EME 303 - FLUID MECHANICS
SUMMARIZED EQUATIONS AND CONCEPTS

Some Useful Conversion Factors

	SI	BGS	EE
Temperature:	$K = {}^{o}C + 273.15$	${}^{o}R = {}^{o}F + 459.67$	${}^{o}R = {}^{o}F + 459.67$
Force:	$1N = (1kg)(1m/s^2)$	$1lb = (1slug)(1ft/s^2)$	$1lbf = (1lbm)(32.2ft/s^2)$
Mass:	kg	$slug$	lbm
Density:	$1\,kg/m^3$	$0.00194\,slug/ft^3$	$0.06243\,lbm/ft^3$
ρ_{water}	$1000kg/m^3$	$1.94slugs/ft^3$	$62.4lbm/ft^3$
Pressure:	$1Pa = 1N/m^2$	$0.0209\,lb/ft^2$	$0.0209\,lbf/ft^2$
Work, energy:	$1J = 1N.m$	$1ft.lb = 778.2Btu$	—
Power:	$1W = 1N.m/s$	$1hp = 550ft.lb/s$	—

General [Topic:1]

$$\mathbf{F} = m\mathbf{a} \text{ or } \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = m \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix}$$

$$p = \rho R T \begin{cases} Isothermal: \dfrac{p}{\rho} = const. \\ Isentropic: \dfrac{p}{\rho^k} = const. \end{cases}$$

$$\tau = \mu \frac{du}{dy}; \quad \nu = \frac{\mu}{\rho}$$

$$E_v = -\frac{dp}{d\mathcal{V}/\mathcal{V}} = \frac{dp}{d\rho/\rho}$$

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

[1:3] Fluid Properties

Recap

Definitions

Dimensional homogeneity

Fluid properties

Mass and Weight $M = \rho \cancel{V}; W = Mg$

Equations of State $p = \rho R T$

Compressibility $E_v = -\dfrac{dp}{d\cancel{V} \,/\, \cancel{V}_0} = +\dfrac{dp}{d\rho \,/\, \rho_0}$

Outline

Wave Speeds $c = \sqrt{gy} \; or \; \sqrt{E_v / \rho}$

http://www.youtube.com/watch?v=629em0mPpUY

Viscosity $\tau = \mu \dfrac{\partial v_x}{\partial y}$

Vapor pressure (airfoil)

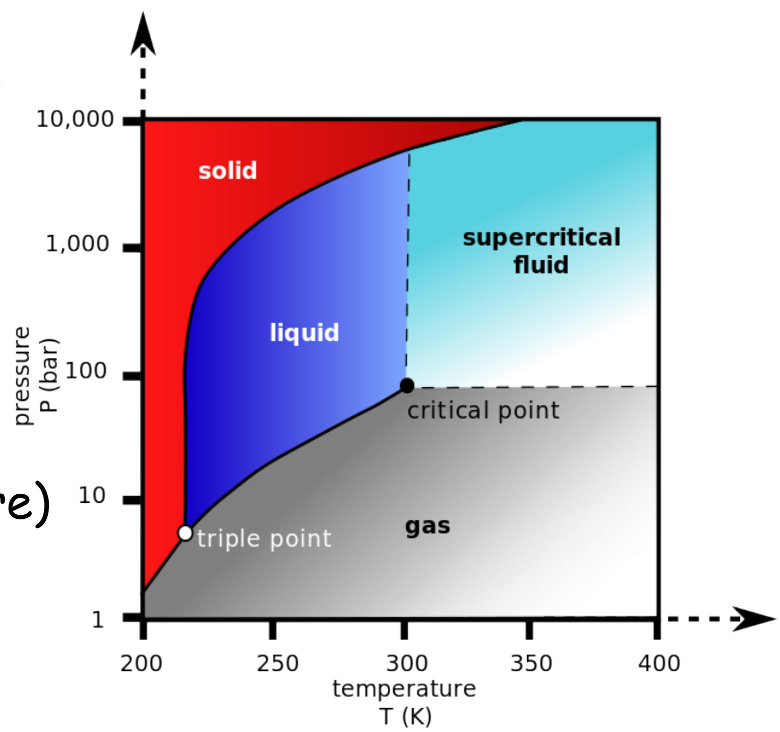
Surface tension (vinometer)

Topic 2 Pressure

Fluid pressure at a point

Incompressible (water)

Compressible (atmosphere)



http://en.wikipedia.org/wiki/Supercritical_fluid

1.7.2 COMPRESSION / EXPANSION OF GASES

Ideal gas law $\quad p = \rho R T$

Compression @ constant temperature (ISOTHERMAL) $(dT = 0)$

$$\frac{p}{\rho} = \text{constant} \qquad (1.14)$$

Compression – Frictionless (ISENTROPIC)

Adiabatic – no heat exchange to surroundings. – no heat loss

$$\frac{p}{\rho^k} = \text{constant} \qquad (1.15)$$

$$k = \frac{\text{Specific heat @ const. pressure}}{\text{Specific heat @ const volume}}$$

Tables 1.7 & 1.8.

Modulii of gases are given as, E_v

since $m = \rho \downarrow \forall \uparrow$ ∴

$$E_v = -\frac{dp}{d\forall/\forall_0} = +\frac{dp}{d\rho/\rho_0} = \frac{dp}{d\rho}\cdot\rho_0$$

Isothermal: $p = \rho \cdot \text{const.} \quad \therefore \quad \frac{dp}{d\rho} = \text{const.} \quad \therefore \quad E_v = \text{const.}\,\rho$

Also $\rho = \frac{p}{\text{const}}$ $\qquad \boxed{\therefore E_v = p}$

Isentropic: Similar procedure $\boxed{E_v = kp}$

MODULUS, E_v, IS DIRECTLY PROP. TO PRESSURE ∴ COMPRESSIBLE !!

The action of a fluid may be described as a series of very thin sheets each of which slip relative to the next.

Through experimentation it has been shown that the velocity gradient (du/dy) times the viscosity (μ) is equal to the shearing stress (τ) between the thin sheets.

