

# **2-DIMENSIONAL FINITE ELEMENTAL MODELING OF MICROWAVE HEATING (FEM) OF A COAL SAMPLE; TO DETERMINE TEMPERATURE RISE WITH TIME AT A CERTAIN POWER DENSITY**

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## **ABSTRACT**

A two dimensional finite element model (FEM) was developed to Predict the temperature distribution in coal particles with the aid of microwave heating. However, transient temperature profile for coal particles of high dielectric loss factor is obtained when exposed to microwave radiation. The heat equation and Maxwell's equation are simultaneously solved using the finite element method to predict the temperature rise in samples at an applied power density. With FEM, a radiation boundary condition was derived and the temperature is dependent on the dielectric properties of the coal sample thereby influencing the heating patterns. The power absorbed was seen to be higher from inside to outside which signifies the heating pattern of microwave radiation. This is such as in thermal runaway seen in pre-treatment of highly dielectric loss materials.

## **1.INTRODUCTION**

FEM was used in numerical problems to solve Partial differential equation (PDE) [1]. Solving transient state problems involves interrelating the PDE into an equivalent ordinary equation, or by the differential equation elimination. In the area of coal grindability, pre-treating with Microwave energy has been discovered to be more efficient than the famous conventional heating [2, 3, 4, 5.]. However, temperature measurement has been the most critical part of this technology [6]. This is due to the different dielectric properties of mineral phases in most materials such as coal. Therefore, this report outlines the use of finite elemental model (FEM) to predict temperature rise and also visualizing the heating profile with respect to power density and time. The many benefit of microwave pretreatment of coal

at such a lower temperature as in this report, are; increased mill capacity, wear reduction, as well as increased rate of comminution. This is as a result of intragranular cracks within coal particles due to microwave heat energy [7, 9, and 10]. One thing for sure is the fact that microwave treatment is time savings and also releases little or no emission [7]. With these advantages, there is need to develop a FEM to predicting temperature rise with respect to time at various power densities ( $\text{W}/\text{m}^3$ ).

## 1.1 THEORY

If the material's thermal and dielectric properties are constant, the problem is said to be linear and Maxwell equation, can be solved irrespective of the heat equation. For this reason, both equations were solved here simultaneously using FEM.

### Geometry:

The theory behind this modeling is based on the assumption that the system is rectangular and isotropic. This is such that the coal particles are in fixed position and also in a closed system with one of the domain sides insulated. This was done to visualize the heating profile towards the insulated region.

### Heat transfer:

- Here, the main mode the power source to the coal particles by means of radiation.
- Secondly, the heat is transferred between the particles at a constant heat flux while having the initial temperature (temperature of samples before microwave treatment) set in the subdomain.

## **2.1 GOVERNING EQUATIONS**

The system can be modeled by coupling two differential equations: the heat equation and the Maxwell-Stefan's equation. These equations emphasized on the mathematical equations governing the behavior of microwave energy within a coal sample. A model of this behavior requires the use of the finite element method (FEM) which entails incorporating thermal and electromagnetic properties. The equations used here are as follows assuming homogeneous and isotropic;

## 2.2 CONSTITUTIVE RELATIONS;

This is a relationship that constitutes the required parameters. Energy balance implies the constitutive relations to be represented as;

Heat Equation

- $V\rho C_p \partial T / \partial t = V(\nabla \cdot k \nabla T) + P$ ..... (1)

Where V = product volume, P = power generated by microwave absorption, ρ = material density, Cp = specific heat capacity, K = thermal conductivity and T = temperature and ∇ = Laplace or vector differential operator.

- Whereas for a 2-D,  $V\rho C_p \partial T / \partial t = V \partial K / \partial x + VK \partial^2 T / \partial x^2 + P$ .....(2)

Maxwell’s equation is used as an electric wave equation which describes space and time for electric field

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \mathbf{E} = 0 \dots\dots\dots (3)$$

Where E = ELECTRIC FIELD

With the aid of frequency response analysis from the structural mechanics modules, the Maxwell’s equation can be used to incorporate frequency (2450Hz), and the relative permittivity (8.854e-12).

