B = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{\pi}{6} D^3 \tag{2}$$

CREEPING FLOW   Table 9.4

$C_D = \frac{24}{Re}$ $Re \leq 1$

$C_D = \frac{D}{\frac{1}{2}\rho u^2 A}$

We assume (because of the smallness of the object) that the flow will be creeping flow ($Re < 1$) with $C_D = 24/Re$ (see Table 9.4) so that

$$\mathcal{D} = \frac{1}{2}\rho_{H_2O} U^2 \frac{\pi}{4} D^2 C_D = \frac{1}{2}\rho_{H_2O} U^2 \frac{\pi}{4} D^2 \left(\frac{24}{\rho_{H_2O} UD/\mu_{H_2O}}\right)$$

or

$$\mathcal{D} = 3\pi\mu_{H_2O} UD \tag{3}$$

$C_D = \frac{24}{Re}$

We must eventually check to determine if this assumption is valid or not. Equation 3 is called Stokes law in honor of G. G. Stokes (1819–1903), a British mathematician and physicist. By combining Eqs. 1, 2, and 3, we obtain

$$SG\, \gamma_{H_2O} \frac{\pi}{6} D^3 = 3\pi\mu_{H_2O} UD + \gamma_{H_2O} \frac{\pi}{6} D^3$$

or, since $\gamma = \rho g$,

$$U = \frac{(SG\rho_{H_2O} - \rho_{H_2O}) g D^2}{18\mu} \tag{4}$$

From Table 1.6 for water at 15.6 °C we obtain $\rho_{H_2O} = 999 \text{ kg/m}^3$ and $\mu_{H_2O} = 1.12 \times 10^{-3}$ N·s/m^2. Thus, from Eq. 4 we obtain

Evaluate, u.

$$U = \frac{(2.3 - 1)(999 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.10 \times 10^{-3} \text{ m})^2}{18(1.12 \times 10^{-3} \text{ N·s/m}^2)}$$

or

$$U = 6.32 \times 10^{-3} \text{ m/s} \qquad \text{(Ans)}$$

Since

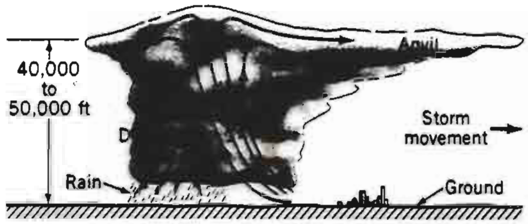
Check $Re \leq 1$.

$$Re = \frac{\rho D U}{\mu} = \frac{(999 \text{ kg/m}^3)(0.10 \times 10^{-3} \text{ m})(0.00632 \text{ m/s})}{1.12 \times 10^{-3} \text{ N·s/m}^2} = 0.564$$

we see that $Re < 1$, and the form of the drag coefficient used is valid.

EXAMPLE 9.11

Hail is produced by the repeated rising and falling of ice particles in the updraft of a thunderstorm, as is indicated in Fig. E9.11. When the hail becomes large enough, the aerodynamic drag from the updraft can no longer support the weight of the hail, and it falls from the storm cloud. Estimate the velocity, U, of the updraft needed to make $D = 1.5$-in.-diameter (i.e., "golf ball-sized") hail.



■ FIGURE E9.11

SOLUTION

As is discussed in Example 9.10, for steady state conditions a force balance on an object falling through a fluid gives

$$\mathcal{W} = \mathcal{D} + F_B$$

where $F_B = \gamma_{air} \forall$ is the buoyant force of the air on the particle, $\mathcal{W} = \gamma_{ice} \forall$ is the particle weight, and $\mathcal{D}$ is the aerodynamic drag. This equation can be rewritten as

$$\tfrac{1}{2}\rho_{air} U^2 \frac{\pi}{4} D^2 C_D = \mathcal{W} - F_B \tag{1}$$

With $\forall = \pi D^3/6$ and since $\gamma_{ice} \gg \gamma_{air}$ (i.e., $\mathcal{W} \gg F_B$), Eq. 1 can be simplified to

$$U = \left(\frac{4}{3}\frac{\rho_{ice}}{\rho_{air}}\frac{gD}{C_D}\right)^{1/2} \tag{2}$$

By using $\rho_{ice} = 1.84 \text{ slugs/ft}^3$, $\rho_{air} = 2.37 \times 10^{-3} \text{ slugs/ft}^3$, and $D = 1.5 \text{ in.} = 0.125 \text{ ft}$, Eq. 2 becomes

$$U = \left[\frac{4(1.84 \text{ slugs/ft}^3)(32.2 \text{ ft/s}^2)(0.125 \text{ ft})}{3(2.37 \times 10^{-3} \text{ slugs/ft}^3)C_D}\right]^{1/2}$$

or

$$U = \frac{64.5}{\sqrt{C_D}} \tag{3}$$

where U is in ft/s. To determine U, we must know C_D. Unfortunately, C_D is a function of the Reynolds number (see Fig. 9.23), which is not known unless U is known. Thus, we must use an iterative technique similar to that done with the Moody chart for certain types of pipe flow problems (see Section 8.5).

From Fig. 9.23 we expect that C_D is on the order of 0.5. Thus, we assume $C_D = 0.5$ and from Eq. 3 obtain

$$U = \frac{64.5}{\sqrt{0.5}} = 91.2 \text{ ft/s}$$

The corresponding Reynolds number (assuming $\nu = 1.57 \times 10^{-4} \text{ ft}^2/\text{s}$) is

$$\text{Re} = \frac{UD}{\nu} = \frac{91.2 \text{ ft/s } (0.125 \text{ ft})}{1.57 \times 10^{-4} \text{ ft}^2/\text{s}} = 7.26 \times 10^4$$

For this value of Re we obtain from Fig. 9.23, $C_D = 0.5$. Thus, our assumed value of $C_D = 0.5$ was correct. The corresponding value of U is

$$U = 91.2 \text{ ft/s} = 62.2 \text{ mph} \qquad \textbf{(Ans)}$$

This result was obtained by using standard sea-level properties for the air. If conditions at 20,000 ft altitude are used (i.e., from Table C.1, $\rho_{air} = 1.267 \times 10^{-3} \text{ slugs/ft}^3$ and $\mu = 3.324 \times 10^{-7} \text{ lb}\cdot\text{s/ft}^2$), the corresponding result is $U = 125 \text{ ft/s} = 85.2 \text{ mph}$.

Clearly, an airplane flying through such an updraft would feel its effects (even if it were able to dodge the hail). As seen from Eq. 2, the larger the hail, the stronger the necessary updraft. Hailstones greater than 6 in. in diameter have been reported. In reality, a hailstone is seldom spherical and often not smooth. However, the calculated updraft velocities are in agreement with measured values.

$W = \frac{\pi D^3}{6} \rho_{ice} g$

$F_B \ll W$

$u = \frac{64.5}{\sqrt{C_D}}$

High Re...

$C_D \approx 0.5$

Evaluate u.