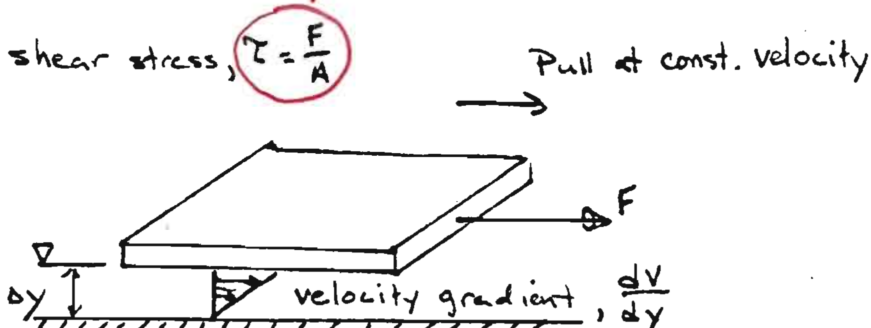
Experimentally determined $\rightarrow$ $$\tau = \mu \frac{du}{dy} \Rightarrow$$

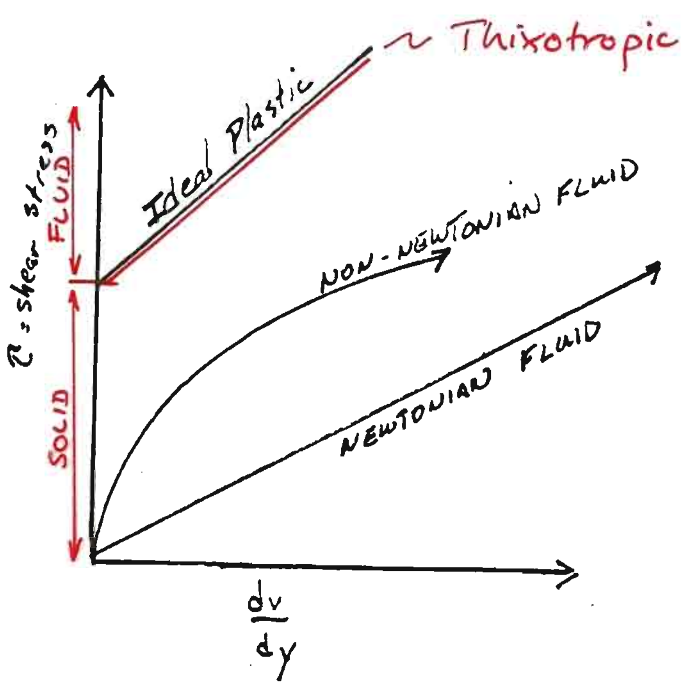
Units ?

$$\mu = \frac{\tau}{dv/dy} \doteq \frac{ML^{-1}T^{-2}}{LT^{-1}/L}$$

$$\mu \doteq ML^{-1}T^{-1}$$

Note: du may be represented by dv
Velocity may be represented as U or V.

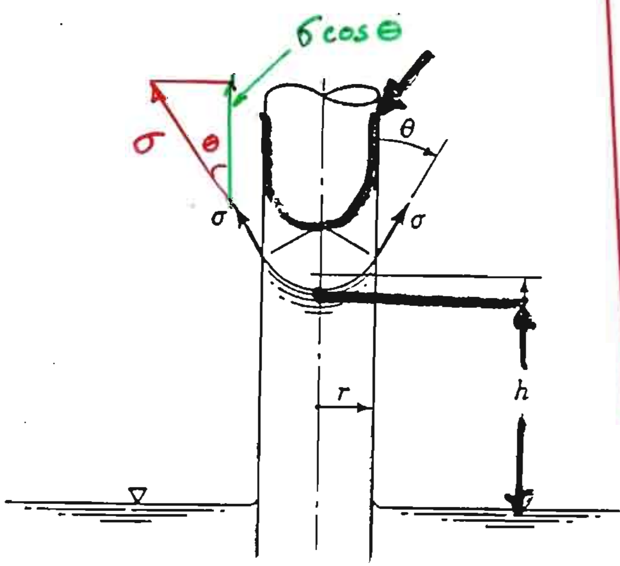




The capillary rise (or depression) as shown in the figure below is expressed as:

$$h = \frac{2\sigma \cos \theta}{\gamma r}$$

σ = surface tension in units of force per unit length
θ = wetting angle
γ = specific weight of the liquid
r = radius of the tube
h = capillary rise (from inflection point of meniscus)



FORCE BALANCE

$$\gamma \underbrace{h \pi r^2}_{\text{Column volume}} = \underbrace{2\pi r}_{\text{Circumference}} \overbrace{\sigma \cos \theta}^{\text{Vertical force}}$$

REARRANGING

$$h = \frac{2\sigma \cos\theta}{\gamma r}$$

Figure 1.8 Capillary rise.

For a clean tube $\theta = 0°$ H_2O, $140°$ Hg

For tube diameters > ½" (12 mm) capillary effects are negligible.

Fluid Statics [2,3]

$$p_x = p_y = p_z = p$$

$$-\nabla p - \gamma\hat{\mathbf{k}} = \rho\mathbf{a} \text{ or } -\begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} p - \gamma \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \rho \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix}$$

Simplifies when: $a_x = a_y = a_z = 0$ to $\frac{dp}{dx} = \frac{dp}{dy} = 0$ and $\frac{dp}{dz} = -\gamma; \frac{dz}{dx} = -\frac{a_x}{g + a_z}$

Rigid body rotation: $\frac{\partial p}{\partial r} = \rho r \omega^2; \frac{\partial p}{\partial \theta} = 0; \frac{\partial p}{\partial z} = -\gamma; \begin{cases} z = \frac{\omega r^2}{2g} + c \\ p = \frac{\rho \omega^2 r^2}{2} - \gamma z + c \end{cases}$

Incompressible fluid: $p = \gamma h + p_0$

Compressible fluid: $\frac{dp}{dz} = -\frac{gp}{RT}$ and integrate w.r.t (p, z).

Manometer rules: $(\uparrow -ve)(\downarrow +ve); \frac{dp}{dx} = \frac{dp}{dy} = 0; p_v$ if evacuated; $\gamma_{gas} \to 0$.

$$F_R = \gamma h_c A; \quad F_R = \sqrt{F_H^2 + F_V^2}$$

F_R acts through center of pressure $\begin{cases} y_R = \frac{I_{xc}}{y_c A} + y_c \\ x_R = \frac{I_{xyc}}{y_c A} + x_c \end{cases}$

$$F_B = \gamma \forall$$

PRESSURE AT A POINT

INCOMPRESSIBLE FLUID

$$\frac{dp}{dz} = -\gamma$$

Integrate as:

$$\int_{p_1}^{p_2} dp = -\int_{z_1}^{z_2} \gamma \, dz$$

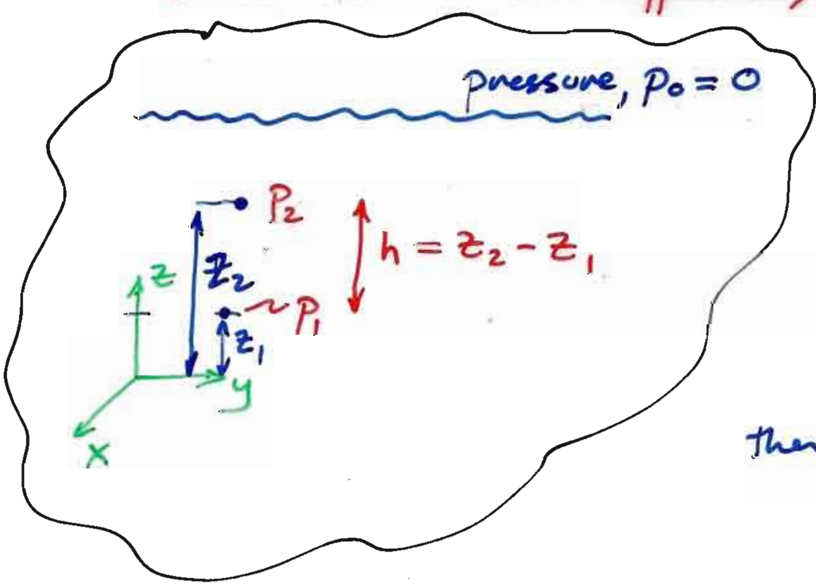
Insert limits as:

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

or:

$$p_1 = \underbrace{\gamma(z_2 - z_1)}_{h} + p_2$$

Static head or head difference, h.