2.3 CONSERVATION EQUATION;

- $\partial V / \partial t + (V \cdot \nabla) V = 1/\rho \nabla \cdot \sigma + g$  such that  $\nabla = (\partial / \partial x_1, \partial x_2, \dots, \partial / \partial x_n)$  .....(4)

Where V = volume, ρ = density, ∇ = vector differential operator, σ = stress, t = time and g = acceleration due to gravity

2.4 BOUNDARY CONDITIONS;

This system, requires the heat transfer equation;  $\partial u / \partial t = k (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2)$ ..... (5)

Where u = v (t, x, y) = temperature as a function of time and space,  
For x = 0 -k ∂T/∂x = 0 t > 0

INITIAL CONDITIONS;

With values at the lower boundary of domain, the initial condition here will be represented by the equation:  $\mu (t_0) = 0$ ..... (6)

This implies that for the initial temperature to coincide with the ambient temperature, T (x, y, 0) = T<sub>A</sub>, dt/dx = 0 at x=0. Where T<sub>A</sub> = the ambient temperature.

**3.1 SOLVING WITH FEM**

The heat and Maxwell equations were simultaneously solved using the FEMLAB (COMSOL Multi physics) to predict and visualize the

temperature rise in coal sample of defined parameters needed for both equations. The power absorbed and temperature distributions are obtained by simultaneously solving Maxwell's equations in the frequency domain with the transient heat equation. Dielectric properties are assumed to be temperature dependent and the heating of coal particles is dependent on the dielectric loss of the mineral phase which in this case is considered even at 3.4.

Heat equation:  $\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + P$ ..... (1)

Boundary condition here is:  $-n \cdot k \nabla T = h (T - T_\infty)$ ... (2) With

Initial condition as  $T(t_0) = 298K$

Maxwell equation:  $(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) E = 0$ ..... (3)

Where  $\nabla$  = the vector differential operator

3.2 Table 1: Data input for the simulation

Material properties	Values
Power density	1500W
Heat source	937.5 W/m <sup>3</sup>
Density	625J/Kg. K
Thermal conductivity	1440.4 W/(m. K)
Relative permittivity	8.854e-12
Frequency	2.450e3 Hz
Dielectric constant	3.4
Heat transfer Coefficient	103.022 W/(m <sup>2</sup> .K)
Emissivity	0.95
Speed of light	3x10 <sup>8</sup> m/s <sup>2</sup>