COMPOSITE DRAG

EXAMPLE 9.13

CARE IN NOTING 3-D AND INTERACTION EFFECTS

A 60-mph (i.e., 88-fps) wind blows past the water tower shown in Fig. E9.13*a*. Estimate the moment (torque), M, needed at the base to keep the tower from tipping over.

SOLUTION

We treat the water tower as a sphere resting on a circular cylinder and assume that the total drag is the sum of the drag from these parts. The free-body diagram of the tower is shown in Fig. E.9.13*b*. By summing moments about the base of the tower, we obtain

$$M = \mathfrak{D}_s\left(b + \frac{D_s}{2}\right) + \mathfrak{D}_c\left(\frac{b}{2}\right) \tag{1}$$

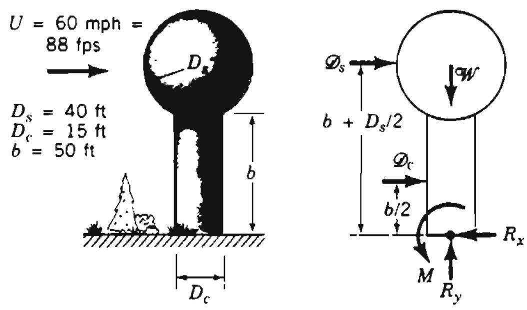
where

$$\mathfrak{D}_s = \frac{1}{2}\rho U^2 \frac{\pi}{4} D_s^2 C_{Ds} \tag{2}$$

and

$$\mathfrak{D}_c = \frac{1}{2}\rho U^2 b D_c C_{Dc} \tag{3}$$

Evaluate Drag components



■ FIGURE E9.13

are the drag on the sphere and cylinder, respectively. For standard atmospheric conditions, the Reynolds numbers are

Need Re. Fig 9.23

$$\mathrm{Re}_s = \frac{UD_s}{\nu} = \frac{(88\ \mathrm{ft/s})(40\ \mathrm{ft})}{1.57 \times 10^{-4}\ \mathrm{ft^2/s}} = 2.24 \times 10^7$$

and

$$\mathrm{Re}_c = \frac{UD_c}{\nu} = \frac{(88\ \mathrm{ft/s})(15\ \mathrm{ft})}{1.57 \times 10^{-4}\ \mathrm{ft^2/s}} = 8.41 \times 10^6$$

The corresponding drag coefficients, C_{Ds} and C_{Dc}, can be approximated from Fig. 9.23 as

$$C_{Ds} \approx 0.3 \quad \text{and} \quad C_{Dc} \approx 0.7$$

Note that the value of C_{Ds} was obtained by an extrapolation of the given data to Reynolds numbers beyond those given (a potentially dangerous practice!). From Eqs. 2 and 3 we obtain

$$\mathfrak{D}_s = 0.5(2.38 \times 10^{-3}\ \mathrm{slugs/ft^3})(88\ \mathrm{ft/s})^2 \frac{\pi}{4}(40\ \mathrm{ft})^2(0.3) = 3470\ \mathrm{lb}$$

and

$$\mathfrak{D}_c = 0.5(2.38 \times 10^{-3}\ \mathrm{slugs/ft^3})(88\ \mathrm{ft/s})^2(50\ \mathrm{ft} \times 15\ \mathrm{ft})(0.7) = 4840\ \mathrm{lb}$$

From Eq. 1 the corresponding moment needed to prevent the tower from tipping is

$$M = 3470\ \mathrm{lb}\left(50\ \mathrm{ft} + \frac{40}{2}\ \mathrm{ft}\right) + 4840\ \mathrm{lb}\left(\frac{50}{2}\ \mathrm{ft}\right) = 3.64 \times 10^5\ \mathrm{ft{\cdot}lb} \qquad \text{(Ans)}$$

The above result is only an estimate because (a) the wind is probably not uniform from the top of the tower to the ground, (b) the tower is not exactly a combination of a smooth sphere and a circular cylinder, (c) the cylinder is not of infinite length, (d) there will be some interaction between the flow past the cylinder and that past the sphere so that the net drag is not exactly the sum of the two, and (e) a drag coefficient value was obtained by extrapolation of the given data. However, such approximate results are often quite accurate.

[13-14]

Open Channel Flows

Open Channel Flow [13-14]

$$R_h = \frac{A}{P}; \quad \mathbf{Re} = \frac{VR_h\rho}{\mu}; \quad \mathbf{Fr} = \frac{V}{\sqrt{gy}}; \quad c = \sqrt{gy}$$

Specific Energy: $E = y + \dfrac{q^2}{2gy^2}$; Specific Momentum: $M = \dfrac{y^2}{2} + \dfrac{q^2}{gy}$

Energy Equation: $y_1 + \dfrac{q_1^2}{2gy_1^2} + z_1 = y_2 + \dfrac{q_2^2}{2gy_2^2} + z_2 + S_f l \quad \rightarrow \quad E_1 = E_2 + (S_f - S_0)l$

$$E_{min} = \frac{3y_c}{2} \text{ at } \mathbf{Fr} = 1$$

$$\frac{dy}{dx} = \frac{S_f - S_0}{1 - \mathbf{Fr}^2}$$

Uniform Flow: $V = \dfrac{\kappa}{n} R_h^{\frac{2}{3}} S_0^{\frac{1}{2}} Q = \dfrac{\kappa}{n} A R_h^{\frac{2}{3}} S_0^{\frac{1}{2}} \kappa = 1(SI) \kappa = 1.49(BGS)$

Hydraulic Jump: $\dfrac{y_2}{y_1} = \dfrac{1}{2}(-1 + \sqrt{1 + 8Fr_1^2}) \; \dfrac{h_L}{y_1} = 1 - \dfrac{y_2}{y_1} + \dfrac{1}{2} Fr_1^2 [1 - (\dfrac{y_1}{y_2})^2]$

Sharp-Crested Weir: $Q = C_{rectangular} \dfrac{2}{3} \sqrt{2gh^3} b; Q = C_{triangular} \dfrac{8}{15} \tan\dfrac{\theta}{2} \sqrt{2g} H^{5/2}$.

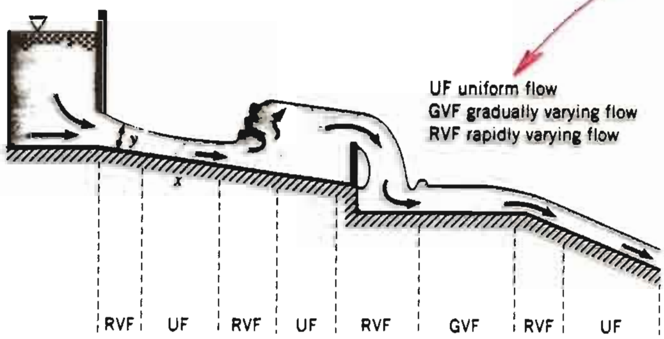
Broad-Crested Weir: $Q = C_{wb} b \sqrt{g} (\dfrac{2}{3})^{3/2} H^{3/2} : C_{wb} = \dfrac{0.65}{(1 + H / P_w)^{1/2}}$.