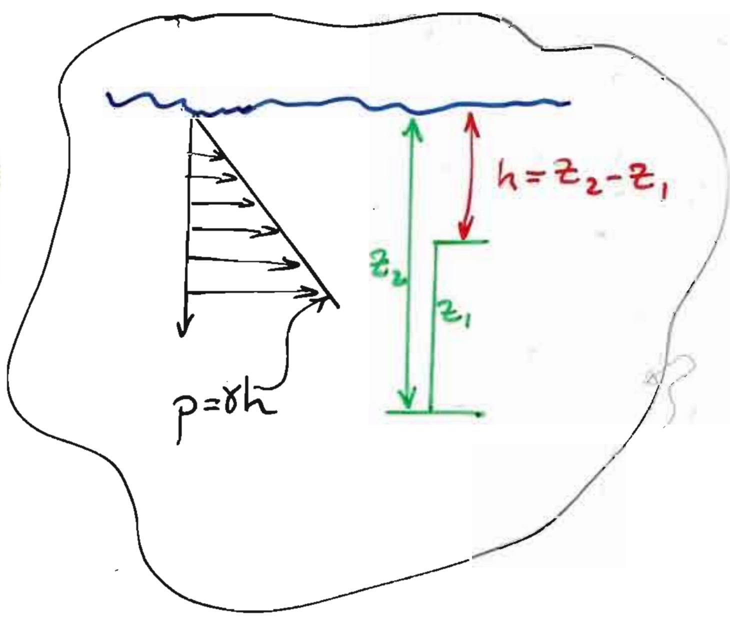


$$\boxed{p_1 = \gamma h + p_2}$$

Set p_2 @ z_2 on surface $\cancel{p_2} \rightarrow 0$

then $p_1 = \gamma h + \cancel{p_2} \rightarrow 0$

Note change in p is linear with depth.



Pressure head; $h = \dfrac{p}{\gamma}$

[2:2] Fluid Statics

Recap

Fluid pressure at a point (static) $p_x = p_y = p_z = p$

$$-\nabla p - \gamma \hat{\mathbf{k}} = \rho \mathbf{a} \text{ or } -\begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} p - \gamma \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \rho \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix}$$

Simplifies when: $a_x = a_y = a_z = 0$ to $\frac{dp}{dx} = \frac{dp}{dy} = 0$ and $\frac{dp}{dz} = -\gamma$

Incompressible (water) Incompressible fluid: $p = \gamma h + p_0$

Compressible (atmosphere) Compressible fluid: $\frac{dp}{dz} = -\frac{gp}{RT}$ and integrate w.r.t (p, z).

Outline

Pressure measurement (manometry)

Manometer rules: $(\uparrow -ve)(\downarrow +ve); \frac{dp}{dx} = \frac{dp}{dy} = 0; p_v$ if evacuated; $\gamma_{gas} \rightarrow 0.$

PRESSURE AT A POINT (Cont'd)

COMPRESSIBLE FLUID – GASES

ρ or $\gamma = f(p, T)$

Important in column.

General equation: $\boxed{\dfrac{dp}{dz} = -\gamma}$

dp is small, even for large dz since γ_{gas} is small!!

$\gamma_{water} = 62.4 \text{ lb/ft}^3$

$\gamma_{air} = 0.0763 \text{ lb/ft}^3$

For large height variations (dz large)

$p = \rho R T$; $\dfrac{dp}{dz} = -\gamma = -\rho g$

$-\rho g = -\dfrac{pg}{RT}$

$$\boxed{\frac{-pg}{RT} = \frac{dp}{dz}}$$

Separate variables:

$$\int_{z_1}^{z_2} \frac{-g}{RT}\, dz = \int_{p_1}^{p_2} \frac{dp}{p}$$

$$-\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T} = (\ln p_2 - \ln p_1) = \ln\left(\frac{p_2}{p_1}\right)$$

How does temperature vary, T.

If T is constant in the range $z_1 \rightarrow z_2$ (ISOTHERMAL)

$$T = T_0$$

Then:

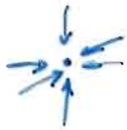
$$-\frac{g}{RT_0}\int_{z_1}^{z_2} dz = \ln\left(\frac{P_2}{P_1}\right)$$

$$\exp\left[-\frac{g(z_2-z_1)}{RT_0}\right] = \exp\left[\ln\left(\frac{P_2}{P_1}\right)\right]$$

$$\boxed{P_2 = P_1 \exp\left[-\frac{g(z_2-z_1)}{RT_0}\right]}$$

Q.E.D.

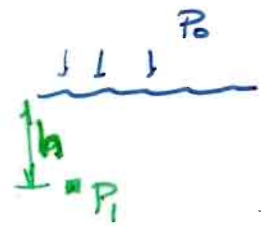
WHAT DO WE KNOW ?

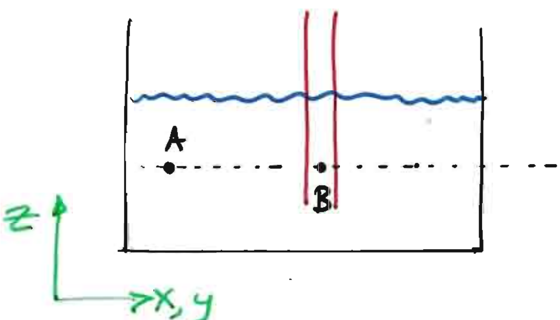


$p_x = p_y = p_z$ @ a point

$$\frac{dp}{dx} = \frac{dp}{dy} = 0 \qquad \frac{dp}{dz} = -\gamma$$

$$p_1 = \gamma h + p_0$$

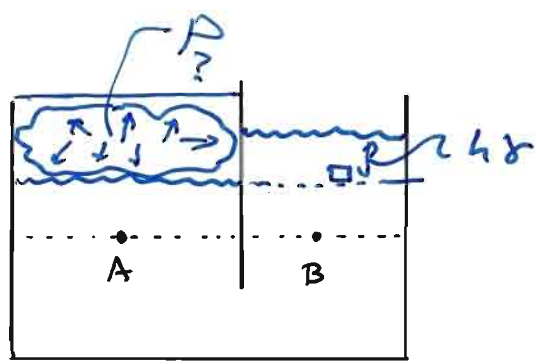




$$\frac{dp}{dx} = \frac{dp}{dy} = 0$$

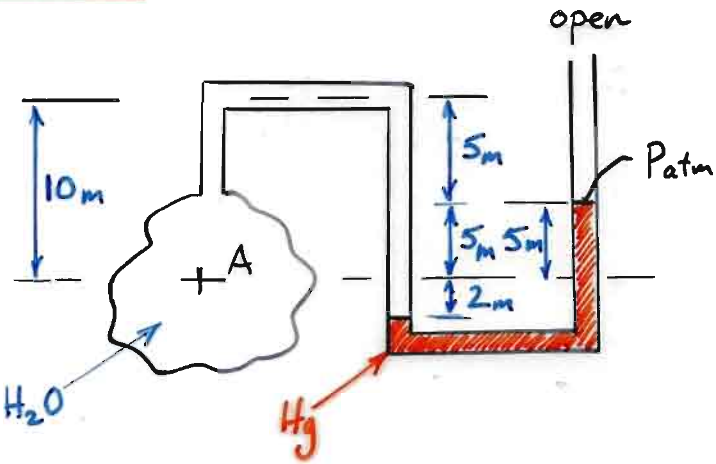
$p_A = p_B$!!