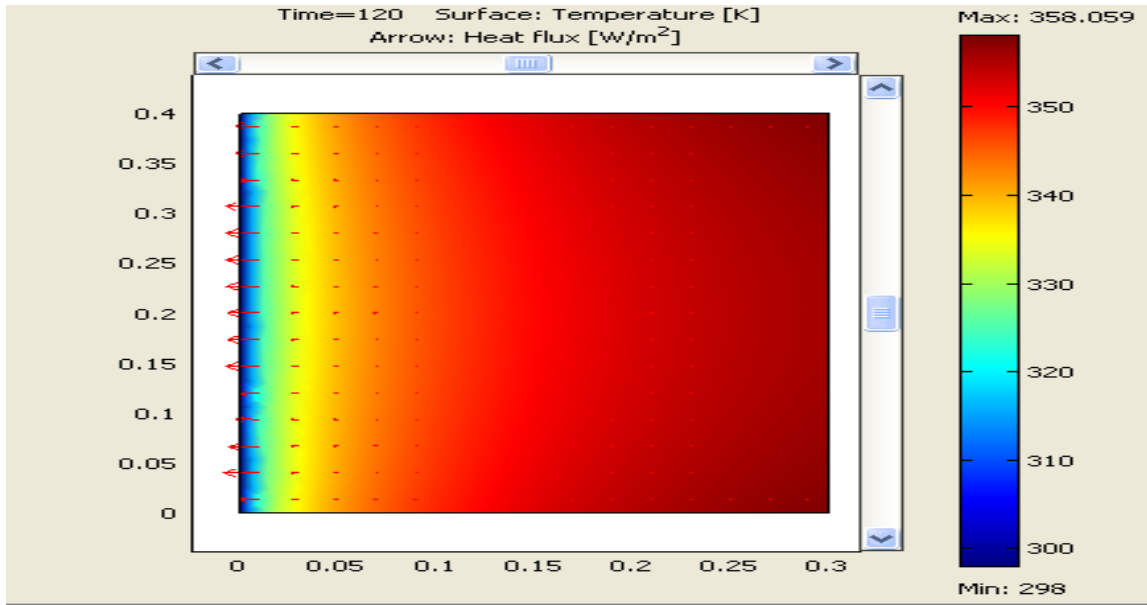


Fig 1: Heating profile of coal particles at time 120 second

As shown in figure 1 above, microwave heating is from inside to outside. This is a basic theory of microwave heating systems; though in this model, the left hand side (0-0.4m) of the geometry is actually insulated to prevent any form heat loss. The material was considered to have a uniform dielectric loss factor of 0.44 and hence temperature distribution was expected to increase from inside to outside.

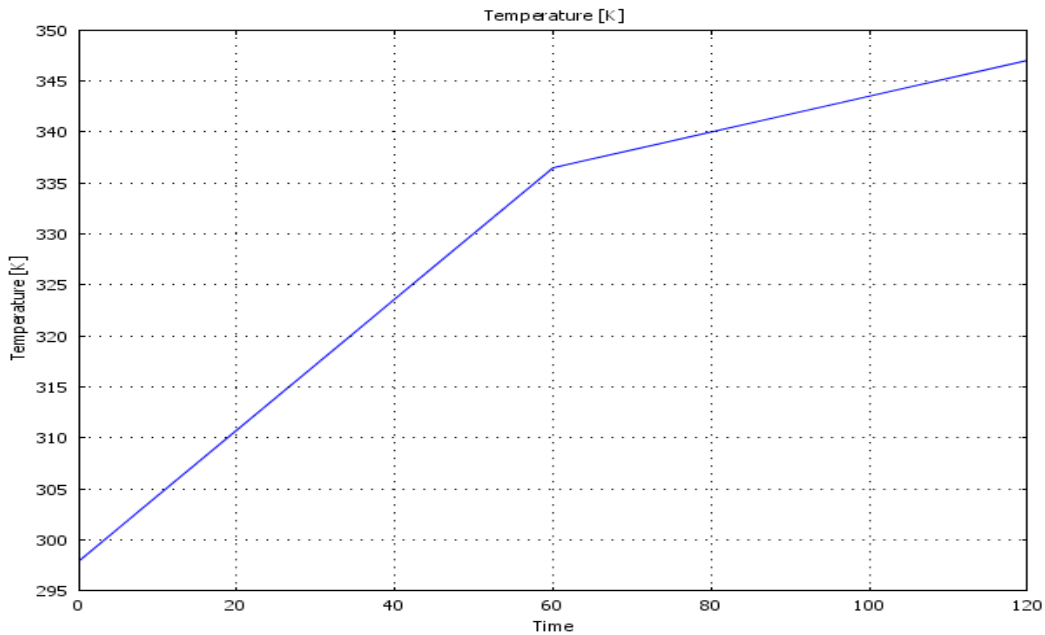


Fig. 2: Cross section of temperature time profile

The temperature in figure 2 increased steadily from room temperature (298K) assigned as the initial temperature at time zero second up to about 358K at 60 seconds. This is referred to as thermal runaway. From 60 seconds the temperature increase begins to flatten as it approaches 120 seconds-which was the maximum time input into the simulation. This actually illustrates the effect of power density of 1500W at 2450 Hz for 120 seconds as solved by FEM.