Underflow Gates: $q = C_d a \sqrt{2gy_1}$

GENERAL DEFINITIONS

- Uniform $\frac{\partial y}{\partial x} = 0$
  no change in depth
- Non uniform $\frac{dy}{dx} \neq 0$
  - GVF $\frac{dy}{dx} \ll 1$
  - RVF $\frac{dy}{dx} \rightarrow ?$

Flow classifications



■ **FIGURE 10.1** **Classification of open-channel flow.**

STEADY -vs- UNSTEADY

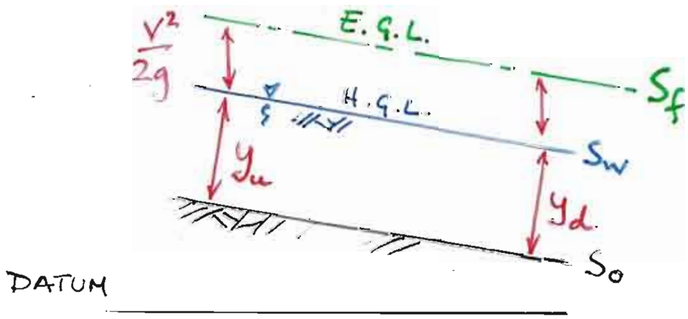
$$\frac{\partial}{\partial t} \rightarrow 0 \implies \text{Steady}$$

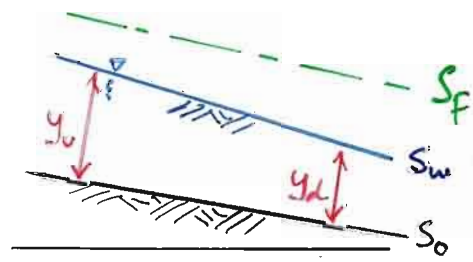
Usually adequate

Nonsteady:
- Dam release / break
- Rainfall / flash flood.
- Tides (quasi-steady)
- Tsunamis

UNIFORM -vs- NONUNIFORM

$$\frac{\partial}{\partial s} \rightarrow 0 \implies \text{Uniform}$$





UNIFORM FLOW

$$y_u = y_d$$

$$S_f = S_w = S_o$$

NON-UNIFORM FLOW

$$y_u \neq y_d$$

$$S_f \neq S_w \neq S_o$$

UNIFORM FLOW

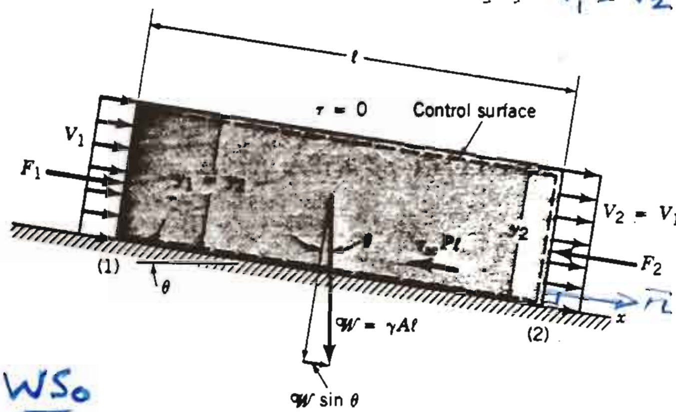
$$\frac{dy}{dx} = 0 \quad \text{i.e. } y_1 = y_2$$

MOMENTUM BALANCE:

$$\Sigma F_x = \rho Q (V_2 - V_1) = 0$$

$$\therefore \ \Sigma F_x = 0$$

$$\therefore \ V_1 = V_2$$



FORCE BALANCE:

$$\underbrace{F_1 - F_2}_{0} - \tau_w P\ell + W\sin\theta = 0$$

Rearranging: $\tau_w = \dfrac{W\sin\theta}{P\ell} = \dfrac{WS_o}{P\ell}$

Small θ: $\sin\theta \approx \tan\theta \approx S_o$

$W = A\ell\gamma \Rightarrow \tau_w = \dfrac{A}{P}\dfrac{\ell}{\ell}\gamma S_o = R_h \gamma S_o$

$R_h = \dfrac{A}{P}$

Turbulent flow $\tau_w = K_L \rho \dfrac{V^2}{2}$ (Darcy-Weisbach formula.)

Combining $V = \sqrt{\dfrac{2\gamma}{K_L\rho}}\sqrt{R_h S_o} = \boxed{C\sqrt{R_h S_o} = V}$

$C =$ Chezy coef.

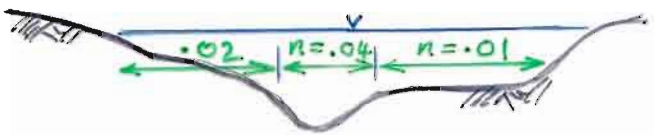
Manning modification SI: $C = \dfrac{R_h^{1/6}}{n}$ English: $C = 1.486\dfrac{R_h^{1/6}}{n}$

$$\boxed{\begin{aligned} V &= \frac{K}{n} R_h^{2/3} S_o^{1/2} \\ Q &= \frac{K}{n} A R_h^{2/3} S_o^{1/2} \end{aligned}}$$

$K = 1$ SI: m/s

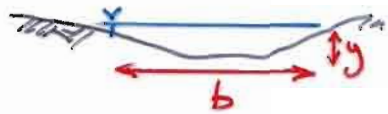
$K = 1.49$ English: ft/s

AVERAGE 'n' VALUES



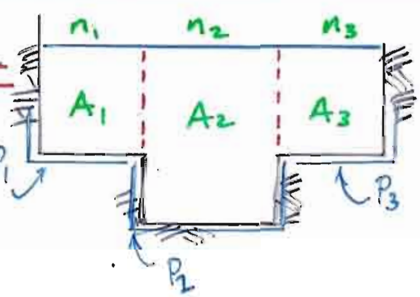
WIDE CHANNEL APPROXIMATION

$$R_h = \frac{A}{P} \cong \frac{by}{b+2y}$$



$$\frac{b}{y} > 10 \qquad \boxed{R_h \simeq y}$$

COMPOUND CHANNEL



$$R_h = \frac{A}{P}$$

$$Q = \frac{K}{n_1} A_1 R_1^{2/3} S_o^{1/2} + \frac{K}{n_2} A_2 R_2^{2/3} S_o^{1/2} + \frac{K}{n} A_3 R_3^{2/3} S_o^{1/2}$$

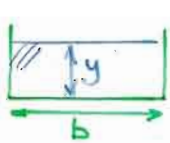
■ **TABLE 10.1**
Values of the Manning Coefficient, *n* (Ref. 6)

Wetted Perimeter	*n*
A. *Natural channels*	
Clean and straight	0.030
Sluggish with deep pools	0.040
Major rivers	0.035
B. *Floodplains*	
Pasture, farmland	0.035
Light brush	0.050
Heavy brush	0.075
Trees	0.15
C. *Excavated earth channels*	
Clean	0.022
Gravelly	0.025
Weedy	0.030
Stony, cobbles	0.035
D. *Artificially lined channels*	
Glass	0.010
Brass	0.011
Steel, smooth	0.012
Steel, painted	0.014
Steel, riveted	0.015
Cast iron	0.013
Concrete, finished	0.012
Concrete, unfinished	0.014
Planed wood	0.012
Clay tile	0.014
Brickwork	0.015
Asphalt	0.016
Corrugated metal	0.022
Rubble masonry	0.025