?



$p_A = p_B$?

EXAMPLE



FROM TABLE 1.6

$$\gamma_w = 9.8 \text{ KN/m}^3$$

$$\gamma_{Hg} = 133 \text{ KN/m}^3$$

Rules : (−ve ↑) (+ve ↓)

$$P_A \underbrace{- 10\gamma_w + 5\gamma_w + 5\gamma_w} + 2\gamma_w - 7\gamma_{Hg} = \cancelto{0}{P_{atm}}$$

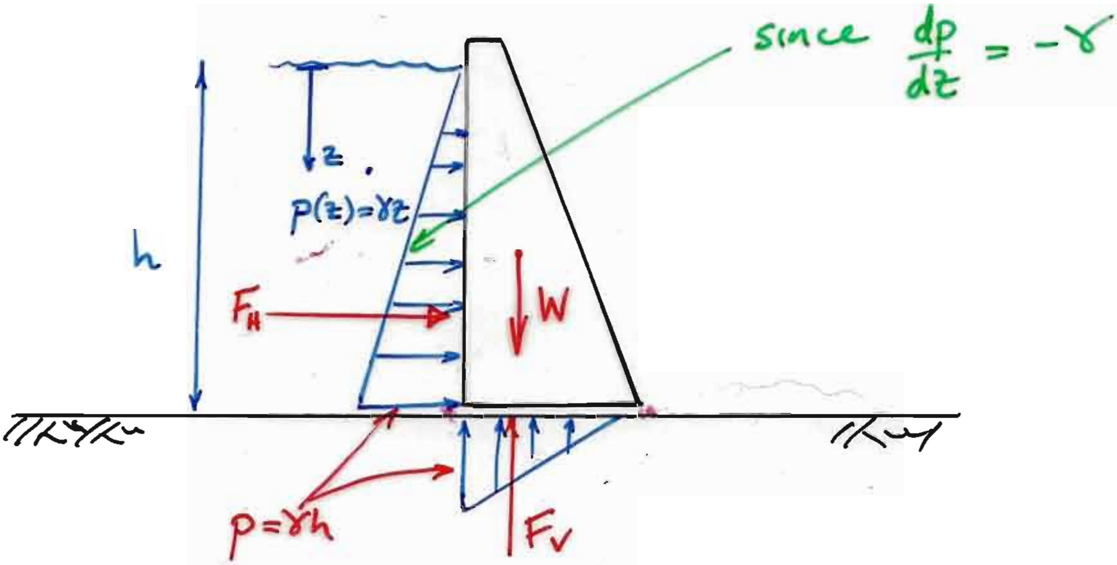
$$P_A + 2\gamma_w - 7\gamma_{Hg} = 0$$

$$P_A = -2\gamma_w + 7\gamma_{Hg} = -2(9.8) + 7(133) \text{ KPa}$$

$$P_A = 911.4 \text{ KPa} \equiv \text{KN/m}^2$$

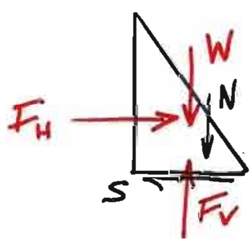
HYDROSTATIC FORCES ACTING ON SURFACES.

Why?



Questions:

1. What is magnitude of force?



$$S = N \tan \phi$$

Resolve vertically: $N = W - F_V$

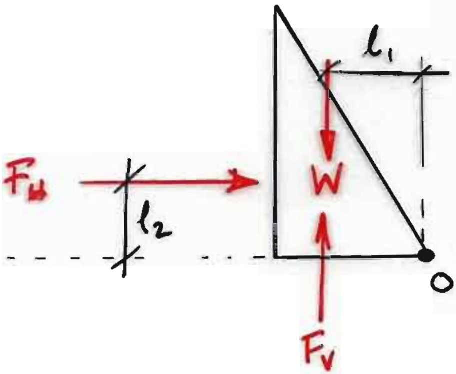
Strength of base $= S = (W - F_V) \tan \phi$

if $F_H > S$ Translational failure.

$F_H \leq S$ Stable equilibrium.

$\therefore$ Need to know force magnitudes!!

2. Where does the force act?



$$\overset{\curvearrowright}{\sum M_O} = 0$$

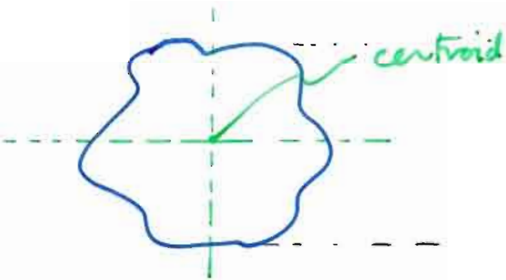
$$F_H l_2 - (W - F_V) l_1 = 0$$

$$F_H = (W - F_V) \frac{l_1}{l_2}$$

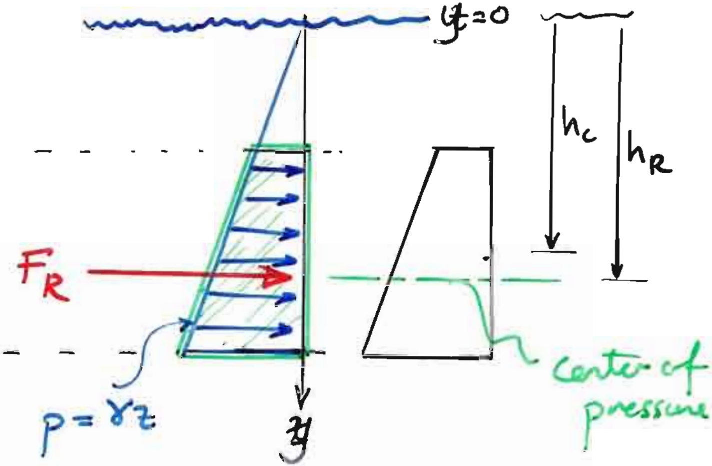
If F_H is larger than this magnitude limit then the dam "fails" by overturning.

∴ Need to know where the forces act!!

CENTER OF PRESSURE



Centroid is the "balance" point of the plate.!



"Center of pressure" is the "balance" point for the pressure distribution

Determine C of Pressure by summing moments around 'x' axis.