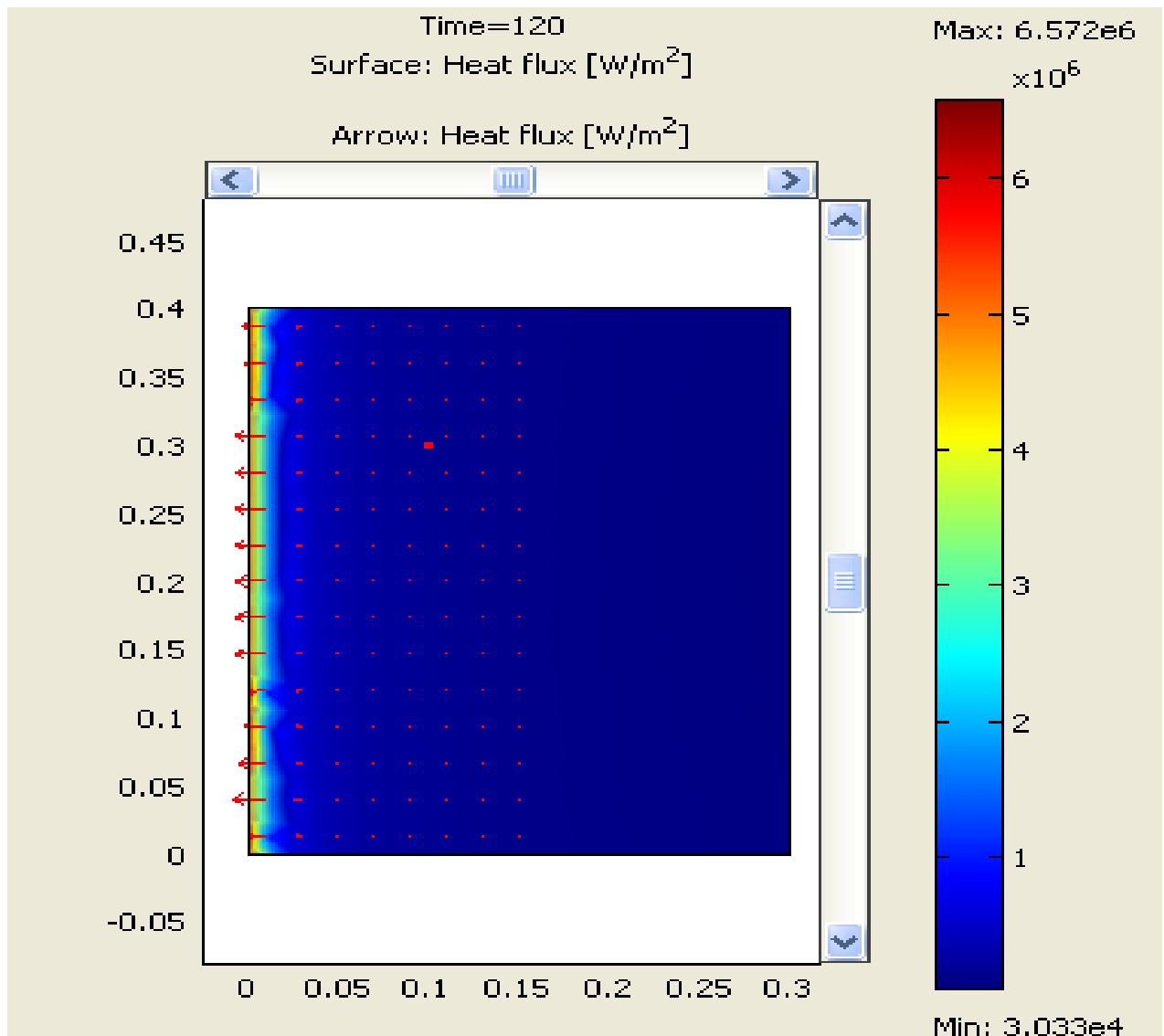


Fig. 3: Heat flux (w/m2) at time 0-120 seconds

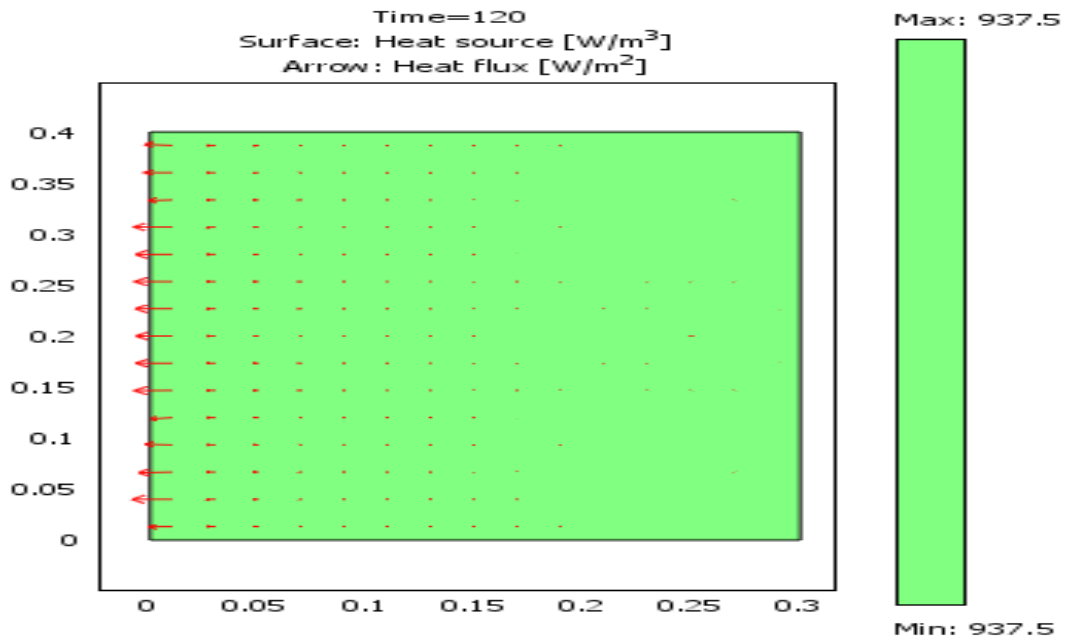


Fig. 4: Heat source distribution within the entire system

It is important in microwave heating, to ensure that the heat source is even within the entire system in order to verify that every surface was exposed the same amount of microwave radiation energy [8]. The use of FEM software was appropriate in this justification