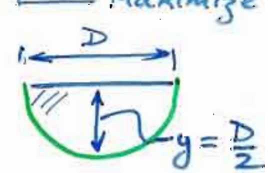
MOST EFFICIENT SECTION

$F_f = \tau P \ell$ ← Minimize

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$ ← Maximize



Rectangle $\quad y = \frac{1}{2} b$



Circle

Triangle

$\frac{1}{2}$ Hexagon

EXAMPLE 10.3

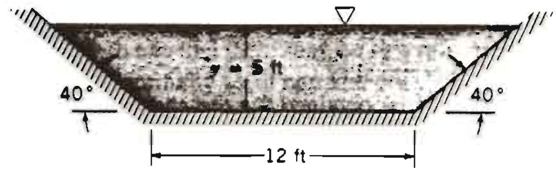
Water flows in the canal of trapezoidal cross section shown in Fig. E10.3. The bottom drops 1.4 ft per 1000 ft of length. Determine the flowrate if the canal is lined with new smooth concrete, or if weeds cover the wetted perimeter. Determine the Froude number for each of these flows.

FIND Q

1. $R_h = \frac{A}{P}$

2. $S_o = \frac{1.4}{1000}$

3. $n = ?$ use table 10.1.



■ FIGURE E10.3

SOLUTION

From Eq. 10.20,

$$\boxed{Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2}} \tag{1}$$

where we have used $\kappa = 1.49$, since the dimensions are given in BG units. For a depth of $y = 5$ ft, the flow area is

$$A = 12 \text{ ft } (5 \text{ ft}) + 5 \text{ ft} \left(\frac{5}{\tan 40^\circ} \text{ ft} \right) = 89.8 \text{ ft}^2$$

so that with a wetted perimeter of $P = 12 \text{ ft} + 2(5/\sin 40^\circ \text{ ft}) = 27.6$ ft, the hydraulic radius is determined to be $R_h = A/P = 3.25$ ft. Note that even though the channel is quite wide (the free-surface width is 23.9 ft), the hydraulic radius is only 3.25 ft, which is less than the depth.

Thus, with $S_0 = 1.4 \text{ ft}/1000 \text{ ft} = 0.0014$, Eq. 1 becomes

$$Q = \frac{1.49}{n} (89.8 \text{ ft}^2)(3.25 \text{ ft})^{2/3}(0.0014)^{1/2} = \frac{10.98}{n}$$

where Q is in ft^3/s.

From Table 10.1, the values of n are estimated to be $n = 0.012$ for the smooth concrete and $n = 0.030$ for the weedy conditions. Thus,

$$Q = \frac{10.98}{0.012} = 915 \text{ cfs} \quad \text{(SMOOTH)} \qquad \text{(Ans)}$$

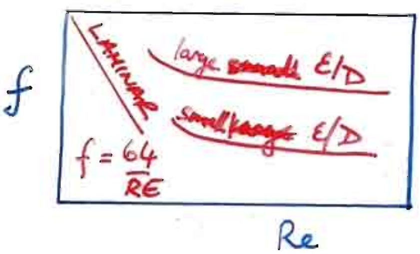
for the new concrete lining and

$$Q = \frac{10.98}{0.030} = 366 \text{ cfs} \quad \text{(ROUGH)} \qquad \text{(Ans)}$$

for the weedy lining. The corresponding average velocities, $V = Q/A$, are 10.2 ft/s and 4.08 ft/s, respectively. It does not take a very steep slope ($S_0 = 0.0014$ or $\theta = \tan^{-1}(0.0014) = 0.080^\circ$) for this velocity.

V ↓ with ↑ ROUGHNESS (ε)

Recall: MOODY for PIPES



∴ f not controlled by Re.

Check Re = 9.4 × 10^5

∴ Turbulent OK.

Note that the increased roughness causes a decrease in the flowrate. This is an indication that for the turbulent flows involved, the wall shear stress increases with surface roughness. [For water at 50 °F, the Reynolds number based on the 3.25-ft hydraulic radius of the channel is $\text{Re} = R_h V/\nu = 3.25 \text{ ft } (4.08 \text{ ft/s})/(1.41 \times 10^{-5} \text{ ft}^2/\text{s}) = 9.40 \times 10^5$, well into the turbulent regime.]

The Froude numbers based on the maximum depths for the two flows can be determined from $\text{Fr} = V/(gy)^{1/2}$. For the new concrete case,

$$\text{Fr} = \frac{10.2 \text{ ft/s}}{(32.2 \text{ ft/s}^2 \times 5 \text{ ft})^{1/2}} = 0.804 \qquad \text{(Ans)}$$

while for the weedy case

$$\text{Fr} = \frac{4.08 \text{ ft/s}}{(32.2 \text{ ft/s}^2 \times 5 \text{ ft})^{1/2}} = 0.322 \qquad \text{(Ans)}$$

In either case the flow is subcritical.

The same results would be obtained for the channel if its size were given in meters. We would use the same value of n but set $\kappa = 1$ for this SI units situation.

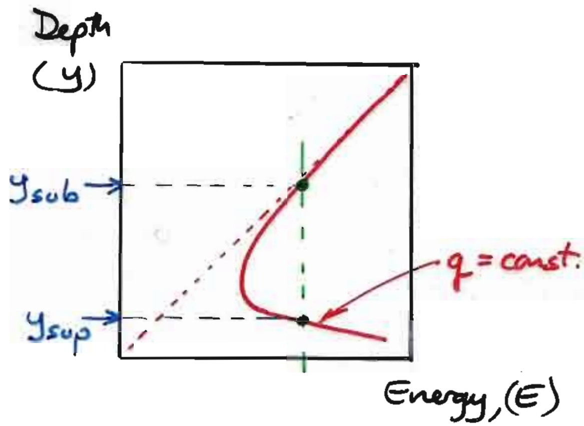
OPEN CHANNEL FLOW CONCEPTS

I GENERIC CONCEPTS

Froude No. $\quad F_R = \dfrac{V}{\sqrt{gy}}$

Energy. $\quad E = y + \dfrac{q^2}{2gy^2}$

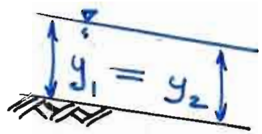
Energy Eqn. $\quad E_1 + z_1 = E_2 + z_2 + h_L$



$$\boxed{\frac{dy}{dx} = \frac{(S_f - S_o)}{(1 - Fr^2)}}$$

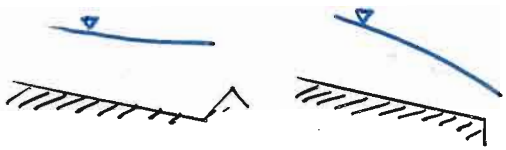
II SPECIFIC CONCEPTS

(i) UNIFORM FLOW $\quad \dfrac{dy}{dx} = 0$



$S_o = S_f$

(ii) GRADUALLY VARIED FLOW $\quad \dfrac{dy}{dx} \ll 1$



(iii) RAPIDLY VARIED FLOW $\quad \dfrac{dy}{dx} \sim 1$

Consideration of:

- Steady flows
- Homogeneous flows

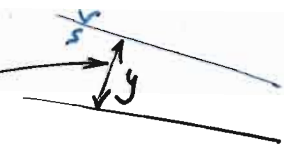
↓

Indexing parameters are:

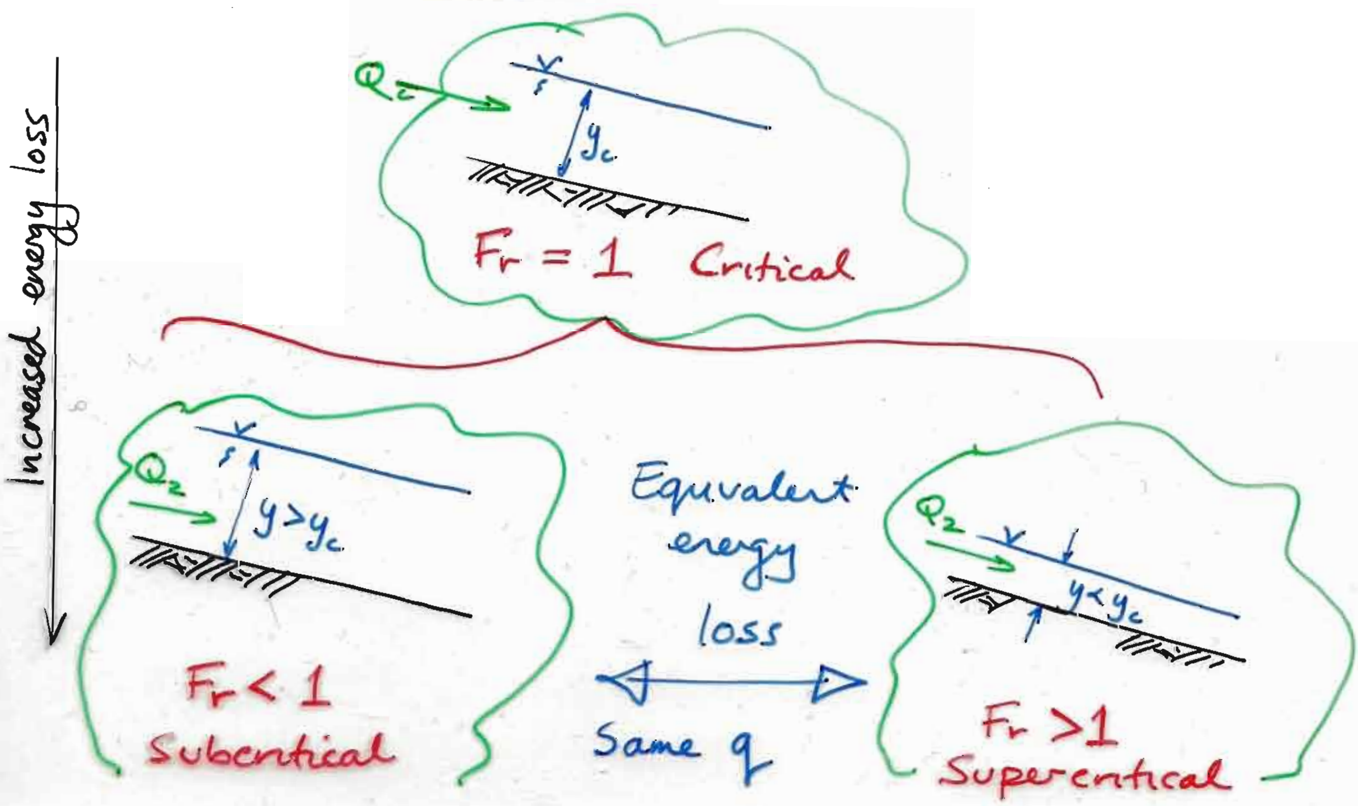
Reynolds No. $Re = \frac{V R_h \rho}{\mu}$

but always turbulent

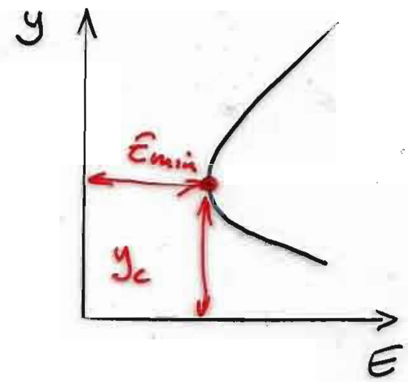
Froude No. $Fr = \frac{V}{\sqrt{(gy)}}$



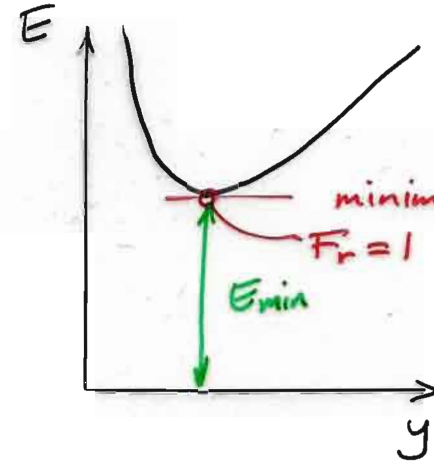
Froude No. indexes the flow



DETERMINE F_R @ y_c



Switch axes $\Longrightarrow$



$$\frac{dE}{dy} = \frac{d}{dy}\left[y + \frac{q^2}{2gy^2}\right] = 1 - \frac{q^2}{gy^3} = 0$$

y_c

Rearrange as: $y_c = \left(\dfrac{q^2}{g}\right)^{1/3}$ * OR $q^2 = y_c^3 g$

Substitute for $\boxed{E_{min} = y_c + \dfrac{q^2}{2gy_c^2} = \dfrac{3y_c}{2}}$

Determine velocity, V_c, at minimum energy, E_{min}.

$$V_c = \frac{q}{y_c}$$

From * we have $\sqrt{y_c^3 g} = q$

$$V_c = \frac{y_c^{3/2} g^{1/2}}{y_c} = \sqrt{gy_c}$$

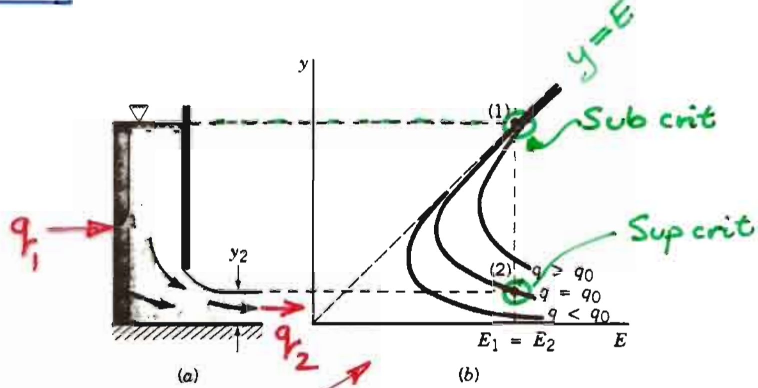
$$F_{R_c} = \frac{V_c}{\sqrt{gy_c}} = 1$$