CENTER OF PRESSURE – X-DIRECTION (X-COORDINATE)

As before:
$$F_R \, x_R = \int_A \gamma \sin\theta \, \underset{\S}{x} \, y \, dA$$

Pressure varies from surface (y), but moment taken about $\underset{\S}{x}$

$$x_R = \frac{\int_A xy \, dA}{y_c A} = \frac{I_{xy}}{y_c A}$$

Parallel axes theorem

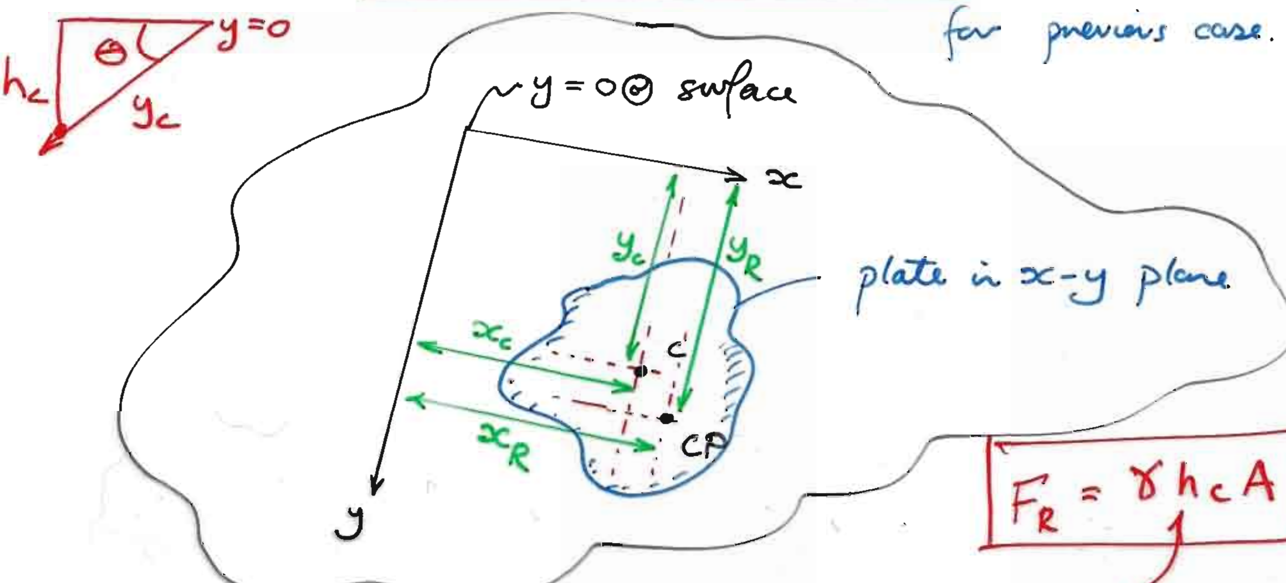
$$\boxed{x_R = \frac{I_{xyc}}{y_c A} + x_c}$$

I_{xyc} = product of moment of inertia with respect to x & y axes.

Compares to:

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c}$$

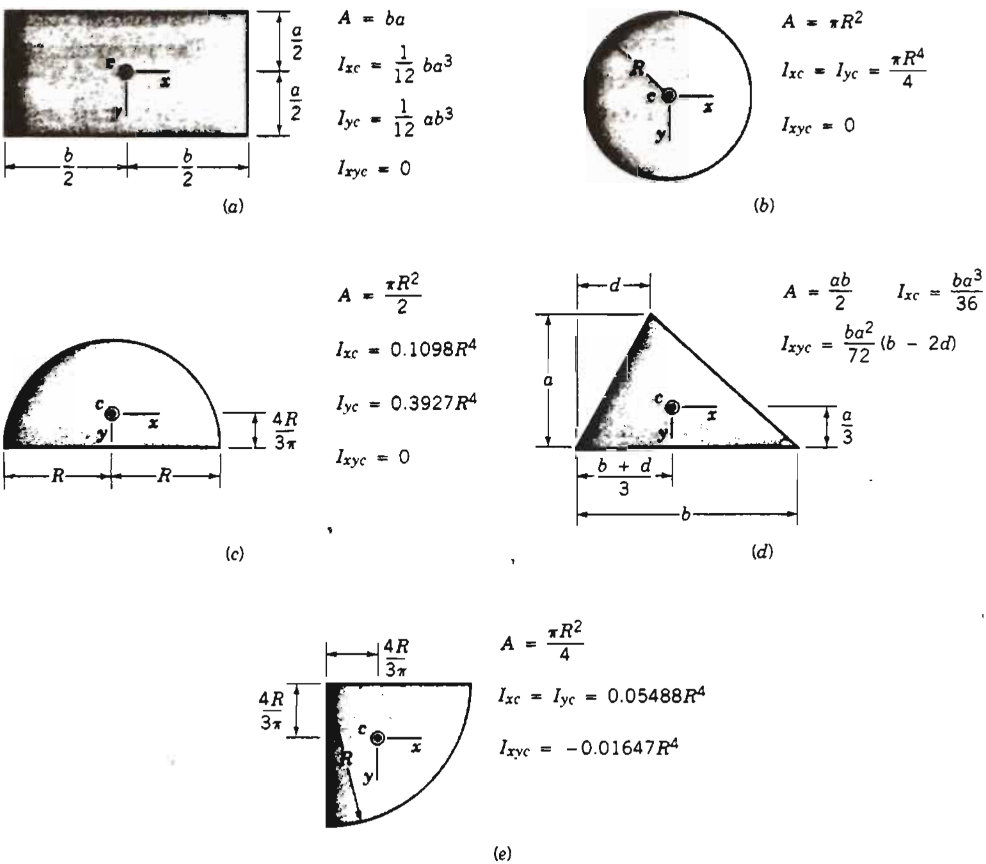
for previous case.



$$\boxed{F_R = \gamma h_c A}$$

$$h_c = y_c \sin\theta$$

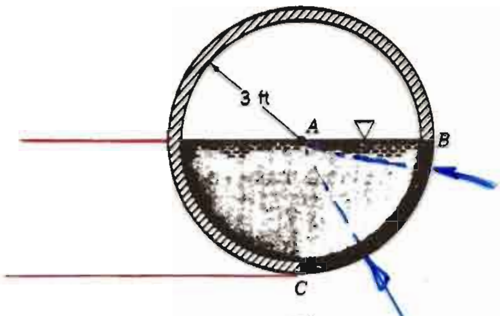
AREAS & MOMENTS OF INERTIA



■ **FIGURE 2.18** Geometric properties of some common shapes.

For any shape symmetric w.r.t. $x = 0$ then $I_{xyc} = 0$

EXAMPLE 2.9 The 6-ft-diameter drainage conduit of Fig. E2.9*a* is half full of water at rest. Determine the magnitude and line of action of the resultant force that the water exerts on a 1-ft length of the curved section *BC* of the conduit wall.