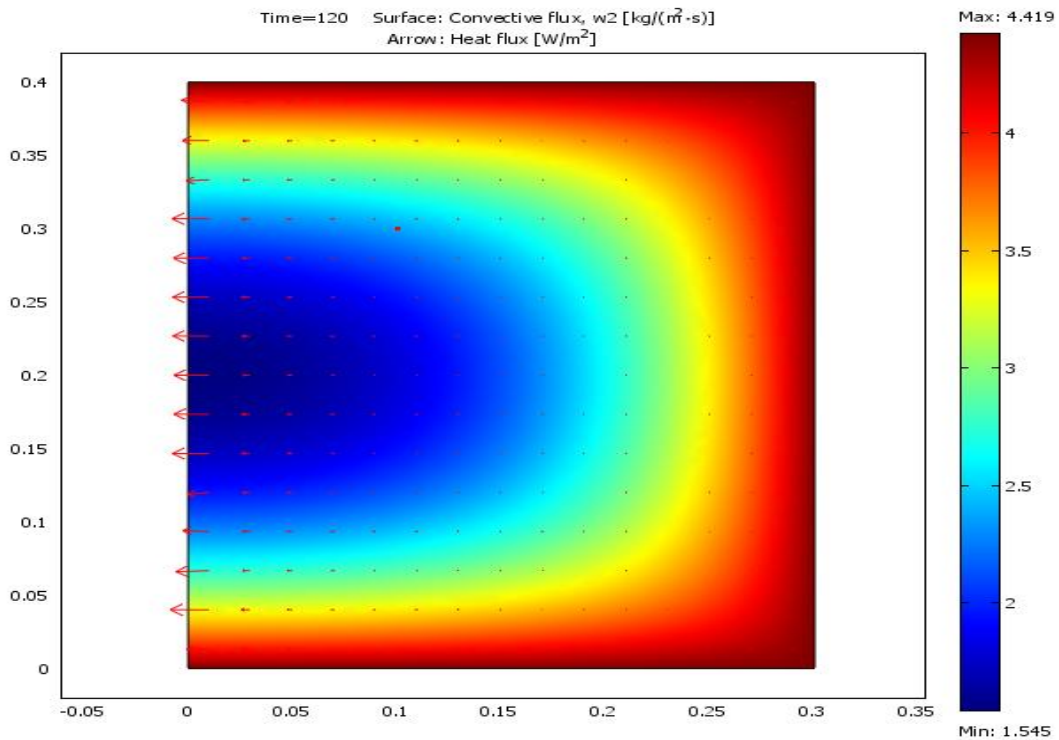


Fig. 5: The convective flux within the system

The convective flux becomes minimal at as it gets to the insulated region; though, the edges of the system still have higher convective flux likely because the heat gets concentrated when microwave becomes concentrated towards the edges. In this simulation, the insulated edge is assumed to disallow heat loss but guess the time stipulated in this model was not enough to heat up the entire system.