DEFINE SPECIFIC ENERGY DIAGRAM FOR INVISCID FLOW BENEATH SLUICE

INVISCID $\longrightarrow$ $S_f = 0$

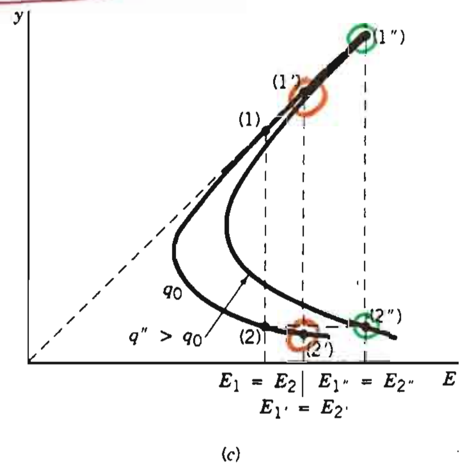
HORIZONTAL BED $\longrightarrow$ $S_o = 0$

$$E_1 = E_2 + (\cancel{S_f}^{\,0} - \cancel{S_o}^{\,0})\,l$$



CONTINUITY: $q_1 = q_2$ ∴ same line

ENERGY: $E = y + \dfrac{q^2}{2gy^2}$.



Note:

1. Changing q changes y_{sup} most as y_{sub} is constrained by $y = E$ line

Changes in configuration

1. Raise upstream level, y_1', but keep q constant by lowering sluice gap, y_2'. $q = q_o$

2. Raise upstream level, y_1'', and allow q to increase by not lowering sluice gap, $y_2'' = y_2$. $q > q_o$

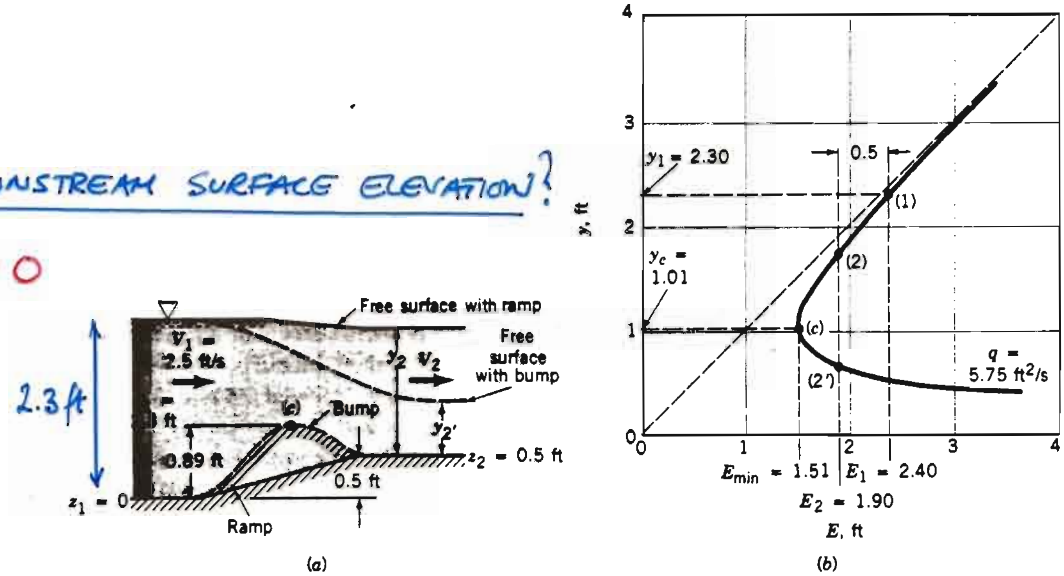
WATER FLOWS UP 0.5 ft RAMP

$q = 5.75\ ft^2/s$

$y_1 = 2.3\ ft$

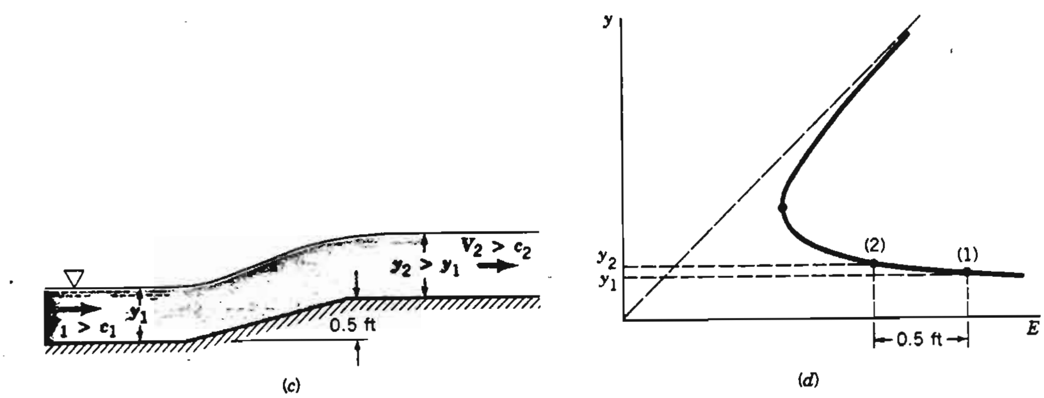
DETERMINE DOWNSTREAM SURFACE ELEVATION?

INVISCID $\therefore h_L = 0$



■ FIGURE E10.2

$E_1 + z_1 = E_2 + z_2 + h_L$



ENERGY EQUATION :

$$y_1 + \frac{V_1^2}{2g} + z_1 = y_2 + \frac{V_2^2}{2g} + z_2 + \underbrace{S_f \ell}_{h_L} \rightarrow 0$$

y_1: 2.3, z_1: 0, y_2: ?, $\frac{V_2^2}{2g}$: ?, z_2: 0.5

DEFINE $\frac{V_1^2}{2g}$ as $\Rightarrow \frac{q_1^2}{y_1^2 2g} = \frac{(5.75)^2}{(2.3)^2 2(32.4)} = 0.0971$

ONLY REMAINING UNKNOWNS y_2, V_2.