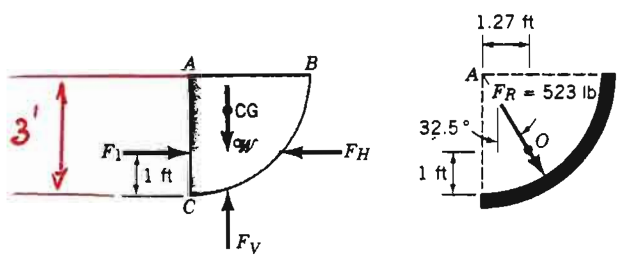


(*a*)

FROM GEOMETRY

$F_1 = F_H$

$F_V = W$



(*b*) (*c*)

$F_1 = \gamma h_c A \Rightarrow$

$F_1 = (62.4)\frac{lb}{ft^3}\left(\frac{3}{2}\right)ft\ (3)ft^2$

Unit length along pipe

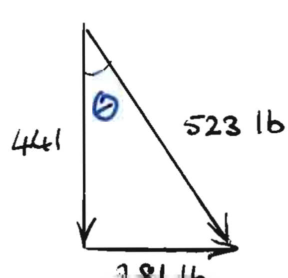
$\boxed{F_1 = 281\ lb} = F_H$

$W = \gamma \frac{\pi r^2}{4} = (62.4)\frac{lb}{ft^3}\left(\frac{\pi}{4}3^2\right)ft^2\ (1)ft = 441\ lb = F_V$

Resultant $F_R = \sqrt{F_H^2 + F_V^2} = 523\ lb$

Direction of resultant: Pressure $\perp$ to conduit wall $\therefore$ all pressure "vectors" pass through 'O'.

Consequently Resultant passes through O.

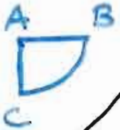


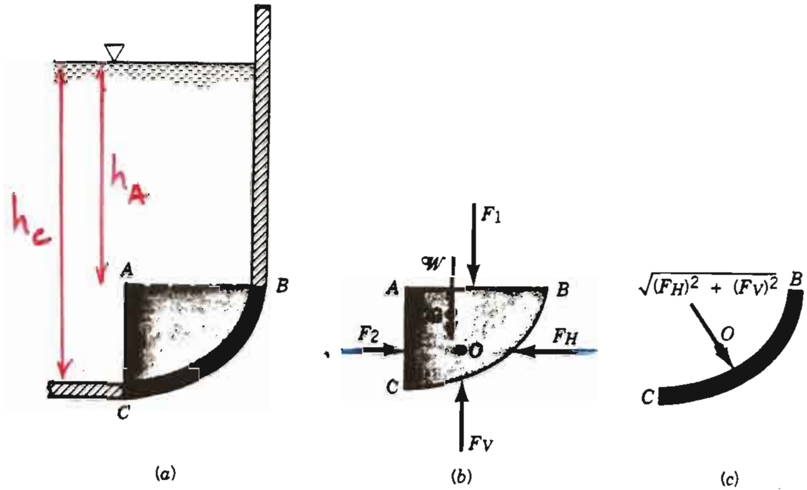
$523 \cos\Theta = 441$

$\cos\Theta = 441/523$

$\Theta = 32.5°$

HYDROSTATIC FORCE ON CURVED SURFACE

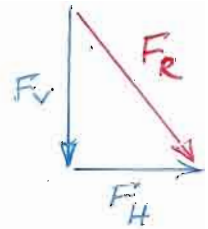
Isolate free body 



■ **FIGURE 2.23** Hydrostatic force on a curved surface.

Horizontal $\Sigma F_H = 0$ $F_H = F_2$

Vertical $\Sigma F_V = 0$ $F_V = F_1 + W$



Resultant, F_R $F_R = \sqrt{F_H^2 + F_V^2}$

(F_R is a magnitude).

Point of action, O

Determined by summing F_H, F_V and F_R (all known) about an appropriate axis.

2.84

2.84 The 9-ft-long cylinder of Fig. P2.84 floats in oil and rests against a wall. Determine the horizontal force the cylinder exerts on the wall at the point of contact, *A*.

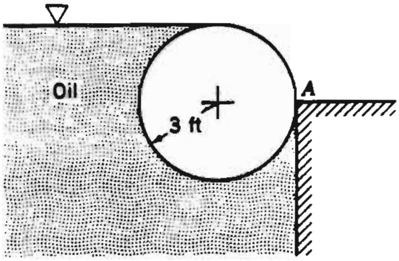
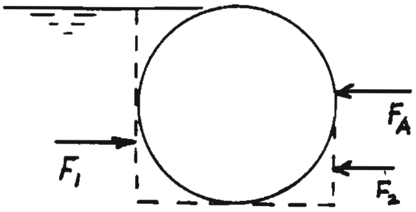


FIGURE P2.84



The horizontal forces acting on the free-body-diagram are shown on the figure. For equilibrium,

$$F_A = F_1 - F_2$$

where F_A is the horizontal force the wall exerts on the cylinder. Since,

$$F_1 = \gamma h_{c1} A_1$$
$$= \left(57.0 \frac{lb}{ft^3}\right)\left(\frac{6 ft}{2}\right)(6 ft \times 9 ft)$$
$$= 9230 \text{ lb}$$

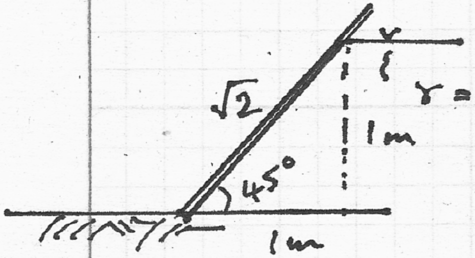
and

$$F_2 = \gamma h_{c2} A_2$$
$$= \left(57.0 \frac{lb}{ft^3}\right)\left(3 ft + \frac{3}{2} ft\right)(3 ft \times 9 ft)$$
$$= 6930 \text{ lb}$$

then

$$F_A = 9230 \text{ lb} - 6930 \text{ lb} = \underline{\underline{2300 \text{ lb} \longrightarrow \text{on the wall}}}$$

Determine the horizontal and vertical forces acting on this gate

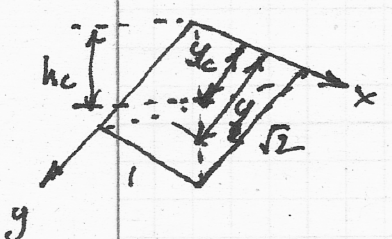


$\gamma = 10\ kN/m^3$

$w = 1m$ into page

Three approaches to solve the same problem – they are equivalent.