## **2.2 VALIDATION**

Validating a model is important as it helps to justify the output of solutions; most especially when compared to previous work done similar to the objected work like in this report. The verification includes two basic steps: (1) testing of computer program; (2) experimental verification. Once again, the equations used to describe these processes, are the Heat and Maxwell's equations respectively and were combined to model the temperature profile using COMSOL 3.2. Hence, each of the equations were evaluated in the model software and compared to previous solutions.

The heat and Maxwell's equation has been solved using FEM to determine temperature and moisture distribution in food materials during microwave heating [5]. Here, the heating profile corresponds with simulated report in this paper.

As reported by Zhou L. et. al. [5], the dielectric loss was 58 being that water is more responsive to microwave and therefore should have higher dielectric loss compared with most materials. This will definitely ensure higher temperature obtained when compared with the model here for the same time period. It was also reported that the power density was 2.89 W/cm<sup>2</sup> for slabs and 3.48W/cm<sup>2</sup> for cylinder. However, considering the power density (W/cm<sup>2</sup>) in this report to be 1.875



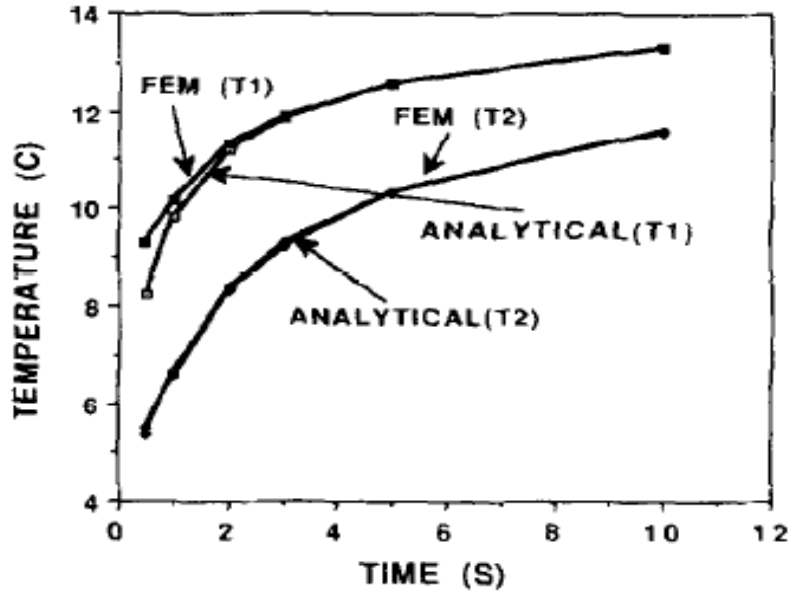


Fig. 6: Comparison between FEM and analytical solution for a semi-infinite 2-Dimensional slab (T1 is the temperature at the first node location; T2 is the temperature at the second node location) [5].