$2.3 + 0.097 - 0.5$

$$\boxed{1.90 = y_2 + \frac{V_2^2}{2g}} \quad (1)$$

ENERGY EQUATION

CONTINUITY: $\quad y_1 V_1 = y_2 V_2$

q_1

$$\boxed{5.75\ ft^2/s = y_2 V_2} \quad (2)$$

Substitute (2) into (1) as $\quad V_2 = \dfrac{5.75}{y_2}$

to give $\quad 1.90 = y_2 + \dfrac{(5.75)^2}{y_2^2\, 2g} \Rightarrow 0 = y_2^3 - 1.9y_2^2 + 0.513$

Solutions $\left\{ \begin{array}{ll} 1.72\ ft & \text{Subcritical} \\ 0.638\ ft & \text{Supercritical} \\ -.466\ ft & \text{False} \end{array} \right\} y_2$

Free surface elevations $\quad y_2 + z_2 = 1.72 + 0.5 = 2.22\ ft$

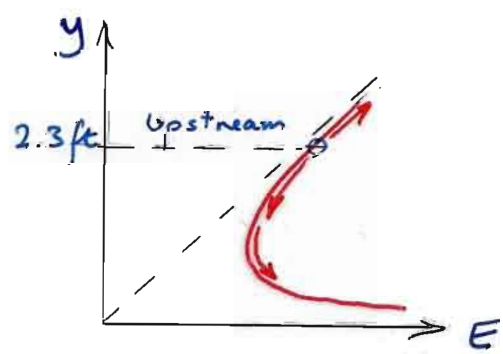
$y_2 + z_2 = 0.64 + 0.5 = 1.14\ ft$

Does subcritical or supercritical flow develop?

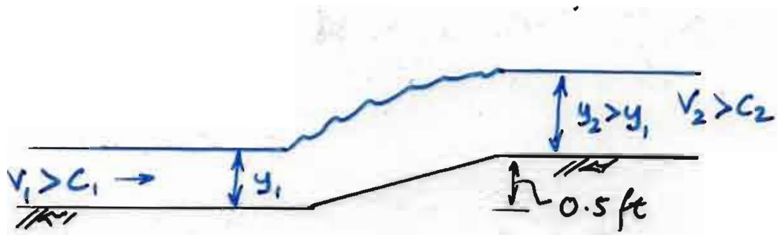
Use definition of energy, E $\quad E = y + \dfrac{q^2}{2gy^2}$

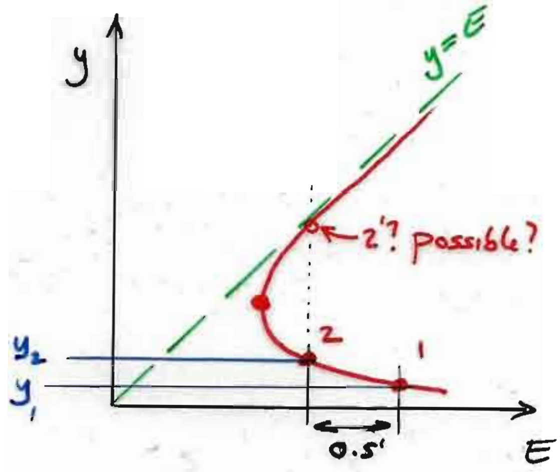
$$E = y + \frac{0.513}{y^2}$$

DRAW SPECIFIC ENERGY DIAGRAM



ALTERNATIVE SCENARIO

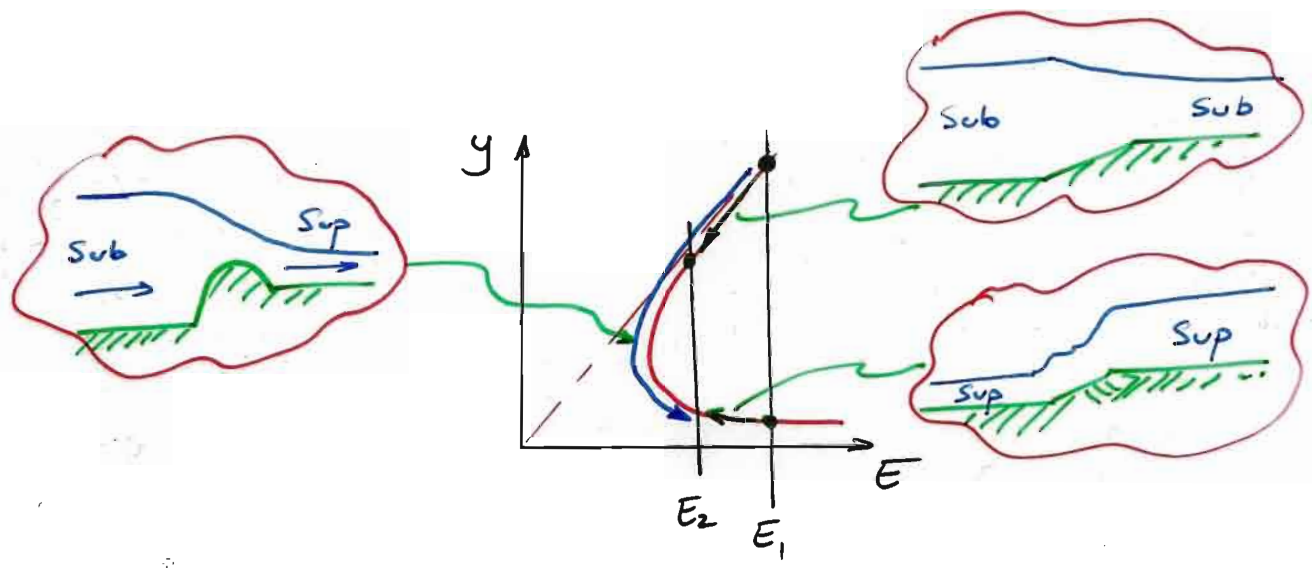




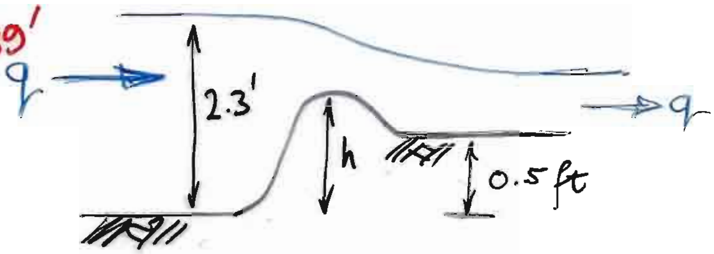
If upstream conditions supercritical.

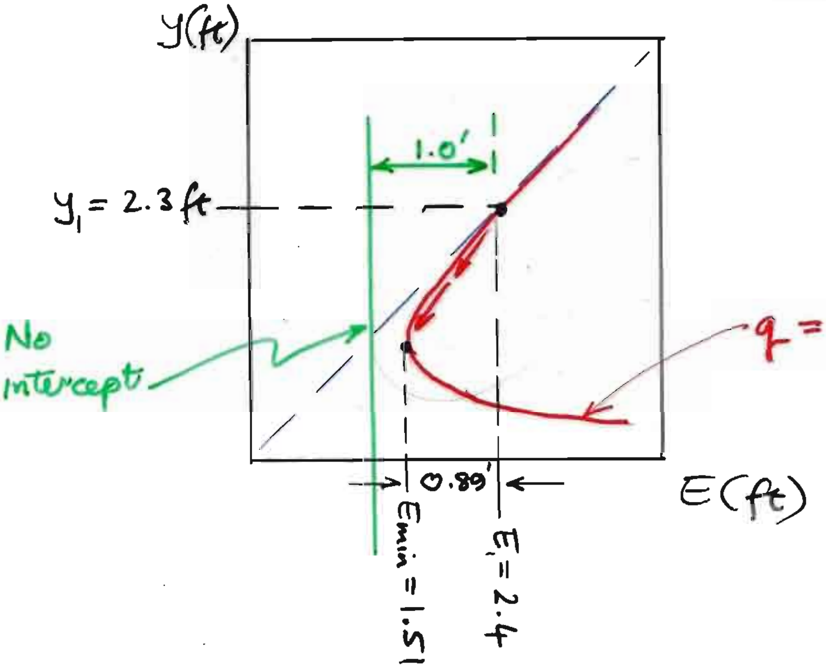
Changing Energy by −0.5 ft ↑ y_2 Flow remains supercritical