Centroid and Center of Pressure Approach

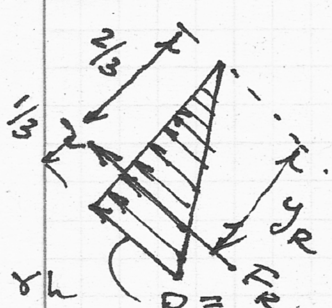


$$F_R = \gamma A h_c = 10\,KN/m^3\,(1\times\sqrt{2})m^2\ \frac{1}{2}m = \underline{\underline{5\sqrt{2}\ kN}}$$

$$y_R = y_c + \frac{I_{xc}}{y_c A},\quad I_{xc} = \frac{1}{12}ba^3$$

$$= \frac{1}{2}\sqrt{2}m + \frac{\frac{1}{12}\,1\,2^{3/2}}{\frac{1}{2}\sqrt{2}\,.(\sqrt{2})\,1} = \underline{\underline{\frac{1}{2}\sqrt{2} + \frac{1}{6}\sqrt{2}\ m}}$$

Pressure Prism Approach



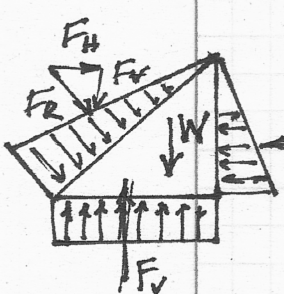
$p = \gamma h$ $\quad p =$

$p = 10\,kN/m^3 \times 1m = 10\ kN/m^2$

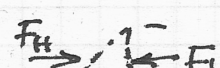
$$F_R = \underbrace{\tfrac{1}{2}p}_{\text{average pressure}} \times A = \frac{10}{2}\,kPa \times \underbrace{\sqrt{2}\times 1}_{A} = \underline{\underline{5\sqrt{2}\ kN}}$$

$$y_R = \frac{2}{3}\sqrt{2}m \equiv \underline{\underline{(\tfrac{1}{2} + \tfrac{1}{6})\sqrt{2}\ m}}$$

Free Body Diagram Approach



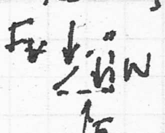
Resolving horizontally:



$F_H = F_h = \gamma A h_c$ (area of vert plane)

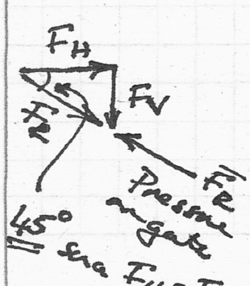
$F_H = F_h = 10\,kN/m^3\,(1\times 1)m^2\,\frac{1}{2}m = 5kN$

Resolving vertically: $F_V = F_{\underline{v}} + W$ or $F_V = F_{\underline{v}} - W$



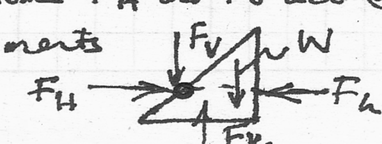
$$F_V = F_{\underline{v}} - W = \gamma A h_c - \gamma \forall$$

$$= 10\,kN/m^3\,(\underbrace{1\times 1}_{A}\times \underbrace{1}_{h_c} - \underbrace{\tfrac{1}{2}(1\times 1\times 1)}_{\forall}) = 5\,kN$$



Resultant: $F_R = \sqrt{F_H^{\,2} + F_V^{\,2}} = \sqrt{5^2 + 5^2} = \sqrt{2\times 25} = \underline{\underline{5\sqrt{2}\ kN}}$

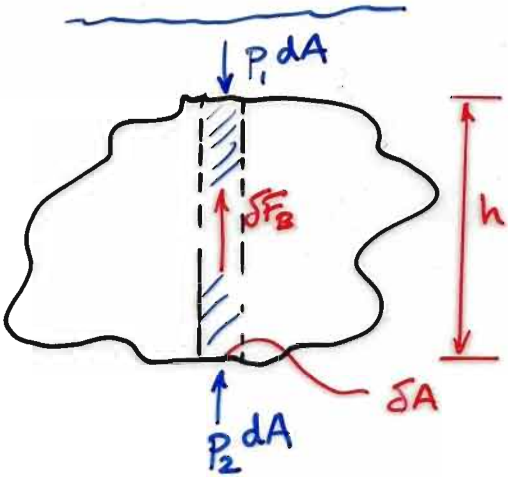
Assume F_H and F_V act @ 2/3 depth and check moments



$W\frac{1}{3} + F_V\frac{2}{3} = F_{\underline{v}}\frac{1}{2}$

$5\frac{1}{3} + 5\frac{2}{3} = 10\frac{1}{2}$ QED.

BUOYANCY, FLOTATION, STABILITY



$$\delta F_B = (p_1 - p_2)\,\delta A$$
$$= \gamma h\,\delta A$$
$$= \gamma\, d\forall$$

Integrating over prism:

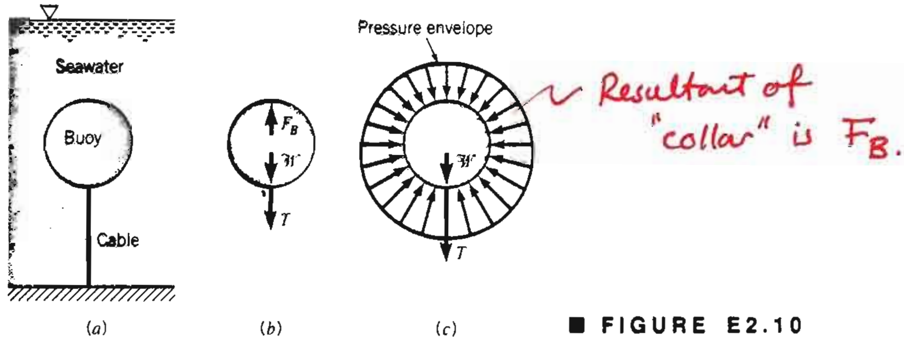
$$F_B = \int_{\forall} \delta F_B = \gamma \int_{\forall} d\forall = \gamma \forall$$

$$\boxed{F_B = \gamma \forall}$$

- ☐ No lateral forces (all cancel)
- ☐ Buoyant force acts through centroid of displaced volume. Center of buoyancy.

EXAMPLE 2.10

A spherical buoy has a diameter of 1.5 m, weighs 8.50 kN, and is anchored to the sea floor with a cable as is shown in Fig. E2.10*a*. Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed as illustrated. For this condition what is the tension of the cable?



■ FIGURE E2.10

Resolving vertically

$$T + W = F_B$$

$$F_B = \gamma \forall$$

$$T = F_B - W = \gamma \forall - W$$

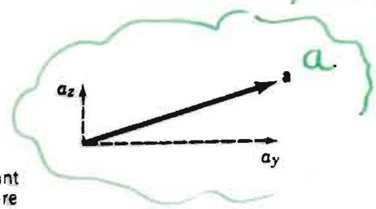
$$= 10.1 \ \mathrm{KN/m^3} \ \frac{4}{3}\pi (0.75)^3 \ \mathrm{m^3} - 8.5 \ \mathrm{KN}$$

$$T = 17.85 \ \mathrm{KN} - 8.5 \ \mathrm{KN}$$

$$\boxed{T = 9.35 \ \mathrm{KN}.}$$

LINEAR MOTION

Acceleration profile.