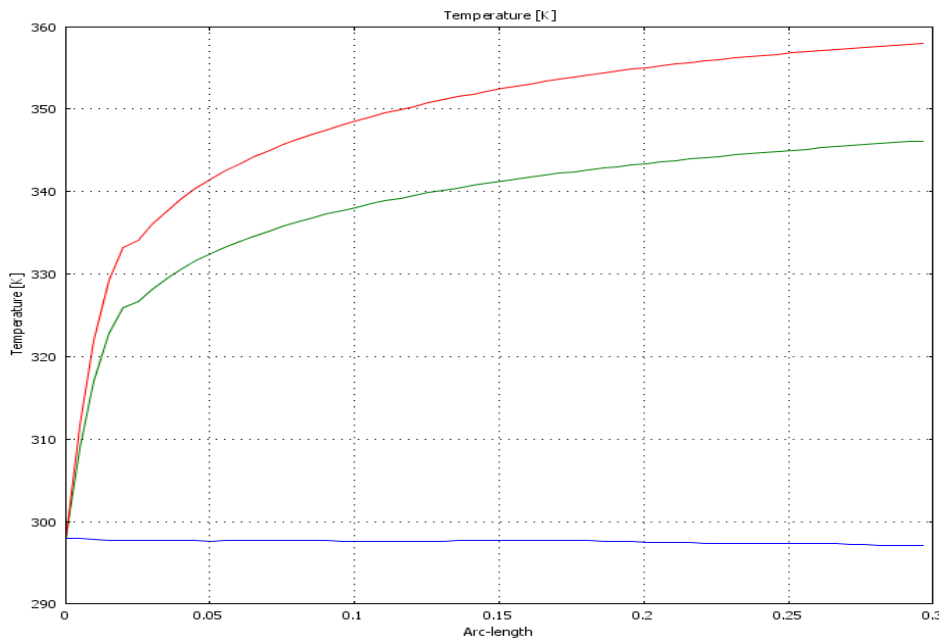


Fig. 7: Temperature profile compared with figure 6

Comparing figure 6 to figure 2 and 7 while considering power densities and time of microwave heating, the temperature is in the same trend and differs with about 10C at each corresponding time. As mentioned earlier, the as reported results by Zhou L., et al. higher power density and dielectric loss were considered compared to the results reported in this paper. Also, Marland S., et al. [9] reported an average heating

rate 0.5C/s to 1C/s for coal readily heated in microwave at frequency of 2.45GHz and 650W power density whereas, the results obtained in this paper has an average of 1.85C/s considering the fact that the power used in the simulation here was 1500W at 2.45GHz.

### 2.3 PARAMETRIC STUDY

The behavior of these systems could be greatly affected by either of these parameters; power density, frequency, heat flux and dielectric loss. Parameters likely to affect the behavior of this system are power density, dielectric loss properties and time.