It is important to know the initial upstream conditions



LARGE 'BUMP' PRESENT ? $h > 0.89'$





What if downstream water elevation (bump) higher than <u>0.89'</u> ?

say $h = 1.0$ ft.

$$E_1 + z_1 = E_2 + z_2 \quad \Rightarrow \quad 1.4 = E_2$$

$E_1 = 2.4'$, $z_1 = 0'$, $z_2 = h = 1.0$ ft

$$1.4 = y_2 + \frac{(5.75)^2}{2g\,y_2^2}$$

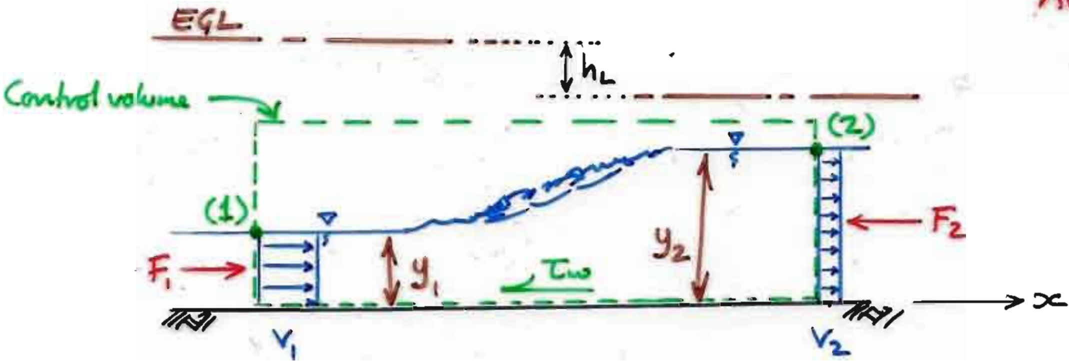
No (+ve) solution – see graph

∴ Flow is "choked" – cannot flow @ $q = 5.75\ ft^2/s$ in this configuration.

HYDRAULIC JUMP

$\frac{dy}{dx} = \infty$ — Discontinuity

Also discontinuity in $\frac{dE}{dx}$.



Neglect τ_w

MOMENTUM:

$$F_1 - F_2 = \rho Q (V_2 - V_1) \qquad \begin{cases} F_1 = \frac{1}{2}\gamma y_1^2 b \\ F_2 = \frac{1}{2}\gamma y_2^2 b \end{cases}$$

$$Q = qb \;; \quad V = \frac{q}{y}$$

SUBSTITUTING:

$$\frac{1}{2}\gamma b \left[y_1^2 - y_2^2 \right] = \frac{\gamma}{g} q b \left(\frac{q}{y_2} - \frac{q}{y_1} \right)$$

$$\boxed{\frac{y_1^2}{2} + \frac{q^2}{g y_1} = \frac{y_2^2}{2} + \frac{q^2}{g y_2}}$$

MOMENTUM:

$$\boxed{M = \frac{y^2}{2} + \frac{q^2}{g y}}$$

MOMENTUM = const.

M = constant through jump.

CONTINUITY:

$$\boxed{y_1 V_1 b = y_2 V_2 b}$$

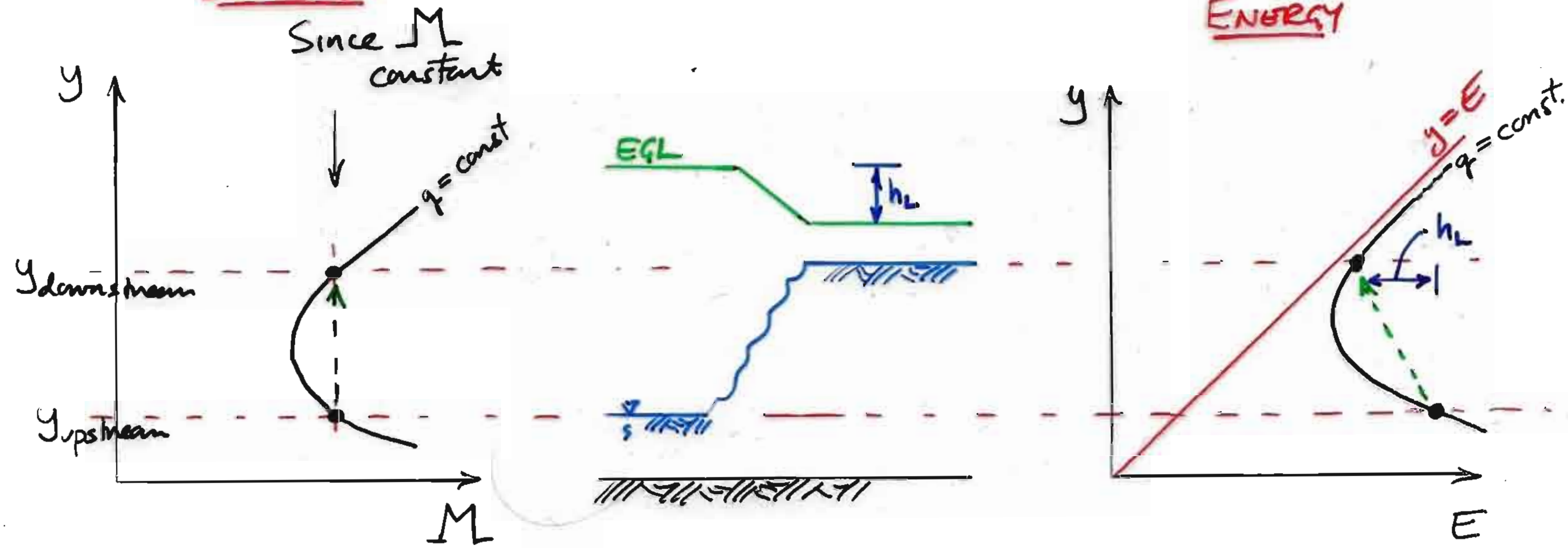
ENERGY EQN:

$$\boxed{y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L}$$

Momentum & Energy Diagrams

Momentum

Energy



$$M = \frac{y^2}{2} + \frac{q^2}{gy}$$

$$E = y + \frac{q^2}{2gy^2}$$

- Plot M for a given q, variable with flow depth, y.
- If upstream M is known then know that $M_{downstream}$ is the same
- Project up on M diagram to determine downstream depth, $y_{downstream}$.