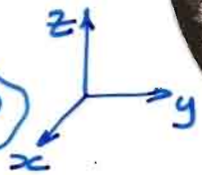


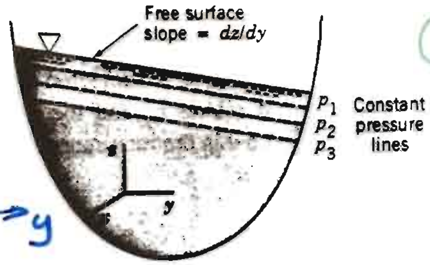
Accelerate:

$a_y \neq 0;\ a_z \neq 0$

$a_x = 0$

$\therefore\ \partial z/\partial x$ surface $= 0$





From basic relations:

$$\left.\begin{aligned} \partial p/\partial x &= -\cancel{\rho a_x}^{\,0} = 0 \\ \partial p/\partial y &= -\rho a_y \\ \partial p/\partial z &= -\rho(g + a_z) \end{aligned}\right\} \quad (1)$$

Evaluate change in pressure, dp, in x, y & z directions

$$dp = \frac{dp}{dx}dx + \frac{dp}{dy}dy + \frac{dp}{dz}dz \qquad (2)$$

Substitute (1) into (2):

$$dp = 0\,dx - \rho a_y\,dy - \rho(g + a_z)\,dz \qquad (3)$$

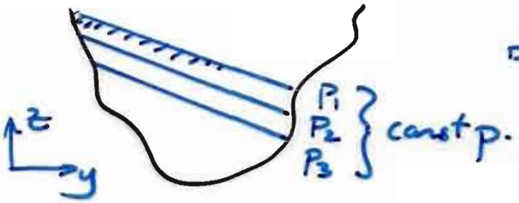
Along a line of constant pressure, $dp = 0$. Setting $dp = 0$ in (3) gives slope of line of constant pressure dz/dy, including free surface.

$$\rho a_y\,dy = -\rho(g + a_z)\,dz$$

$$\boxed{\frac{dz}{dy} = -\frac{a_y}{(g + a_z)}} \qquad (2.28)$$

Equation (2.28):

- $\frac{dz}{dy}$ is constant for constant accelerations a_y & a_z.
- Surfaces of constant pressure of inclination $dz/dy = \text{const}$.

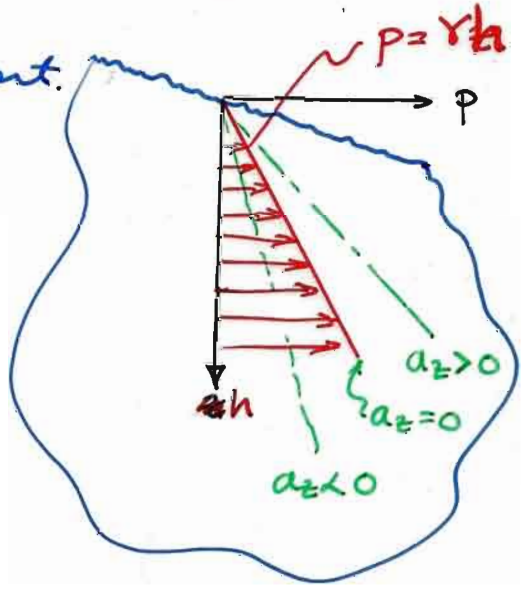


Free surface dz/dy

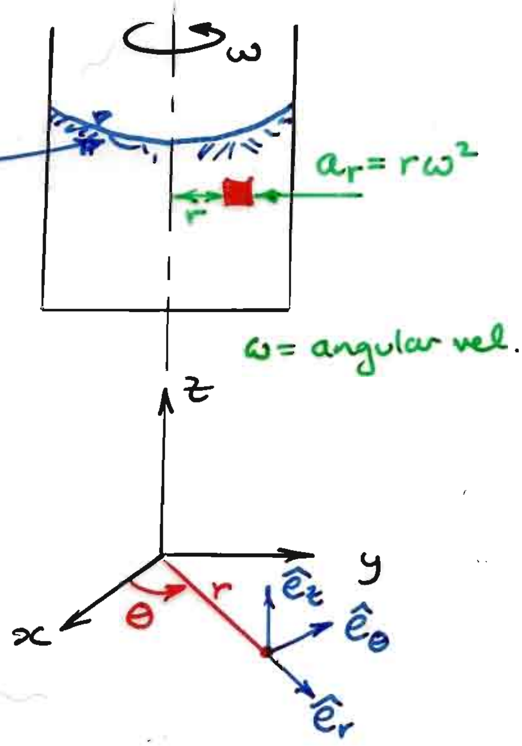
Pressure gradient down from the free surface is given by (1) as: $\frac{\partial p}{\partial z} = -\rho(g + a_z)$

If $a_z = 0$ then $\partial p/\partial z = -\gamma$ ∴ "hydrostatic"

If $a_z \neq 0$ then extra component.



RIGID BODY ROTATION



Basic relations:

$$\left.\begin{aligned} \partial p/\partial r &= \rho r \omega^2 \\ \partial p/\partial \theta &= 0 \\ \partial p/\partial z &= -\gamma \end{aligned}\right\} \quad (2.30)$$

Evaluate change in pressure, dp.

$$dp = \frac{dp}{dr}dr + \cancelto{0}{\frac{dp}{d\theta}}d\theta + \frac{dp}{dz}dz \qquad (1)$$

Setting $dp = 0$ for equi-pressures, isobars then

$$\cancelto{0}{dp} = \rho r \omega^2\, dr - \gamma\, dz \qquad (2)$$

Rearrange for

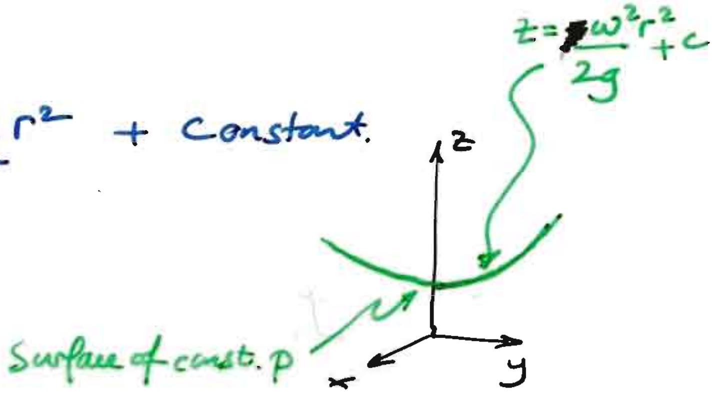
$$\frac{dz}{dr} = \frac{\rho r \omega^2}{\underbrace{\gamma}_{\rho g}} = \frac{r\omega^2}{g} \qquad (3)$$

Integrating (3).

$$\int dz = \frac{\omega^2}{g}\int r\, dr$$

$$z = \frac{\omega^2}{g}\,\frac{1}{2}r^2 + \text{Constant.}$$

gives surface of equal pressure (parabolic)



With lines of constant pressure defined by (parabolic). $z = \frac{\omega^2 r^2}{2g} + c$

Integrate equation (2).

$$dp = \rho r \omega^2 \, dr - \gamma \, dz$$

$$\int dp = \rho \omega^2 \int r \, dr - \gamma \int dz$$

$$\boxed{p = \frac{\rho \omega^2 r^2}{2} - \gamma z + \underline{Const}}$$

To solve, define p @ specified r_0, z_0 and define Const

Then resubstitute to solve p @ any r and z.

- For r = constant, p varies linearly, since $p = 0$ @ surface and sets const. magnitude.