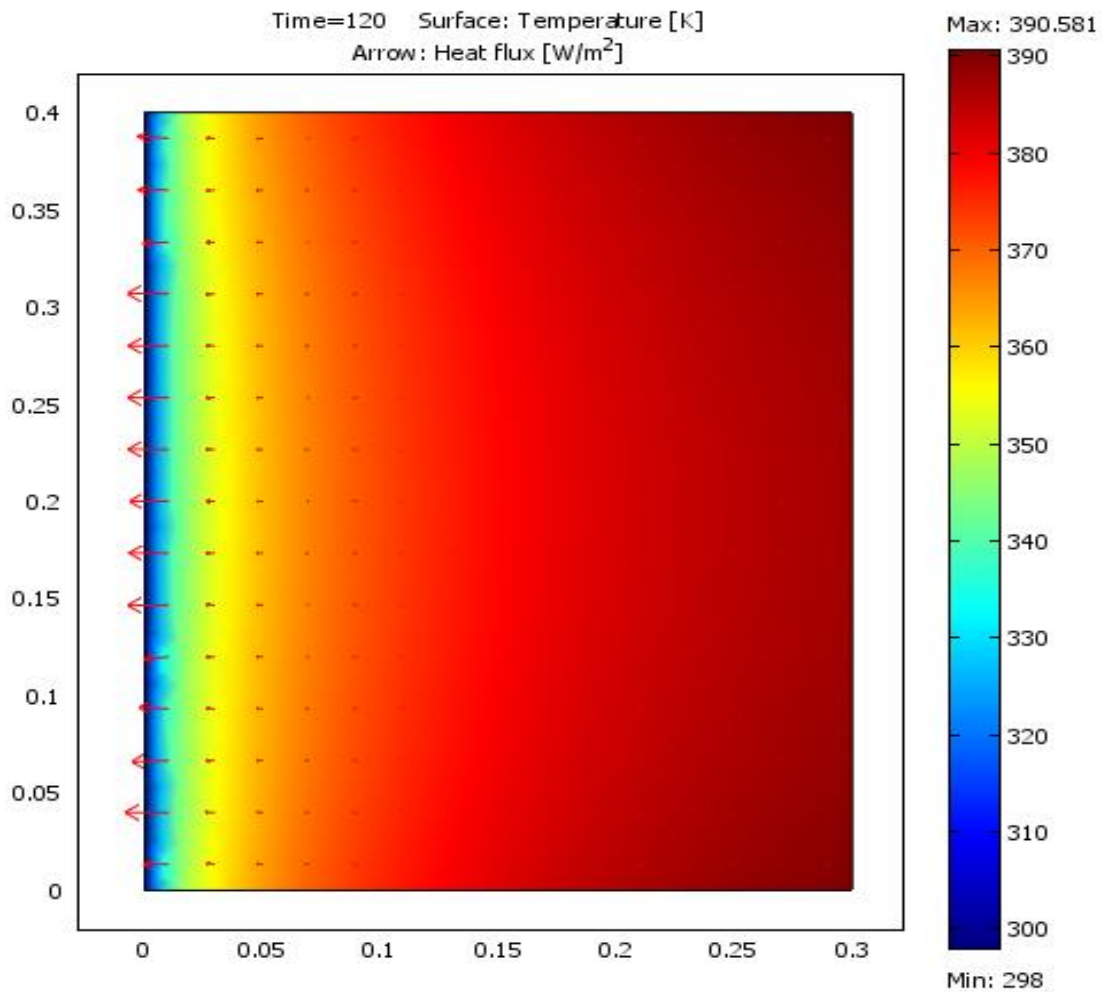


Fig. 7: Parametric study of temperature distribution at higher heat flux and frequency of 28GHz, power 2000W, TIME 120S.

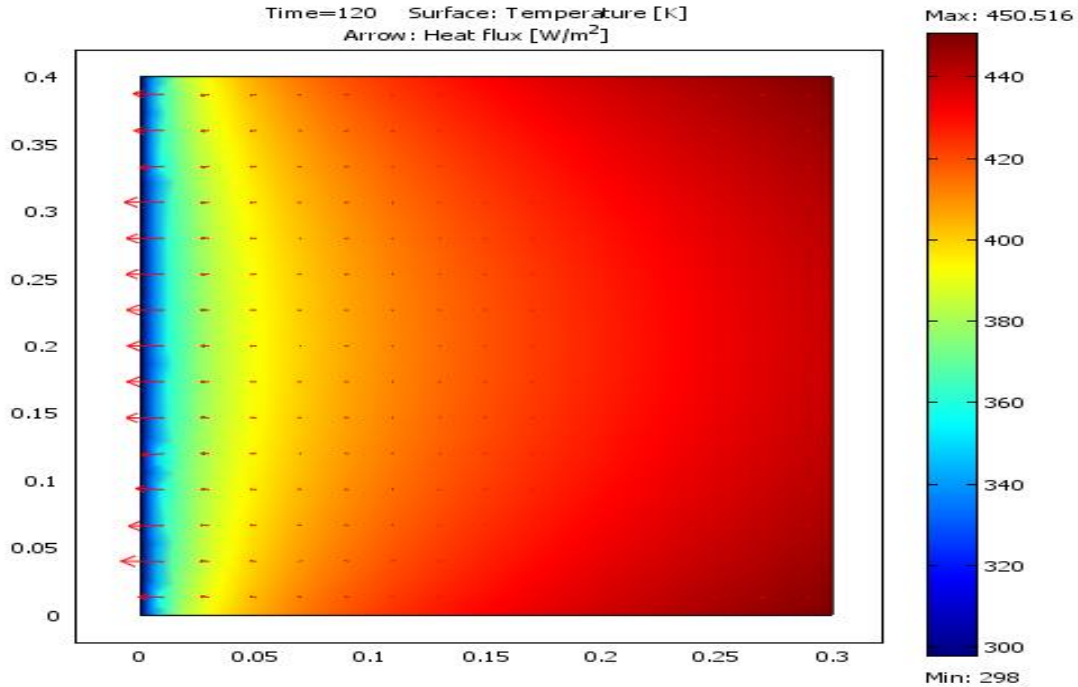


Fig. 8: Parametric study of thermal conductivity of 200 W/ (m.K), time 120s, power 1500W

## **2.4 CONCLUSION**

The heating rate predicted and visualized has proved FEM to be a reliable tool in modeling engineering systems. It is quite clear here that at very short time exposure with relevant parameters, we can see the nature of temperature increase with time. Most importantly, at a lower temperature increase like in the results presented here, coal particles are known to develop cracks within their inter-granular structure as a result of thermal stress [9]. This enhances easy grinding; thereby saving energy cost and as well perfect temperature for coal cleaning without altering its calorific values.

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